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Young

ELEMENTS
OF
PLANE AND SPHERICAL
TRIGONOMETRY,
WITH
ITS APPLICATIONS TO THE PRINCIPLES
OF
NAVIGATION AND NAUTICAL ASTRONOMY;
WITH THE
LOGARITHMIC AND TRIGONOMETRICAL TABLES.

BY J. R. YOUNG,
AUTHOR OF "THE ELEMENTS OF ANALYTICAL GEOMETRY," "ELEMENTS OF THE DIFFERENTIAL AND INTEGRAL CALCULUS," &c.

TO WHICH ARE ADDED
SOME ORIGINAL RESEARCHES IN
SPHERICAL GEOMETRY;
BY T. S. DAVIES, F.R.S.E., F.R.A.S., &c.

REVISED AND CORRECTED BY
J. D. WILLIAMS,
AUTHOR OF "KEY TO HUTTON'S MATHEMATICS," &c.

A NEW EDITION.

PHILADELPHIA:
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THE AMERICAN EDITION.

The extensive circulation and rapid sale which attended the republication, in this country, of Mr. Young's Treatise on Algebra, revised and corrected by Mr. Ward, the Elements of Geometry, by M. Floy, Jr., Analytical Geometry, by the Editor of the present Treatise, and also the Elements of the Differential and Integral Calculus, by Michael O'Shannessy, all of New-York,—together with the increasing demand for all his other Treatises upon the elements of abstract science, have induced me to attempt, by the publication of the present work, to place, as it were, the key-stone of the arch of Elementary Mathematics.

No further merit is claimed, in this edition, than that of a careful correction of all the errors occurring in the original and republished typography; as also of having altered the first part of the valuable Trigonometrical Tables connected with the work, so as to correspond with the improved plan, which was not adopted by the author, as will be seen by his preface, until after some few sheets were stereotyped, and consequently past all recall.

A consideration of the rapidly advancing state of analytical science amongst us, must be soul-reviving to every lover of his country:—And that her sons may continue daily to increase in a scientific knowledge of the mysterious principles hidden so long in the vast book of nature, is the fervent aspiration of one of Science's most humble votaries.

JOHN D. WILLIAMS.

New-York, 1833.

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PREFACE.

It is the design of this treatise to establish the theory of Plane and Spherical Trigonometry analytically, and to present that theory, together with some of its most interesting and valuable applications, in a form fitted for elementary instruction.

Of late years several analytical works on Trigonometry have been published in this country; but, as they are confined almost entirely to the theory of the subject, it may be questioned whether, to many young students, they prove much else than so many collections of mere algebraical exercises. Yet a book upon so practical a subject as Trigonometry, ought undoubtedly to be something more than this, and ought not to be considered as complete when the various calculations which the science involves, and which its name implies, are wholly omitted.

The symbolical expression of a practical rule, in algebraic language, will often, to the young student, but indistinctly point out the numerical operation. Those much occupied in mathematical instruction, know full well that a learner may readily yield his assent to every step of an algebraic process, be fully satisfied as to the truth of the result to which it leads, may even clearly see a valuable truth involved in it, and may yet be very far from perceiving how to turn it to account in any case of actual calculation. Indeed, algebraical formulas, transform them as we will, cannot always be made to indicate the best mode of arithmetical arrangement; and yet much, as regards facility of operation, depends upon this arrangement in many parts of practical mathematics, but especially in Trigonometry.

In the present volume, therefore, both the theory and the practice of the science have been introduced, every particular formula being illustrated by examples of the numerical calculation, arranged in the proper form. This plan of combining practice with theory, in works like the present, was always adopted by the earlier English writers, and it is to be regretted that recent authors have, in their admiration of foreign methods, departed so widely, in this respect, from the example of their predecessors, dwelling so much as they do upon the symbols, and so little upon the things signified.

In addition to the practical illustration of formulas, a distinct part of the work is devoted to the principles of Navigation and Nautical Astronomy, in which will be found a very short and convenient method of clearing the Lunar Distance, for the purpose of ascertaining the Longitude at Sea. This method is probably new, although, as the analytical expression for it occurs during the investigation of the well-known formula of *Borda*, it is equally probable that it has been noticed before.

The supplement appended to the treatise is from the pen of my valued and accomplished friend, *T. S. Davies*, *Esq.* Fellow of the Royal Society of Edinburgh, and of the Royal Astronomical Society of London. It will be found to contain several new and interesting researches, which cannot fail to prove acceptable both to the inquiring student and to the more advanced analyst.

J. R. YOUNG.

January 1, 1833.

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PART I.

ELEMENTS OF PLANE TRIGONOMETRY.

CHAPTER I.

EXPLANATION OF THE TRIGONOMETRICAL LINES.

(*Article* 1.) Plane Trigonometry is that branch of pure mathematics of which the primary object is to determine the several parts of a plane triangle from having certain other dependent parts given.

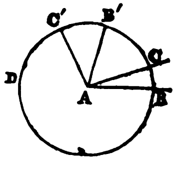
By the *parts* of a plane triangle we mean these six things, viz. the three sides and the three angles, and if any three of these six be given, provided only that a side be among them, the other three may always be determined either by geometrical construction, as shown in the Elements of Geometry, or by numerical computation, as will be seen hereafter.

From the foregoing definition it appears that quantities of two kinds, perfectly distinct from each other and admitting of no comparison, are concerned in Trigonometry, viz. *straight lines* and *angles*.

By means of certain happy contrivances, however, the whole business of trigonometry, and, indeed, the general theory of angular magnitude is conducted by help of linear quantities only; the angles themselves not entering into the computations, but certain straight lines dependent upon them and serving as indexes to them.

(2.) Before we explain the nature of these trigonometrical lines, it will be necessary first to show how angular magnitude is measured.

In order to this we must remark that when straight lines are submitted to calculation, all those which are concerned in the same inquiry must be measured in reference to one common standard of measure, called the *linear unit;* the choice of which unit is, however, arbitrary. Thus if we estimate any one of the lines concerned in any inquiry in feet, all must be estimated in feet, and the linear unit adopted will be *a foot*, which is represented by the *numeral unit* 1. Also if one of the lines is measured in yards all must be measured in yards, the linear unit being then a yard, which, as before, is represented in the calculation by the numeral unit 1. As far as the accurate representation of the lines are concerned, it is obviously a matter of indifference what length be assumed for the linear unit, for the length of any line will always be expressed numerically by that number which denotes the units it contains, but, for the purpose of facilitating computation, some scales of measure are often preferable to others.



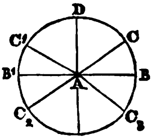
(3.) Let now BAC be any angle concerned in any inquiry; then having chosen the linear unit AB, describe the circumference BCD about the centre A. The arc BC may be taken for the measure or representative of the angular magnitude CAB: for let there be any other angle B′ AC′ about the same centre A; then we know, by Geometry, that the angle BAC is to the angle B′AC′ as the intercepted arc BC to the intercepted arc

B′C′, (Geometry, p. 103); hence, as the intercepted arcs always vary as the angles, the former may, obviously, be taken to represent the latter.

It is usual to consider the circumference of every circle to consist of 360 equal parts, called *degrees* of that circle; an arc consisting of any number of these, 24 for instance, is called an arc of 24 degrees, and represented for brevity thus, 24°; moreover each degree is supposed to consist of 60 equal parts, called *minutes*, and each minute of 60 equal parts called *seconds*. To express any number of minutes, we mark one accent over the number, and to express seconds we mark two; thus, 24° 16′ 26″, is 24 degrees 16 minutes 26 seconds. What we say of circular arcs applies equally to the angles which they measure, so that we call that an angle of 20° whose sides include an arc of 20° or the eighteenth part of a whole circumference.

Let us now speak of the trigonometrical lines before adverted to, and which are introduced for the purpose of reducing the entire theory of angular magnitude to the investigation of linear quantities only; we must, first, however, mention one or two further particulars respecting the arcs to which these lines refer.

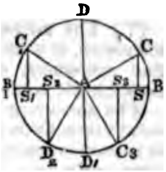
(4.) The arc CD which must be *added* to BC to make up a quadrant, or 90° is called the *complement* of the arc BC; and every arc will have a complement, even those which are themselves greater than 90°, provided we consider the arcs measured in the direction BCD, &c. as positive, and those measured in the opposite direction as negative; thus the complement CD of the arc BC commences at C where BC terminates, and may be considered as generated by the motion of C, the extremity of the radius AC, in the direction CD; but the complement C_1D of the arc BC_1, commencing in like manner at the extremity C_1 of the proposed arc, must be generated by the motion of C_1 in the opposite direction, and the angular magnitude BAC_1, will here be diminished by the motion of AC_1, in generating the complement; the complement of BAC_1, or of the arc BC_1, is, therefore, with propriety considered as negative. Calling the arc BC, or BC_1, ω, the complement will be 90° — ω; thus the complement of 24° 16′ 4″ is 65° 43′ 56″, and the complement of 120° 36′ 10″ is —30° 36′ 10″.



The arc CB_1, which must be added to BC to make up a semicircle, or 180°, is called the *supplement* of the arc BC. If the arc is greater than 180°, as the arc BC_2, its supplement C_2 B_1 measured in the reverse direction is negative. The expression for the supplement of any arc or angle ω is, therefore, 180° — ω; thus the supplement of 110° 30′ 20″ is 69° 29′ 40″, and the supplement of 200° 25′ is — 20° 25′.

In the same manner as the complementary and supplementary arcs are considered as positive or negative, according to the direction in which they are measured, so are the arcs themselves positive or negative; thus, still taking B for the commencement of the arcs, as BC is positive BC_3 will be negative. In the doctrine of triangles we consider only positive angles or arcs, and the magnitudes of these are comprised between $\omega = 0$ and $\omega = 180°$; but in the general theory of angular quantity, we consider both positive and negative angles, according as they are situated above or below the fixed line AB from which they are measured, as the angles CAB, C_3AB; moreover, an angle may consist of any number of degrees whatever, thus if the revolving line AC set out from the fixed line AB and make n revolutions, and a part the angular magnitude generated is measured by n times 360°, plus the degrees in the additional part.

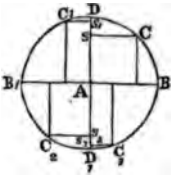
Of the Sine.



(5.) The sine of an arc or of the angle which it **measures, is the** perpendicular, from one extremity of the arc, upon the diameter passing through the other extremity: thus CS is the sine of the arc BC; $C_1 S_1$ is the sine of the arc BC_1; $C_2 S_2$ is the sine of the arc BC_2; $C_3 S_3$ the sine of the arc BC_3, &c. If the proposed arc were a quadrant, or 90°, the sine DA would be equal to the radius, and, therefore, its numerical value would be 1; the same would be the case if the arc consisted of three quadrants, or 270°, or indeed of any odd number of quadrants; for all other arcs the numerical value of the sine will be a proper fraction or decimal. These, it must be observed, are the trigonometrical values of the sines, which are estimated according to the scale $AB = 1$; but it should be remarked that when we know the value of the sine of an arc agreeably to this scale, its value agreeably to any other scale is at once obtained by proportion; thus let R be any value assumed for the radius, and let us write the sine corresponding in capitals, SINE; then 1 : sine :: R : SINE = R × sine, so that the sine of an arc, corresponding to any assumed radius, is found by multiplying its trigonometrical sine by that radius; and, on the contrary, the sine according to any value of the radius being known, the trigonometrical sine is found by dividing it by that radius; the number, in fact, which expresses the trigonometrical sine being the ratio of the geometrical line itself to the radius, whatever this may be. What we have said of the sine will be easily seen to apply to the other trigonometrical lines. As with the arcs so with the sines; those which lie in opposite directions take opposite signs, those above the fixed line B_1 B being regarded as positive, and those below as negative; so that the sines in the first and second quadrants are positive, those in the third and fourth negative, while in the fifth and sixth they are again positive, and so on.

Every arc or angle has the same sine as its supplement; thus if $B_1 C_1$ is equal to BC it is obvious that BC_1 will be the supplement of BC, and the sine CS of the latter must be equal to the sine $C_1 S_1$ of the former.

Of the Cosine.



(6.) The cosine of an arc or angle is the sine of its complement: **thus** the cosine of the arc BC is the line Cs, which is, obviously, the sine of the arc DC, the complement of BC. As the several sines are arranged on opposite sides of the diameter B_1 B, so the cosines are arranged on opposite sides of the diameter DD_1; those on the right of DD_1 being regarded as positive, and those opposite as negative; hence in the first quadrant, the cosines are positive, in the second negative, in the third negative, in the fourth positive, and so on; the cosine of an arc is equal to the cosine of its supplement, but has a different sign.

When the arc is 0 the sine is 0, but the cosine BA is 1; when the arc is 90, the sine DA is 1, but the cosine is 0; when the arc is 180° the sine is 0, but the cosine is $B_1 A = -1$; when the arc is 270° the sine $D_1 A$ is -1, but the cosine is 0; and when the arc is 360° the sine is 0, and the cosine 1, as at first, and so on.

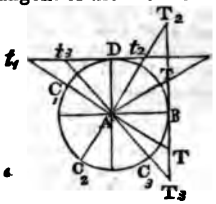
It is plain that the cosine of an arc is always equal to that part of the radius which is intercepted between the sine of that arc and the centre.

Thus referring to the figure in (5) AS is equal to the cosine of BC, and AS_2 to the cosine of BC_1C_2, or of BC_3C_2.

Of the Tangent, Cotangent, Secant, and Cosecant.

(7.) The tangent of an arc, and, therefore, of the angle which it measures, is a line drawn from one extremity of the arc, touching it at that extremity, and terminating in the diameter produced, drawn through the other extremity: thus BT is the tangent of the arc BC.

The cotangent is the tangent of the complement: thus D*t* is the cotangent of the arc BC. It is easy to trace the changes which these two lines undergo as the arc BC increases from 0, for which value the tangent is obviously 0, and the cotangent infinite. Observing the same rules here as for the sine and cosine, we see that in the first quadrant the tangent and cotangent are both positive, in the second the tangent BT_1 and cotangent Dt_1 are both negative; in the third the tangent BT_2 and cotangent Dt_2 are both positive; and in the fourth the tangent BT_3 and cotangent Dt_3 are both negative, and so on; but as we shall soon see, the signs of the tangent and cotangent may always be at once inferred from those of the sine and cosine.



The secant of an arc is that portion of the prolonged diameter limiting the tangent, which is included between the centre and tangent; and the cosecant is the secant of the complement. Thus in the last figure AT is the secant of the arc BC, and A*t* the cosecant.

In the four trigonometrical lines, sine, cosine, tangent and cotangent, we have seen that each is posited in one or other of two directly opposite directions, and that, therefore, one or other of the opposite signs + and —, prefixed to the numerical value of any such line, served to point out the proper direction for any particular value of the arc or angle. But as the secant and cosecant continually vary in direction, as well as in magnitude with the arc or angle, the geometrical position of either of these lines does not so clearly indicate to us the sign with which it should be represented. The proper sign, however, is always readily ascertained from knowing the signs of the sine and cosine, for upon these two lines all the others depend, as we shall shortly show.

(8.) Besides the six trigonometrical lines now defined there are three others, sometimes, although but seldom, employed; these are the *versed sine* or *sagitta*, the *coversed sine*, and the *suversed sine*. The versed sine of an arc BC (see fig. to art. 5) is the line BS between the commencement of the arc and the sine; it is always equal to the radius minus, the cosine, and, therefore, is always positive. The coversed sine is the versed sine of the complement, so that the coversed sine of BC is D*s* (see fig. to art. 6;) also the suversed sine is the versed sine of the supplement. As the versed sine of any arc must be positive, it follows that the coversed sine and suversed sine must always be positive.

(9.) The following is the way in which the trigonometrical lines, connected with any arc or angle ω, are expressed in computation;

The sine of ω is expressed thus,	sin. ω
cosine of ω . .	cos. ω
tangent of ω . .	tan. ω
cotangent of ω . .	cot. ω
secant of ω . .	sec. ω
cosecant of ω . .	cos. ω

versed sine of ω . . vers. ω
coversed sine of ω . . covers. ω
suversed sine of ω . . suvers. ω

From knowing the numerical value of any one of these lines, those of all the others may be obtained; thus, let the sine be given, then since the radius sine and cosine always form a right-angle triangle, of which the hypotenuse is the radius $= 1$, (see the fig. in art. 5,) we have $\cos.\ \omega = \sqrt{1 - \sin.^2 \omega}$. Again, since the triangle formed by the radius, sine, and cosine, is always similar to that formed by the secant, tangent, and radius, and to that formed by the cosecant, radius, and cotangent, as the student will at once see by sketching these lines for any arc, it follows, from the proportionality of the sides of similar triangles, that

$$\tan.\ \omega = \frac{\sin.\ \omega}{\cos.\ \omega}, \quad \cot.\ \omega = \frac{\cos.\ \omega}{\sin.\ \omega} = \frac{1}{\tan.\ \omega}$$

$$\sec.\ \omega = \frac{1}{\cos.\ \omega}, \operatorname{cosec.}\ \omega = \frac{1}{\sin.\ \omega} = \sqrt{1 + \cot.^2 \omega};$$

and, from these expressions, we at once see that the signs of the several lines, as well as their numerical values, are deducible from those of the sine and cosine.

Now the numerical expression for sin. ω, for all values of ω, from $\omega = 0$ to $\omega = 90°$, (between which limits every possible value is comprised) are actually computed by methods to be hereafter explained, and thence the values of the other trigonometrical lines are deduced. These values are then arranged as in table III, at the end, and form a *table of natural sines, cosines, &c.* By help of such a table we may readily find the values of the same lines, computed to any other radius R; for as observed at (5) we shall merely have to multiply the tabular value by R. Writing; therefore, for distinction sake, the words sin., cos., &c. in capitals, when the value of the radius is other than unity, the foregoing equations are the same as $\frac{\text{TAN. }\omega}{\text{R}} = \frac{\text{SIN. }\omega}{\text{COS. }\omega}, \ \frac{\text{COT. }\omega}{\text{R}} = \frac{\text{R}}{\text{TAN. }\omega}$

$$\frac{\text{SEC. }\omega}{\text{R}} = \frac{\text{R}}{\text{COS. }\omega}, \ \frac{\text{COSEC. }\omega}{\text{R}} = \frac{\text{R}}{\text{SIN. }\omega};$$

and thus by substituting in any trigonometrical formula $\frac{\text{SIN. }\omega}{\text{R}}, \frac{\text{COS. }\omega}{\text{R}}$ &c. for sin. ω, cos. ω, &c. the formula will become generalized so as to hold good for any value of the radius whatever.

(10.) It is obvious that when any trigonometrical formula is thus generalized every term in it will be the same abstract number as in the original formula; whatever powers or roots of the lines enter the formula they will always be divided by the same powers or roots of the radius R. The denominators will all be removed by multiplying each term by the highest power of R which enters, and the result will necessarily be a homogeneous expression; that is, every term will have the same dimensions, or will involve as factors the same number of lines. Hence, in order to generalize any trigonometrical formula, or to render it independent of any particular value of R, it will be necessary merely to introduce into the several terms such powers of R as will render them all of the same dimension. For example, the following formula, viz.

$$\sin.\ (\text{A} + \text{B}) = \sin.\ \text{A} \cos.\ \text{B} + \sin.\ \text{B} \cos.\ \text{A};$$

in which the term on the left is of one dimension, and the terms on the right are each of two dimensions, will become homogeneous by introducing the factor R into the left hand member, so that when this is the value of the radius instead of unity, the formula will be

$$R \sin. (A+B) = \sin A \cos. B + \sin. B \cos. A;$$

each term being the product of two lines.

In like manner the formula $\cos. 4 A = 8 \cos.^4 A - 8 \cos.^2 A + 1$, becomes when the radius is R instead of unity

$R^3 \cos. 4 A = 8 \cos.^4 A - 8 R^2 \cos.^2 A + R^4$; the powers of R being introduced so as to render each term of four dimensions.

From the preceding definitions and remarks the following simple properties are immediately deducible, viz.

1. The sine of an arc is equal to half the chord of twice that arc.

2. The chord of 60° being equal to the radius (Geom. p. 122), therefore, the sine of 30°, or the cosine of 60°, is equal to half the radius.

3. Hence, from the expression for the secant at the top of the preceding page, the secant of 60° is equal to the diameter of the circle.

4. The tangent of 45° is equal to the cotangent, and, therefore, to the radius, (see fig. to art. 7.)

(11.) We shall terminate this introductory chapter with a table exhibiting the correlative values of the trigonometrical lines, situated in different quadrants; it is readily constructed from the values of the sine and cosine, by help of the relations in (9), bearing in mind that an arc and its supplement have the same sine.

Table of the Correlative Values of the Trigonometrical Lines.

arc.	sin.	cos.	tan.	cot.	sec.	cosec.
0°	0	1	0	∞	1	∞
ω	+ sin. ω	+ cos. ω	+ tan. ω	+ cot. ω	+ sec. ω	+ cosec. ω
90°	1	0	∞	0	∞	1
90° + ω	+ cos. ω	− sin. ω	− cot. ω	− tan. ω	− cosec. ω	+ sec. ω
180°	0	− 1	0	− ∞	− 1	− ∞
180° + ω	− sin. ω	− cos. ω	+ tan. ω	+ cot. ω	− sec. ω	− cosec. ω
270°	− 1	0	− ∞	0	− ∞	− 1
270° + ω	− cos. ω	+ sin. ω	− cot. ω	− tan. ω	+ cosec. ω	− sec. ω
360°	0	1	0	∞	1	∞

This last line is the same as the first; and any line will, obviously, remain unaltered if we add to the corresponding arc a whole circumference or any number of circumferences. If we take ω negatively, we may extend the table as follows:

− ω	− sin. ω	+ cos. ω	− tan. ω	− cot. ω	+ sec. ω	− cosec. ω
90° −	+ cos. ω	+ sin. ω	+ cot. ω	+ tan. ω	+ cosec. ω	+ sec. ω
180 − ω	+ sin. ω	− cos. ω	− tan. ω	− cot. ω	− sec. ω	+ cosec. ω
270 − ω	− cos. ω	− sin. ω	+ cot. ω	+ tan. ω	− cosec. ω	− sec. ω
360 − ω	− sin. ω	+ cos. ω	− tan. ω	− cot. ω	+ sec. ω	− cosec. ω

and by continuing this series of arcs the same values of the trigonometrical lines would obviously recur as before.

It is obvious that the cosine of a negative arc, whether less or greater than a quadrant, is the same as the cosine of the same arc, taken positively; but the sine of a negative arc, although the same in magnitude as that of an equal positive arc, has an opposite sign: hence, by the equations at (9), the sine, tangent, cotangent, and cosecant, will have opposite signs to those of the same arc taken positively; but the cosine and secant will have the same signs.

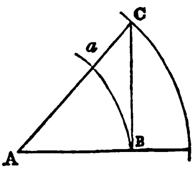
CHAPTER II.

FORMULAS AND RULES FOR THE SOLUTION OF PLANE TRIANGLES.

(12.) We shall now proceed to investigate rules for the solution of all the cases of plane triangles.

Right-angled triangles.

As right-angled triangles are those whose several parts are the most easily determined we shall consider them first.



Let ABC be any right-angled plane triangle, and with AB as a radius describe the arc Ba. If AB were unity BC would be the tangent, and AC the secant of the angle A; as it is, however, these lines are equal to AB times the trigonometrical tangent and secant (5), that is, BC = AB tan A, AC = AB sec. A.

Also, by taking the hypotenuse for the radius, we have BC = AC sin. A, AB = AC cos. A.

These four equations, together with the geometrical property $AC^2 = AB^2 + BC^2$, enables us to solve every case of right-angled triangles.

(13.) In applying these formulas, it must be remembered that the trigonometrical lines which they involve are according to the scale of radius = 1; they are computed and registered in the tables of *natural sines and tangents.* The tables of *logarithmic-sines and tangents* are not, however, computed to this radius, on account of the inconvenience which would attend the continual use of negative indices in all the sines and cosines; but they are computed to a radius of 10^{10}. Hence, in all formulas of trigonometry, intended for logarithmic computation, the radius R must always be introduced, so as to make the terms homogeneous; and, although in the formulas which will be hereafter given, we shall but seldom encumber the expressions by actually inserting in them R, and its powers, yet the computist must not fail to take account of them in the logarithmic process.

Introducing R into the foregoing equations we may write them thus:

$$\frac{R}{AB} = \frac{\tan.A}{BC}, \quad \frac{R}{AB} = \frac{\sec.A}{AC}; \quad \frac{R}{AC} = \frac{\sin.A}{BC}, \quad \frac{R}{AC} = \frac{\cos.A}{AB};$$

and all these equations may be comprehended in a single rule expressed as below. As the tabular radius

: the radius in the figure
:: any tabular line
: the corresponding line in the figure;

and from this it immediately follows that

Any tabular line
: corresponding line in the figure,
:: any other tabular line
: corresponding line in the figure;

which proportion, obviously, comprehends the former.

It appears from this rule that when we want to find a side, we must begin the proportion with a given tabular line, that is, either with the tabular radius, of which the logarithm is 10, or else with the tabular sine, cosine, &c. of a given angle; but when we want to find an angle then we must invert this proportion, beginning with a given side which must be made the geometrical radius, as no other tabular line but the radius will be given, seeing that angles are in this case unknown.

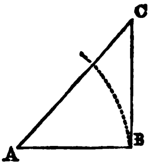
(14.) In operating with logarithms, the logarithm of the first term of the proportion must be subtracted from the sum of the logs. of the other two, to obtain the logarithm of the sought fourth term; and thus the logarithmic process will consist of five lines or rows of figures. If, however, the first term, or that to be subtracted were 10, we might save a line, by adding the two other logs. together, and rejecting 10 in the index; when the first term is not 10 we may still save a line by the following artifice, viz. instead of putting down for the first term the log. given by the table, put down its deficiency from the number 10, which may be done with as much readiness as transcribing the number itself, provided we begin at the left-hand figure and subtract each in succession from 9, till we come to the last significant figure, which must be taken from 10; we shall thus have instead of the logarithm, what is called its *arithmetical complement*, which, being added in with the other two terms, rejecting 10 from the index, must give the same result as if we had subtracted the log. of the first term from the sum of the other two. An example or two will fully illustrate what has now been said.

EXAMPLES.

(15.) 1. Given the angles and the base to find the perpendicular and hypotenuse, viz. A = 53° 8′, AB = 288.

i. *To find the Perpendicular BC.*

As a side is here required, we must begin with a tabular line; we shall choose for simplicity the tabular radius, to save a reference to the table, as we know the log. of this to be 10. Taking then the known line AB for radius in the figure, we have



Rad. . . 10
: AB 288 2·4593925
: : tan. A 53° 8′ 10·1249898

: BC 384·05 2·5843823;

BC might have been found by making any other side radius, although not quite so easily, as we should then have had to seek out in the table the tabular line for the first term, corresponding to the known line AB; thus if AC had been made radius, then the tabular line we should have commenced with, would have been that corresponding to AB, viz. the cosine of the angle A. If CB had been made radius we should have commenced with the cotangent of A, that is, the tangent of C, for such would be the tabular lines corresponding to BA.

ii. *To find the Hypotenuse AC.*

Preserving the same radius we have,

Rad. . . 10
: AB 288 2·4593925
: : sec. A 53° 8′ 10·2218814

: AC 480·036 2·6812739.

If we had made AC radius, the proportion would have been cos. A : AB : : rad. : AC. By way of showing the use of the arithmetical complement, let us determine AC by this proportion

cos. A 53° 8′ arith. comp. 0·2218814
: AB 288 . . . 2·4593925
: : Rad. 10

: AC 480·036 . . 2·6812739

2. Given the two perpendicular sides to find the hypotenuse and angles, viz. AB = 472, BC = 765, (see last fig.)

I. *To find the Angle* A.

We must here, agreeably to the rule, begin with a given side, say AB, which we shall make radius.

	AB	472 arith. comp.	7·3260580
:	Rad.	. .	10
: :	BC	765 .	2·8836614
:	tan. A*	58° 19′ 32″	10·2097194

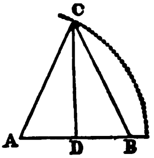
II. *To find the Hypotenuse.*

Here we must begin with a tabular line; we shall choose the radius

	Rad.	. .	10
:	AB	472 .	2·6739420
: :	sec. A	58° 19′ 32″	10·2797645
:	AC	898·89	2·9537065.

Or without employing the angle A we may determine AC by the formula. $AC = \sqrt{AB^2 + BC^2}$.

3. Given two sides and the included angle of an isosceles triangle ABC to find the other parts AC = BC = 288, ACB = 78° 12′.



Let the perpendicular CD be drawn, then since it will bisect the angle C, we shall have given in the right-angled triangle ADC, AC = 288, ACD = 39° 6′ ∴ A = 90° — 39° 6′ = 50° 54′; hence to find AD, we have by making AC radius,

	Rad.	. .	10
:	AC	288	2.4593925
: :	cos. A	50° 54′	9·7998062
:	AD	181·635	2·2591987

∴ AB = 363·270.

4. Given the base AB = 53·42, and the perpendicular BC 75·18, to find the hypotenuse and angles?

A = 54° 36′ 14″, C = 35° 23′ 46″, AC = 72·23.

5. Given the hypotenuse AC = 643·7, and the base AB = 473·8 to find the other parts? A = 42° 36′ 12″, C = 47° 23′ 48″, BC = 35·87.

6. Given the angle A = 37° 2′ 43″, and the hypotenuse AC = 173·2 to find the other parts? C = 52° 57′ 17″, AB 138·24, BC = 104·34.

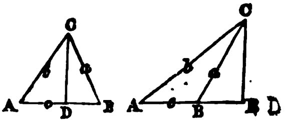
(16.) We shall now proceed to investigate rules and formulas for the solution of triangles in general.

Oblique-Angled Triangles.

Let ABC be any plane triangle, and let us denote the angles by the capital letters A, B, C, at their vertices, and the sides opposite to them by the small letters *a*, *b*, *c*.

From either vertex, as C, draw the perpendicular CD to the opposite side.

Then the sine of A to the radius *b* will, obviously, be the line C D, and the value of this sine in terms of the trigonometrical sine of the same angle to radius 1 is (art



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* For the method of determining the angle corresponding to any tabular number to seconds, see the introductory explanation prefixed to the tables.

5,) $CD = b \sin. A$. In like manner the sine of B to the radius a, is the same line CD, whose value, therefore, in term of the trigonometrical sine, is $CD = a \sin. B$; consequently, by equating these two values of CD, we have $a \sin. B = b \sin. A$

$\therefore \dfrac{a}{b} = \dfrac{\sin. A}{\sin. B}$. This equation immediately furnishes us with an important rule, which may be expressed as follows.

Any side of a triangle is to any other side as the sine of the angle, opposite to the former, is to the sine of the angle opposite to the latter.

Whenever, therefore, we know two sides and an angle opposite to one of them, or two angles and a side opposite to one of them, the other three parts of the triangle may always be determined by help of this rule.

The cosine of A, to the radius b, is the line AD; and, therefore, AD, in terms of the trigonometrical cosine of A, is $AD = b \cos. A$. In like manner the cosine of B to the radius BC, is BD, which, in terms of the trigonometrical cosine, is $BD = a \cos. B$; if the angle B is obtuse, as in the second of the above diagrams, cos. B will be negative; hence whether it be acute or obtuse we shall have for the side AB the expression $c = a \cos. B + b \cos. A$; in which the proper signs of the cosines are supposed to be involved in their expressions.

If instead of drawing the perpendicular from C we had drawn it from B, it is easy to see the result we should have obtained; for then considering B the vertical angle instead of C, or supposing the triangle to be turned about till B actually becomes the vertical angle, then commencing at the vertex, the arrangement of the angles will now be B, C, A; these, therefore, should respectively be substituted for C, A, B, in the above formula; also the arrangement of the sides will be a, b, c, instead of b, c, a, as at first, so that these letters must be replaced by the former: consequently, our equation will become $b = c \cos. A + a \cos. C$.

If, on the contrary, A be made the vertical angle, then the order of the angles will be A, B, C, and of the sides c, a, b, and these must supply the places, of C, A, B, and b, c, a, in the first formula, so that we shall then have $a = b \cos. C + c \cos. B$. Collecting these equations together we have,

$$\left.\begin{array}{l} a = b \cos. C + c \cos. B \\ b = c \cos. A + a \cos. C \\ c = a \cos. B + b \cos. A \end{array}\right\} \ldots . (1);$$

and these equations contain the whole theory of plane trigonometry. They involve all the six parts of a triangle, the three angles, and the three sides; and, as the equations are three in number, any three of the parts, considered as unknown quantities, may be determined, provided only the other three are known; but fewer than three being given will not be sufficient to determine the others, as then there would be a greater number of unknowns than of equations.

We must remark too that the three given quantities must not be the three angles simply, because the three other quantities a, b, c, severally enter the three terms of each equation, so that if we were to multiply each equation, by any assumed factor whatever, m, the values resulting from the elimination of A, B, C, would, obviously, be the same for ma, mb, mc, as for a, b, c; thus, showing that the data are not sufficient to *determine* any triangle, but belong equally to innumerable triangles, all, however, similar to each other.

(17.) It appears then that the solutions to all the cases of plane triangles are derivable from the equations (1), under different hypotheses, as to the three unknown quantities, and we might now with but little trouble proceed to deduce these solutions, one after another, from these equations: thus suppose the three sides a, b, c, were given, then multiply the first equation by a, the second by b, and the third by c, we have

$$a^2 = ab \cos. C + ac \cos. B$$

$$b^2 = bc\cos. A \text{ [illegible] } a\cos. C$$
$$c^2 = ac\cos. B \text{ [illegible] } c\cos. A;$$

and subtracting each of these from the sum of the other two, we get

$$\left.\begin{array}{l} b^2+c^2-a^2 = 2\,bc\cos. A \therefore \cos. A = \dfrac{b^2+c^2-a^2}{2\,bc} \\[2ex] a^2+c^2-b^2 = 2\,ac\cos. B \therefore \cos. B = \dfrac{a^2+c^2-b^2}{2\,ac} \\[2ex] a^2+b^2-c^2 = 2\,ab\cos. C \therefore \cos. C = \dfrac{a^2+b^2-c^2}{2\,ab} \end{array}\right\} \cdots\cdots (2);$$

and thus the values of the cosines of the required angles become known, and by searching in the table of natural sines and cosines we shall find the angles to which they belong.

It is necessary to remark here that in almost every trigonometrical calculation it is advisable to conduct the operation by means of logarithms, in order to avoid lengthy and tiresome multiplications, divisions, and extractions; so that it becomes a matter of consequence to express all our general rules and formulas in a form, adapted as much as possible to logarithmic calculation, that is, the operations indicated by the formulas should be those of multiplication, division, involution, and evolution, and not those of addition and subtraction.

The formulas just deduced for the angles of a triangle, when the sides are given, do not appear in a form adapted to logarithmic computation; and the same would be found to be the case with the various other formulas directly deducible from the general equations (1); nor would it be easy, without the aid of other and independent properties, to convert these expressions into the desired form. Although, therefore, it is true, as we have stated above, that formulas for all the cases of plane trigonometry may be deduced from the equations (1), yet, on account of the inconvenient form these formulas assume, it becomes necessary for us to seek assistance from other sources. Now there exist two general trigonometrical formulas, which may be considered as forming the foundation of the whole theory of angular magnitude, and which, in conjunction with what is laid down above, will enable us to deduce formulas suited to logarithmic calculation for all the cases of plane triangles.

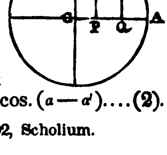
(18.) There are various ways of investigating these formulas; we shall adopt that which appears to us the most simple and general.

It was given by *M. Sarrus* in the *Annales des Mathematiques*, tom. xi.

Given the sines and cosines of two arcs or angles, to find the sine and cosine of their sum and difference.

Let AM $= a$, and AN $= a'$, be any two arcs of the circle, the radius being unity, then drawing the chord of the arc NM $= a - a'$, we shall have from the triangle NMG right angled at G.

MN^2 = NG^2 + MG^2 = (CQ — CP)^2 + (PM — NQ)^2; which may be written thus, chd.$^2\,(a - a') = (\cos.\,a' - \cos.\,a)^2 + (\sin.\,a - \sin.\,a')^2$.



By actually squaring the expressions in the right-hand member of this equation, and recollecting that

$$\sin.^2 a + \cos.^2 a = 1,\ \sin.^2 a' + \cos.^2 a' = 1,$$

we have chd.$^2\,(a - a') = 2 - 2\cos.\,a\cos.\,a' - 2\sin.\,a\sin.\,a'$. . . . . (1).

Suppose now that $a' = 0$, then we have chd.$^2\,a = 2 - 2\cos.\,a$.*

As this expression is true for any arc whatever, it is true for the arc $a - a'$, so that chd.$^2\,(a - a') = 2 - 2\cos.\,(a - a')$. . . . (2).

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* This property is also proved in the Geometry, p. 92, Scholium.

cos. a cos. a' + sin. a sin. a'

Comparing together the sec[illegible]members of (1) and (2) we obtain $\cos. (a - a') = \cos. a \cos. a' +$ [illegible] $a \sin. a'$ . . . . (I).

As this is true for all values [illegible] a', it is true when $a - a'$ is put for a', so that $\cos. a' = \cos. a \cos. (a - a') + \sin. a \sin. (a - a')$; in which equation, if we substitute the value of $\cos. (a - a')$ given by (I), we have $\cos. a' = \cos. a^2 \cos. a' + \cos. a \sin. a \sin. a' + \sin. a \sin. (a - a')$; from which, by putting for $\cos.^2 a$ its value $1 - \sin.^2 a$, we get $\sin. (a - a') = \sin. a \cos. a' - \sin. a' \cos. a$ . . . . (II).

Lastly, putting $(a + a')$ for a, in the equations (I) and (II), we have,

$$\cos. a = \cos. (a + a') \cos. a' + \sin. (a + a') \sin. a'.$$
$$\sin. a = \sin. (a + a') \cos. a' - \cos. (a + a') \sin. a'.$$

In order to obtain from these equations the expressions for $\sin. (a + a')$, and $\cos. (a + a')$, multiply the first by $\sin. a'$; the second by $\cos. a'$, and add, and we thus get, $\sin. (a + a') = \sin. a \cos a' + \sin. a' \cos. a$. (III).

Multiply the first by $\cos. a'$, the second by $\sin. a'$, and subtract, and we get $\cos. (a + a') = \cos. a \cos. a' - \sin. a \sin. a'$ . . . . (IV).

The four general formulas thus deduced may be written as follows:

$$\left.\begin{array}{l} \sin. (a \pm a') = \sin. a \cos. a' \pm \sin. a' \cos. a \\ \cos. (a \pm a') = \cos. a \cos. a' \mp \sin. a \sin a' \end{array}\right\} \quad . \; . \; . \; . \; . \text{(A)}.$$

(19.) The first of these immediately furnish the two following, viz.

$$\sin. (a + a') + \sin. (a - a') = 2 \sin. a \cos. a'$$
$$\sin. (a + a') - \sin. (a - a') = 2 \sin. a' \cos. a; \text{ from which}$$
$$\frac{\sin. (a + a') + \sin. (a - a')}{\sin. (a + a') - \sin. (a - a')} = \frac{\sin. a.}{\cos. a.} \cdot \frac{\cos. a'}{\sin. a'} = \frac{\tan. a}{\tan. a',} \text{(art. 8).}$$

If, therefore, we put

$$a + a' = A,\ a - a' = B \ \therefore\ a = \tfrac{1}{2} (A + B),\ a' = \tfrac{1}{2} (A - B),$$

we shall have $\dfrac{\sin. A + \sin. B}{\sin. A - \sin. B} = \dfrac{\tan. \frac{1}{2} (A + B)}{\tan. \frac{1}{2} (A - B)}$. Now we have already seen that in any plane triangle $\sin. A : \sin. B :: a : b$

$$\therefore \sin. A + \sin. B : \sin. A - \sin. B :: a + b : a - b;$$

consequently, from the equation above, $\dfrac{a + b}{a - b} = \dfrac{\tan. \frac{1}{2} (A + B)}{\tan. \frac{1}{2} (A - B)}$; that is to say, *in any plane triangle the sum of any two sides is to their difference as the tangent of half the sum of the opposite angles is to the tangent of half their difference.*

By help of this rule we may determine the remaining parts of the triangle, when we know two sides a, b, and the included angle C; for knowing C we know also $\frac{1}{2} (A + B) = \frac{1}{2} (180° - C)$; and $\frac{1}{2} (A - B)$ is determined by this rule; therefore, as the half sum added to the half difference of two quantities gives the greater, and subtracted gives the less; we thence readily obtain the angles A and B, and then the third side c, by (16.) We have thus deduced commodious rules fitted for logarithmic computation, for the solution of the first two cases of plane triangles: it remains to furnish a rule for the third case.

(20.) Referring to the expression for cos. A at (17), it is plain that since $b^2 + c^2 = (b + c)^2 - 2bc$, and therefore, $b^2 + c^2 - a^2 = (b + c + a)(b + c - a) - 2bc$; that expression may be put under the form $\cos. A = \dfrac{(b + c + a)(b + c - a)}{2bc} - 1$.

Now supposing the arcs a, a', in equation (A), to be equal to each other, and to $\frac{1}{2}$ A, we have from the second of them

$$\cos. A = \cos.^2 \tfrac{1}{2} A - \sin.^2 \tfrac{1}{2} A$$
$$1 = \cos.^2 \tfrac{1}{2} A + \sin.^2 \tfrac{1}{2} A$$
$$\text{by addition, } \cos. A = 2 \cos.^2 \tfrac{1}{2} A - 1$$
$$\text{by subtraction, } \cos. A = 1 - 2 \sin.^2 \tfrac{1}{2} A;$$

by substituting the first of these values in the foregoing equation, and putting for brevity S for the sum of the three sides of the triangle, we have $\cos.\frac{1}{2}A = \sqrt{\frac{\frac{1}{2}S(\frac{1}{2}S - a)}{bc}}$ . . . . (1).

We can just as readily obtain a second formula by means of the other expression for cos. A; for substituting it in equation (2), art. (17), we have $2\sin.^2\frac{1}{2}A = 1 - \frac{b^2 + c^2 - a^2}{2bc} = \frac{a^2 - b^2 - c^2 + 2bc}{2bc} =$

$$= \frac{a^2 - (b - c)^2}{2bc} = \frac{(a + b - c)(a - b + c)}{2bc};$$

consequently, $\sin.\frac{1}{2}A = \sqrt{\frac{(\frac{1}{2}S - c)(\frac{1}{2}S - b)}{bc}}$ . . . (2);

and by dividing this expression by the former we get a third formula, viz. $\tan.\frac{1}{2}A = \sqrt{\frac{(\frac{1}{2}S - b)(\frac{1}{2}S - c)}{\frac{1}{2}S(\frac{1}{2}S - a)}}$ . . . (3).

(21.) We thus have three distinct formulas for the determination of the angles of a triangle when the three sides are given, and all of them are adapted to logarithmic computation. It is not, however, always a matter of indifference which of these formulas we employ, as in certain cases one may be preferable to another. Thus, if we knew beforehand, or could foresee that the sought angle ½ A would be very nearly equal to 90°, then it would be improper to employ the formula (2), because we should be very likely to commit error in taking out the angle, seeing that for an angle very near 90° the seven first decimals in the sine coincide with those in the sines of several other angles in its vicinity, or which differ each from the proposed angle by only a few seconds.

If the logarithmic tables, which we employ, are calculated to seconds, as the large tables, of *Taylor* or of *Bagay*, then the sought angle when near 90°, may be accurately determined to the nearest second, either from its cosine or from its tangent, as the values of these trigonometrical lines, at this part of the table differ considerably from each other, even when the arcs are nearly equal. But if the table employed is not calculated to seconds, then the sought angle, when near 90°, should be determined from its cosine, and not from its tangent; because in approaching to 90° the tangents increase by very unequal differences, and, as, in proportioning for the seconds, we proceed on the supposition that the tangents increase equally through 60′, we shall be in danger of committing error in thus determining the seconds. As the cosines decrease more regularly towards the extremity of the quadrant than the tangents increase, it will, therefore, be safest to determine such arcs from their cosines.

Sept 25

When the sought angle is very small it will be best to determine it from its sine; although the tangent may be used with safety.

Solution of Plane Triangles in general.

(22.) We shall now proceed to apply the rules and formulas which we have just investigated to the several cases of plane triangles, repeating the rule at the head of each case.

CASE 1. When a side and its opposite angle are among the given parts.

RULE.—Sine of given angle,	RULE 2. Also, any given side,
: its opposite side	: sine of its opposite angle
:: sine of any other angle	:: any other side
: its opposite side.	: sine of its opposite angle.

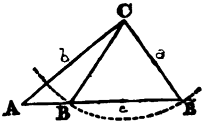
As the same sine belongs both to an angle and to its supplement, it may seem doubtful in determining an angle of a triangle from its sine, whether to take the acute angle given by the tables or the obtuse angle which is its supplement.

The following precepts will remove all doubt on this point.

1. If the given angle is obtuse the sought angle must be acute. This is obvious, because a triangle cannot have two obtuse angles.

2. If the given angle be acute, and the side opposite to it greater than the side opposite to the sought angle, this must be acute; for the greater angle must be opposite to the greater side.

3. But when the side opposite to the given angle is less than that opposite to the sought angle, this may be either acute or obtuse, so that two triangles exist under the proposed conditions, and the problem in question admits, therefore, of two solutions. The annexed diagram shows that with two given sides AC, CB, and the acute angle A, opposite to one of them, we may always construct two triangles, ABC, AB′C; where the angle B, opposite to the other given side in the one triangle, will be the supplement of the corresponding angle B′ in the other, provided CB is less than CA.



EXAMPLES.

(23.) 1. In the triangle ABC are given AB=137, AC=153, B=78° 13′, to find the remaining parts.

I. *To find the Angle* C.

As AC	153	arith. comp.	7·8153086
: sin. B	78° 13′		9·9907502
:: AB	137		2·1367206
: sin. C	61° 13′ 47″		9·9427794.

The obtuse angle, which is the supplement of this, is not admissible, because the side opposite to the given angle is greater than the side opposite the required one.

II. *To find the side* CB.

The angle A is equal to 180°—(B+C)=180°—139° 26′ 47″ = 40° 33′ 13″; therefore,

As sin. B	78° 13′	arith. comp.	0·0092498
: AC	153		2·1846914
:: sin. A	40° 33′ 13″		9·8130198
: CB	101·617		2·0069610.

2. In the plane triangle ABC are given AC=216, CB=117, and A=22° 37′, to find the rest.

I. *To find the Angle* B.

As BC	117	arith. comp.	7·9318141
: sin. A	22° 37′		9·5849685
:: AC	216		2·3344538
: sin. B	45° 13′ 55″ or 134° 46′ 5″		9·8512364.

The angle B is, in this example, ambiguous, because the side opposite the given angle is less than that opposite the required one.

II. *To find the third side* AB.

The angle C is equal to 180°—(A+B)=112° 9′ 5″, provided we take B acute; therefore,

As sin. A	22° 37′	arith. comp.	0·4150315
: BC	117	. . . . .	2·0681859
:: sin. C	112° 9′ 5″	. . . .	9·9667005
: AB	281	. . . . .	2·4499179

SCHOLIUM.

In each of the foregoing examples where two sides, and an angle opposite to one, are given, we have found it necessary to find the angle opposite to the other given side before we could apply the rule to the determination of the third side; so that the determination of this third side requires two proportions, and there is no logarithmic method which will lead us to it by a shorter process. It may, however, be deduced directly from the formula at (17), viz. $\cos. A = \frac{b^2 + c^2 - a^2}{2bc}$;

which gives $c = AB = b \cos. A \pm \sqrt{a^2 - b^2 \sin.^2 A}$;
which expression is, however, not adapted to logarithmic computation.

3. In the plane triangle ABC are given

$$A = 44^\circ\, 13'\, 24'',\ B = 79^\circ\, 46'\, 38'',\ AB = 368,$$

to find the rest.

I. *To find the side* AC.

The angle C is equal to 180° — (A + B) = 55° 59′ 58″, therefore,

As sin. C	55° 59′ 58″	arith. comp.	0·0814286
: AB	368	. . .	2·5658478
: : sin. B	79° 46′ 38″	. .	9·9930503
: AC	436·844	. .	2·6403267

II. *To find the side* CB.

As sin. C	55° 59′ 58″	arith. comp.	0·0814286
: AB	368	. . .	2·5658478
: : sin A	44° 13′ 24″	. .	9·8435174
: CB	309·595	. .	2·4907938

4. In the plane triangle ABC are given AB = 408 yards, A = 74° 14′, B = 49° 23′; to find the other two sides,

C 56° 23 AC = 371·9 yards and BC = ~~416·02~~ 471.50 yards.

5. In the plane triangle ABC are given AB = 408 yards, A = 58° 7′, B = 22° 37′; to find the other two sides,

C 79° 16′ AC = 158·98 yards and BC = 351·03 yards.

6. In the plane triangle ABC are given AB = 318, BC = 195, A = 32° 40′; to find the angle C, C = 61° 40′ 3″, or 118° 19′ 57″.

CASE II. (24.) When two sides and the included angle are given.

RULE, (ART 19.)

As the sum of the two given sides,
: their difference,
: : tangent of half the sum of the opposite angles
: tangent of half their difference.

Having thus found the half difference of the unknown angles, we obtain the angles themselves, by first adding and then subtracting this half difference from the half sum. The angles being thus known, as well as two sides, the third side is found by the first case.

The student will find a more compendious method of solution for this case in Prob. I., Part IV.; but the rule here given will be more easily remembered.

EXAMPLES.

1. In the triangle ABC are given AB $= 137$, AC $= 153$, A $= 40° 33' 12''$; to find the other parts.

I. *To find the other two Angles.*

The sum of the other two angles is $(B + C) = 180° - A = 139° 26' 48''$, therefore

As $AB + AC$	290 .	arith. comp. 7·5376020
: $AC \sim AB$	16 . .	1·2041200
:: tan. $\frac{1}{2}(B + C)$	$69° 43' 24''$ . .	10·4324460
: tan. $\frac{1}{2}(B \sim C)$	$8° 29' 37''$.	9·1741680

By adding $78° 13' 1''$ = greater angle B.
By subtracting $61° 13' 47''$ = less angle C.

II. *To find the third Side* BC.

As sin. B	$78° 13' 1''$.	arith. comp. 0·0092493
: AC	153 . . .	2·1846914
:: sin. A	$40° 33' 12''$ . . .	9·8130173
: BC	101·616 . . .	2·0069580.

2. In the triangle ABC are given AC $= 378$, BC $= 526$, C $= 32° 18' 26''$; to find the other parts.

I. *To find the Angles.*

The sum of the angles A, B is $(A + B) = 180° - 32° 18' 26'' = 147° 41' 34''$.

As $AC + BC$	904 .	arith. comp. 7·0438316
: $AC \sim BC$	148 . .	2·1702617
:: tan. $\frac{1}{2}(A + B)$	$73° 50' 47''$ . .	10·5381278
: tan. $\frac{1}{2}(A \sim B)$	$29° 29' 34''$.	9·7522211

By adding, $103° 10' 21''$ = greater angle B.
By subtracting, $44° 31' 13''$ = less angle A.

II. *To find the side* AB.

As sin. A	$44° 31' 13''$.	arith. comp. 0·1541818
: BC	526 . . .	2·7209857
:: sin. C	$32° 18' 26''$ . . .	9·7279143
: AB	400·942 . . .	2.6030818.

If we wish to obtain the third side of the triangle immediately, without first finding the angle, we may do so by means of the formula at (17), adverted to in the scholium to last case; but as the computation will not be adapted to logarithms, it will in general be the shortest method to proceed as above, by two proportions.

3. In the triangle ABC are given AB $= 1637$, AC $= 2065$, A $= 132° 7' 12''$; to determine the remaining parts.

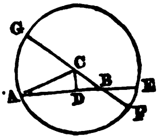
$B = 26° 52' 42\frac{1}{7}''$, $C = 21° 0' 5\frac{6}{7}''$, $BC = 3387·974$.

4. In the triangle ABC are given AB $= 1686$, BC $= 960$, B $= 128° 4'$; to find the rest.

$A = 18° 21' 20''$, $C = 33° 34' 40''$, $AC = 2400·364$.

CASE III. (25.) When the three sides are given.

A rule for this case, easy to be remembered, may be deduced



from the following simple geometrical investigation.

Take the longest side AB of the triangle for base, and demit upon it the perpendicular CD from the vertex, which will necessarily fall within the base. With centre C and radius CA equal to the longer of the two sides AC, CB, describe a circle, and produce the sides AB, BC, to meet the circumference; then it is plain that

$$\text{GB} = \text{AC} + \text{CB},\ \text{BF} = \text{AC} - \text{CB},\ \text{BE} = \text{AD} - \text{DB}.$$

Now (Geom. Prop. 24, book 6).

$$\text{GB}\cdot\text{BF} = \text{AB}\cdot\text{BE} \therefore \text{AB}\cdot(\text{AD} - \text{DB}) = (\text{AC} + \text{CB})\,(\text{AC} - \text{CB})$$

$\therefore$ AB : AC + CB :: AC − CB : AD − DB hence the following rule.

RULE I.

Consider the longest side of the triangle as the base, and demit upon it a perpendicular from the opposite vertex, dividing the base into two segments; then say, As the base,

: the sum of the other two sides,
:: the difference of those sides
: the difference of the segments of the base.

Having thus the sum and difference of the segments, each segment becomes known, and, therefore, in each of the two right-angled triangles into which the proposed is divided, there will be known the base and hypotenuse, and this is enough to determine all the other parts.

RULE II. (ART. 20.)

$$\cos.\tfrac{1}{2}\text{A} = \sqrt{\frac{\tfrac{1}{2}\text{S}\,(\tfrac{1}{2}\text{S} - a)}{bc}},\quad \sin.\tfrac{1}{2}\text{A} = \sqrt{\frac{(\tfrac{1}{2}\text{S} - c)\,(\tfrac{1}{2}\text{S} - b)}{bc}}$$

$$\tan.\tfrac{1}{2}\text{A} = \sqrt{\frac{(\tfrac{1}{2}\text{S} - b)\,(\tfrac{1}{2}\text{S} - c)}{\tfrac{1}{2}\text{S}\,(\tfrac{1}{2}\text{S} - a)}}$$

Both these rules are adapted to logarithmic computation, and this last is much the shortest; when, however, the three sides are small numbers, it will be best to operate without logarithms, by means of the formula (20), $\cos.\ \text{A} = \dfrac{(b+c+a)\,(b+c-a)}{2\,bc} - 1.$

In applying the logarithmic formulas in Rule 2 to the determination of any particular angle, it will generally be best, when this angle is opposite to the longest side of the triangle, to use the first formula, and when it is opposite to the shortest side to use the second; the third may be used when the required angle is opposite to the mean side. If two sides of the triangle are equal, then, of course, neither of these formulas will be used, as the unknown parts will be more readily found as in Example 3, p. 17.

EXAMPLES.

1. The three sides of the triangle ABC are AB = 1637, AC = 2065, BC = 3387·974; required the angle A.

$a = 3387{\cdot}974$
$b = 2065$ arith. comp. 6·6850799
$c = 1637$ arith. comp. 6·7859513

2)7089·974

$\tfrac{1}{2}\text{S} = 3544{\cdot}987$. 3·5496146
$\tfrac{1}{2}\text{S} - a = 157{\cdot}013$. 2·1959356
2)19·2165814
$\cos.\tfrac{1}{2}\text{A} = 66^\circ\ 3'\ 36''$. 9.6082907
$\therefore \text{A} = 132^\circ\ 7'\ 12.$

2. The three sides of the triangle ABC are AB = 98, BC = 95·12, AC = 162·34; to determine the angle A.

Using the third formula in the second rule, we have

$a = 95{\cdot}12$
$b = 162{\cdot}34$
$c = 98$

$2)355{\cdot}46$

$S = 177{\cdot}73$ arith. comp. 7·7502393
$\tfrac{1}{2}S - a = 82{\cdot}61$ arith. comp. 8·0829674
$\tfrac{1}{2}S - b = 15{\cdot}39$. 1·1872386
$\tfrac{1}{2}S - c = 79{\cdot}73$ 1·9016218

2)18·9220671

tan. $\tfrac{1}{2}$ A = 16° 7′ 26¾″ . 9·4610335

$\therefore$ A = 32° 14′ 53½″.

3. In the triangle ABC are given AC = 6, AB = 5.523, BC = 1·372; required the angle A.

Applying the second formula to this example we have

$a = 1{\cdot}372$
$b = 6$ arith. comp. 9·2218487
$c = 5{\cdot}523$ arith. comp. 9·2578250

$2)12{\cdot}895$

$\tfrac{1}{2}S = 6{\cdot}4475$
$\tfrac{1}{2}S - b = {\cdot}4475$. 1·6577930
$\tfrac{1}{2}S - c = {\cdot}9245$ . . $\bar{1}{\cdot}9659069$

2)18·0963736

sin. $\tfrac{1}{2}$ A = 6° 24′ 55″ . 9·0481868

$\therefore$ A = 12° 49′ 50″.

4. The three sides of a plane triangle are AB = 137, AC = 153, BC = 101·616; required the three angles,

A = 40° 33′ 12″, B = 78° 13′ 1″, C = 61° 13′ 47″.

5. The three sides of a plane triangle are AB = 1686, BC = 960, AC = 2400·364; required the angle B,

B = 128° 4′.

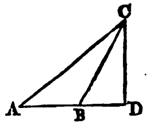
6. Required the angles when the sides are 4, 5, and 6.

The angles are 41° 24′ 35″, 55° 46′ 16″, and 82° 49′ 9″.

CHAPTER III.

APPLICATION OF PLANE TRIGONOMETRY TO THE MENSURATION OF HEIGHTS AND DISTANCES.

PROBLEM. 1.



A person on one side of a river observes an obelisk on the opposite side, and, being desirous to ascertain its height, he took with a quadrant the angle B = 55° 54′, which the obelisk subtended at the place where he stood, then going back the distance BA = 100 feet, he again measured the subtended angle, and found it to be A = 33° 20′; what was th height of the obelisk?

In the triangle of ABC are given the angle A = 33° 20′, the angle ACB = 55° 54′ — 33° 20′ = 22° 34′, and the side AB; and, therefore BC may be found by Case I. of oblique angled triangles. Again, in the triangle BCD, we shall have given the side BC, and the angle B to find CD, which belongs to Case I. of right angled triangles.

The actual computation, however, will be shortened by combining these two rules in a single formula, thus for the first

$\mathrm{BC} = \dfrac{\mathrm{AB\ sin.\ A}}{\mathrm{sin.\ ACB}}$, and from the second CD = BC sin CBD

$$\therefore \mathrm{CD} = \frac{\mathrm{AB\ sin.\ A\ sin.\ CBD}}{\mathrm{sin.\ ACB}}$$

sin. ACB 22° 34′ arith. comp. 0·4159424
sin. A 33° 20′ . 9·7399748
sin. CBD 55° 54′ . 9·9186620
AB 100 . 2

CD 118·57* 2·0739792.

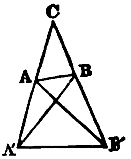
The problem may be solved still more readily as follows.

If we take CD for radius, DB will be the tangent of the angle DCB, and DA the tangent of DCA, therefore, AB is the difference of those tangents; but by referring to the table of natural tangents, we find that to radius 1

nat. tan. 56° 40′ = 1·5204261
nat. tan. 34° 6′ = 6770509

difference = ·8433752
∴ ·8433752 : 1 :: 100 : 118·57, as before.

PROBLEM II.



A person at A wishes to know his distance from an inaccessible object at C, but he has no instrument for taking angles. He, therefore, sets up a staff at A, from which he measures the distance AA′ = 60 feet, so that when he stands at A′ the staff and the object appear in the same straight line A′C; he in like manner, measures another distance BB′ = 86 feet, from a second station B, 38 feet from the former A, and he finds the diagonal distances AB′, BA′, to be respectively 97 feet and 81 feet. From these data it is required to determine the distance of A from the object C.

All the three sides of the triangle A′AB are given, therefore to find the angle A′AB we have, by using the first formula at (25),

A′B = 81
A′A = 60 arith. comp. 8·2218487
AB = 38 arith. comp. 8·4202164

2)179

½ S = 89·5 . 1·9518230
½ S — A′B = 8·5 . 0·9294189

2)19·5233070

cos. ½ A AB = 54° 41′ 56″ 9.7616535
∴ A′AB = 109° 23′ 52″ ∴ CAB = 70° 36′ 8″.

---

* To the height of the object thus determined the height of the observer's eye, or of the instrument, must be added.

Again, by applying the same formula to the triangle B′BA, we have

B′A = 97
B′B = 86 arith. comp. 8·0655015
AB = 38 arith. comp. 8·4202164

2)221

110·5 . 2·0433623
13·5 . 1·1303338

2)19·6594140

cos. ½ B′BA = 47° 29′ 50″ . 9·8297070
∴ B′BA = 94° 59′ 40″ ∴ CBA = 85° 0′ 20″
∴ C = 180 − (CAB + CBA) = 24° 23′ 40″.

Consequently, in the triangle ABC, we have all the angles and one side AB given; hence, by Case I.

sin. C 24° 23′ 40″ arith. comp. 0·3840330
: AB 38 . . . 1·5797836
:: sin. B 85° 0′ 20″ . 9·9983479

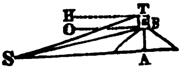
: AC 91·657 . . 1·9621645.

PROBLEM III.

At the top of a castle, which stood on a hill near the sea-shore, the angle of depression HTS, of a ship at anchor, was observed to be 4° 52′; at the bottom of the castle the angle of depression OBS was 4° 2′. Required the horizontal distance AS of the vessel, and the height of the hill above the level of the sea, the height of the castle being 60 feet.

As TH, BO, are parallel to AS, we have TSA = 4° 52′, and BSA = 4° 2′. Bearing this in mind we have

$$\text{In } \Delta\,\text{TSB}, \ \frac{\text{SB}}{\text{BT}} = \frac{\text{sin. ATS}}{\text{sin. TSB}}$$



In Δ BSA, AS = SB cos. BSA, AB = SB sin. BSA

$$\therefore \text{AS}\cdot\text{R} = \frac{\text{BT sin. ATS cos. BSA}}{\text{sin. TSB}};\ \text{AB}\cdot\text{R} = \frac{\text{BT sin. ATS sin. BSA}}{\text{sin. TSB}};$$

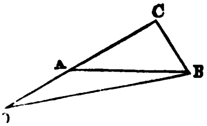
hence the logarithmic operation will be

sin. TSB 0° 50′ arith. comp. 1·8373192 . . . . 1·8373192
sin. ATS 85° 8′ . . . 9·9984315 . . . . 9·9984315
cos. BSA 4° 2′ . . . 9·9989230 sin BSA . 8·8471827
BT 60 . . . . 1·7781513 . . . . 1·7781513

AS 4100·4 . . 3·6128250, AB 289·12 . 2·4610847.

PROBLEM IV.

The distances of three objects A, B, C, from each other, are as follow, viz. AB = 462 yards, AC = 328 yards, and BC = 297 yards; a person at D, wishing to know his distance from each object, takes the angle ADB, and finds it to be 34° 16′ 21″; it is required to determine DA, DC, and DB.



As the three sides of the triangle ABC are given we may find the angle CAB, and, consequently, the supplemental angle DAB, so that we shall have in the triangle DAB the two angles D, A and the side AB to find the rest. The computation will, therefore, be as follows.

I. *To find the angle* CAB.

BC = 297		
AC = 328	arith. comp.	7·4841262
AB = 462	arith. comp.	7·3353580
2)1087		
543·5	.	
215·5	.	2·3334473
81·5	.	2·9111576
		2)19·0640891
sin. ½ CAB = 19° 54′ 14″	.	9·5320445

∴ DAB = 180° — 39° 48′ 28″ = 140° 11′ 32″ ∴ DBA = 15° 32′ 7″.

II. *To find* AD.

As sin. D	24° 16′ 21″	arith. comp.	0·3860770
: AB	462	. .	2·6646420
:: sin. B	15° 32′ 7″	. .	9·4278619
: AD	301·01	. .	2·4785809

∴ DC = DA + AC = 629·101 yards.

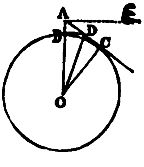
III. *To find* BD.

As sin. D	24° 16′ 21″	arith. comp.	0·3860770
: AB	462	. .	2·6646420
:: sin. A	140° 11′ 32″	.	9·8063252
: BD	719·522	. .	2·8570442

Hence we have the three distances, viz. DA = 301·01, DC = 629·101, DB = 719·522.

PROBLEM V.

Suppose that from the top of a mountain, three miles high, the angle of depression of the remotest visible point of the earth's surface is taken and found to be 2° 13′ 27″; it is required thence to determine the diameter of the earth, supposing it to be a perfect sphere.



Let O be the centre of the earth, BA the mountain, AC the visual ray or line touching the earth's surface in C. Draw the tangent BD, and join OD, OC; then the angle of depression EAC being given, we have also the angle BAD, the complement of it, equal to 87° 46′ 33″. Also since the tangents BD, CD, are equal, (Geom. p. 106,) we have the angle BOD = DOC = ½ comp. A = 1° 6′ 49½″, and, therefore, BDO = 88° 53′ 16½″.

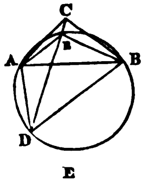
Now in the right-angled triangle ABD we have BD = AB tan. A; and in the right-angled triangle OBD, OB = BD tan. BDO; hence by substitution, OB = AB tan. A tan. BDO; the computation is, therefore, as follows:

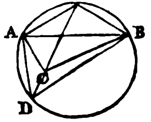
AB =	3	. .	0·4771213
tan. A	87° 46′ 33″	.	11·4107381
tan. BDO	88° 53′ 16½″	.	11·7119309
OB	3979·15	. .	3·5997903;

hence the diameter is 7958·3 miles.

PROBLEM VI.

Given the distances between three objects A, B, C, and the angles subtended by these distances at a point D in the same plane with them; to determine the distance of D from each object.





Let a circle be described about the triangle ADB, and join AE, EB, then will the angles ABE, BAE, be respectively equal to the given angles ADE, BDE, (Geom. p. 52); thus all the angles of the triangle AEB are known, as also the side AB; we may find, therefore, the remaining sides AE, EB. Again, the sides of the triangle ABC being known, we may find the angle BAC; hence the angle CAE becomes known, so that in the triangle CAE we shall have the two sides AE, AC, and the included angle given, from which we may find the angle AEC in fig. 1, or the angle ACE in fig. 2, and thence its supplement AED or ACD; this with the given side AE and angle ADE, in the first figure, or with the given side AC and angle ADC in the second, will enable us to find AD, one of the required lines, and thence DC and DB, the other two.

Or the solution may be conducted more analytically as follows.

Put x for the angle DAC, and x' for the angle DBC; also call the given angles ADC, BDC, α and α', then a, b, c, representing as usual the sides opposite to A, B, C, we have

$$\frac{\sin. \alpha}{\sin. x} = \frac{b}{\text{DC}}, \frac{\sin. \alpha'}{\sin. x'} = \frac{a}{\text{DC}} \; \ldots . \; (1) \; \therefore \frac{\sin. \alpha \sin. x'}{\sin \alpha' \sin. x} = \frac{b}{a}$$

$$\therefore a \sin. \alpha \sin. x' = b \sin. \alpha' \sin. x \; \ldots . \; (2).$$

This is one equation between the unknown quantities x, x'. Another is easily obtained; for since the four angles of the quadrilateral ADCB make up four right angles or 360°, we have $x + x' + \alpha + \alpha' + \text{ACD} + \text{BCD} = 360°$; the two latter angles may be considered as known, since in the triangle ABC the angle C is determinable from the three given sides; therefore all the terms in the first member of this equation are known except x and x'. Call the sum of these known quantities β, and we shall thus have $x' = \beta - x$, and, consequently, by substitution, equation (2) becomes, $a \sin. \alpha \sin. (\beta - x) = b \sin. \alpha' \sin. x$

$$= a \sin. \alpha (\sin. \beta \cos. x - \cos. \beta \sin. x);$$

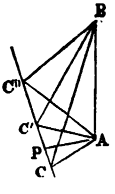
or dividing by $\sin. x$, $b \sin. \alpha' = a \sin. \alpha (\sin. \beta \cot. x - \cos. \beta)$

$$\therefore \cot. x = \frac{b \sin. \alpha'}{a \sin. \alpha \sin. \beta} + \frac{\cos. \beta}{\sin. \beta} = \frac{b \sin. \alpha'}{a \sin. \alpha \sin. \beta} + \cot. \beta.$$

The first term of this second member may be easily calculated by logarithms, and this added to the natural cotangent of β gives the nat. cot. of x, and thence x' is known from the equation $x' = \beta - x$, and CD from either of the equations (1).

PROBLEM VII.

Given the angles of elevation of an object taken at three places on the same horizontal straight line, together with the distances between the stations; to find the height of the object and its distance from either station.



Let AB be the object, and C, C', C'', the three stations, then the triangles BCA, BC'A, BC''A, will all be right angled at A; and, therefore, to radius BA, AC, AC', AC'', will be the tangents of the angles at B, or the cotangents of the angles of elevation; hence putting $\alpha, \alpha', \alpha''$, for the angles of elevation, x for the height of the object, and a, b, for the distances C C', C C'', we shall have $\text{AC} = x \cot. \alpha$, $\text{AC}' = x \cot. \alpha'$, $\text{AC}'' = x \cot. \alpha''$.

Now if a perpendicular AP be drawn from A to C C'', we shall have (Geom. p. 35,) from the triangle ACC'

$AC^2 = AC'^2 + C'C^2 - 2C'C \cdot C'P$; and from the triangle AC'C''

$AC''^2 = AC'^2 + C''C'^2 + 2C''C' \cdot C'P$; that is, we shall have the two equations $x^2 \cot.^2 \alpha = x^2 \cot.^2 \alpha' + a^2 - 2a \cdot C'P$.

$x^2 \cot.^2 \alpha'' = x^2 \cot.^2 \alpha' + b^2 + 2b \cdot C'P$. in order to eliminate C'P, multiply the first by b, the second by a, and add and we shall have

$$x^2 (b \cot.^2 \alpha + a \cot.^2 \alpha'') = (a + b)\, x^2 \cot.^2 \alpha' + ab\,(a + b)$$

$$\therefore x = \sqrt{\frac{ab\,(a+b)}{b \cot.^2 \alpha + a \cot.^2 \alpha'' - (a + b \cot.^2 \alpha')}}$$

If the three stations are equidistant, then $a = b$, and the expression becomes $$x = \frac{a}{\sqrt{\tfrac{1}{2} \cot.^2 \alpha + \tfrac{1}{2} \cot.^2 \alpha'' - \cot.^2 \alpha'}}.$$

The height AB being thus determined, the distances of the stations from the object are found by multiplying this height by the cotangents of the angles of elevation.

PROBLEM VIII.

Three objects A, B, and C, whose distances are AC = 8 miles, BC = $7\frac{1}{5}$ miles, and AB = 12 miles are visible from one station D, in the line joining A and B, at which point the line joining A and C subtends an angle of 107° 56' 13''. Required the distances of the objects from the station. AD = 5 miles, DC = 4·892 miles, DB = 7 miles.

PROBLEM IX.

Suppose the angle of elevation of the top of a steeple to be 40° when the observer's eye is level with the bottom, and that from a window 18 feet directly above the first station, the angle of elevation is found to be 37° 30'. Required the height and distance of the steeple.

Height = 210·44 feet. Distance 250·79 feet.

PROBLEM X.

In order to determine the horizontal distance between two remote objects A, B, a base line A' B' of 536 yards was measured, and then a flagstaff being set up at each extremity, these four angles were taken from them, viz. at A' the angular distance between A and B, 57° 40', and the angular distance between B and B', 40° 16', also at B' the angular distance between A and B, 71° 7', and the angular distance between A' and A, 42° 22'. Required the distance between the objects.

939·52 yards.

PROBLEM XI.

Three objects A, B, C, are in the same straight line, and of known distances from each other, viz. AB = 3·626 yards, and BC = 8·374 yards, the angular distance of A, B, from a station D, where all the objects are visible, is 19°, and the angular distance of B, C, is 25°. Required the distance of each object from the place of observation.

DA = 9·471 yards, DB = 10·861, DC = 16·848.

PROBLEM XII.

At three points in the same horizontal straight line the angles of elevation of an object was found to be 36° 50', 21° 24' and 14°, the middle station being 84 feet from each of the others. Required the height of the object. 53·964 feet.

PROBLEM XIII.

There are three towns A, B, and C, whose distance apart are as follow: from A to B six miles; from A to C, 22 miles; and from B to C, 20 miles. A messenger is despatched from B to A, and has to call at a town D in a direct line between A and C. Now in travelling from B to D, he walks uniformly at the rate of 4 miles an hour, and from D to A at the rate of 3 miles an hour. Supposing him to perform his journey in 3 hours, it is required to determine the position of the town D.

The distance of D from A is 4·72 miles.

The student who has the practical applications of Plane Trigonometry more immediately in view, may pass over the following chapter, on the theory of the trigonometrical lines, and proceed to the first chapter of part III., which contains the application of Trigonometry to Navigation

CHAPTER IV.

INVESTIGATION OF TRIGONOMETRICAL FORMULAS.

(26.) The formulas hitherto investigated are those only which are immediately connected with the business of plane trigonometry, properly so called, that is, with the solutions of the several cases of plane triangles. Having disposed of all these cases, we shall now proceed to develop the theory of the trigonometrical lines more at large, dismissing all considerations of the sides of triangles.

The following general expressions have already been established, viz.

$$\left.\begin{array}{l}\sin.(A+B)=\sin.A\cos.B+\sin.B\cos.A\\ \sin.(A-B)=\sin.A\cos.B-\sin.B\cos.A\end{array}\right\}\ ..\ (1).$$

$$\left.\begin{array}{l}\cos.(A+B)=\cos.A\cos.B-\sin.A\sin.B\\ \cos.(A-B)=\cos.A\cos.B+\sin.A\sin.B\end{array}\right\}\ ..\ (2).$$

From these equations we get

1. By addition,

$$\left.\begin{array}{l}\sin.(A+B)+\sin.(A-B)=2\sin.A\cos.B\\ \cos.(A+B)+\cos.(A-B)=2\cos.A\cos.B\end{array}\right\}\ ..\ (3).$$

2. By subtraction,

$$\left.\begin{array}{l}\sin.(A+B)-\sin.(A-B)=2\cos.A\sin.B\\ \cos.(A-B)-\cos.(A+B)=2\sin.A\sin.B\end{array}\right\}\ ..\ (4).$$

It is worth while to remark here that if we make $A=60°$, then since $\cos.60°$ A $\frac{1}{2}$, (p. 14,) the first of these formulas furnish the equation

$$\sin.B=\sin.(60°+B)-\sin(60°-B)\ ..\ (V);$$

which is a useful expression in the work of computing tables.

3. By multiplication,

$$\sin.(A+B)\sin.(A-B)=\sin.^2A\cos.^2B-\sin.^2B\cos.^2A$$
$$\cos(A+B)\cos.(A-B)=\cos.^2A\cos.^2B-\sin.^2A\sin.^2B.$$

Or eliminating $\cos.^2A$, $\cos.^2B$, from each of these equations by means of the conditions $\sin.^2A+\cos.^2A=1$; $\sin.^2B+\cos.^2B=1$; the second members of them become, respectively,

$\sin.^2A-\sin.^2A\sin.^2B-\sin.^2B+\sin.^2B\sin.^2A$, or $\sin.^2A-\sin.^2B$; and,

$1-\sin.^2B-\sin.^2A+\sin.^2A\sin.^2B-\sin.^2A\sin.^2B$, or $\cos.^2B\sin.^2A$; so that

$$\left.\begin{array}{l}\sin.(A+B)\sin(A-B)=\sin.^2A-\sin^2B=(\sin A+\sin.B)(\sin A-\sin.B)\\ \cos(A+B)\cos(A-B)=\cos^2B-\sin^2A=(\cos.B+\sin.A)(\cos.B-\sin A)\end{array}\right\}(5$$

4. By division, $\dfrac{\sin.(A+B)}{\sin.(A-B)}=\dfrac{\sin.A\cos.B+\sin.B\cos.A}{\sin.A\cos.B-\sin.B\cos.A}$

$$\frac{\cos.(A+B)}{\cos.(A-B)}=\frac{\cos.A\cos.B-\sin.A\sin.B}{\cos.A\cos.B+\sin.A\sin.B}$$

$$\frac{\sin.(A\pm B)}{\cos.(A\pm B)}=\frac{\sin.A\cos.B\pm\sin.B\cos.A}{\cos.A\cos.B\mp\sin.A\sin.B}$$

The right hand number of these equations will assume other useful forms by dividing both numerator and denominator of each by certain expressions: thus, let the divisors for the first equation be

$$\text{cos. A cos. B, sin. A sin. B, sin. A cos. B;}$$

those for the second, cos. A sin. B, sin. A cos. B, cos. A cos. B; and those for the third the same as those for the first; we shall then have

$$\left.\begin{aligned}
&\frac{\text{sin.}(A+B)}{\text{sin.}(A-B)}=\frac{\text{tan. A}+\text{tan. B}}{\text{tan. A}-\text{tan. B}}=\frac{\text{cot. B}+\text{cot. A}}{\text{cot. B}-\text{cot. A}}=\frac{1+\text{cot. A tan. B}}{1-\text{cot. A tan. B}}\\
&\frac{\text{cos.}(A+B)}{\text{cos.}(A-B)}=\frac{\text{cot. B}-\text{tan. A}}{\text{cot. B}+\text{tan. A}}=\frac{\text{cot. A}-\text{tan. B}}{\text{cot. A}+\text{tan. B}}=\frac{1-\text{tan. A tan. B}}{1+\text{tan. A tan B}}\\
&\frac{\text{sin.}(A\pm B)}{\text{cos.}(A\pm B)}=\frac{\text{tan. A}\pm\text{tan. B}}{1\mp\text{tan. A tan. B}}=\frac{\text{cot. B}\pm\text{cot. A}}{\text{cot. A cot. B}\mp 1}=\frac{1\pm\text{cot. A tan. B}}{\text{cot. A}\mp\text{tan. B}}
\end{aligned}\right\}(6).$$

The last of these immediately gives

$$\left.\begin{aligned}
&\text{tan.}(A+B)=\frac{\text{tan. A}+\text{tan. B}}{1-\text{tan. A tan. B}},\ \text{tan.}(A-B)=\frac{\text{tan. A}-\text{tan. B}}{1+\text{tan. A tan. B}}\\
&\text{cot.}(A+B)=\frac{\text{cot. A cot. B}-1}{\text{cot. B}+\text{cot. A}},\ \text{cot.}(A-B)=\frac{\text{cot. A cot. B}+1}{\text{cot. B}-\text{cot. A}}
\end{aligned}\right\}(7).$$

If $A = 45^\circ$, then tan. A = cot. A = 1, therefore,

$$\text{tan.}(45^\circ+B)=\frac{1+\text{tan. B}}{1-\text{tan. B}},\ \text{tan.}(45^\circ-B)=\frac{1-\text{tan. B}}{1+\text{tan. B}}$$

$$\text{cot.}(45^\circ+B)=\frac{\text{cot. B}-1}{\text{cot. B}+1},\ \text{cot.}(45^\circ-B)=\frac{\text{cot. B}+1}{\text{cot. B}-1}$$

$$\therefore \text{tan.}(45^\circ+B)-\text{tan.}(45^\circ-B)=\frac{4\ \text{tan. B}}{1-\text{tan.}^2\text{B}}\quad . \; . \; . \quad (8)$$

$$\text{cot.}(45^\circ-B)-\text{cot.}(45^\circ+B)=\frac{4\ \text{cot. B}}{\text{cot.}^2\text{B}-1}\quad . \; . \; . \quad (9).$$

Such are the most useful theorems respecting the sums and differences of two unequal arcs, and they may be converted into other expressions involving three or more arcs by simply substituting $B + C + D + \&c$ for B. We shall briefly consider the case of three arcs, or angles, because of a curious property belonging to them whenever they make up either 180° or 90°.

Let A, B, C, be any three arcs, and consider A + B as one, then by equa. (1)

$$\begin{aligned}
\text{sin.}(A+B+C)&=\text{sin.}(A+B)\ \text{cos. C}+\text{cos.}(A+B)\ \text{sin. C}\\
&=(\text{sin. A cos. B}+\text{cos. A sin. B})\ \text{cos. C}+\\
&\quad(\text{cos. A cos. B}-\text{sin. A sin. B})\ \text{sin. C},\\
\text{cos.}(A+B+C)&=\text{cos.}(A+B)\text{cos. C}-\text{sin.}(A+B)\text{sin. C}\\
&=(\text{cos. A cos. B}-\text{sin. A sin B})\text{cos. C}-\\
&\quad(\text{sin. A cos. B}+\text{cos. A sin. B})\ \text{sin. C}.
\end{aligned}$$

Let now the sum of the three arcs be 180°, or, indeed, any multiple of 180°, then the sine of this sum will be 0, so that the first of these equations gives sin. A cos B cos. C + cos. A sin. B cos. C +

$$\text{cos. A cos. B sin. C} = \text{sin. A sin. B sin. C};$$

dividing both sides of this equation by cos. A cos. B cos. C, we have

$$\frac{\text{sin. A}}{\text{cos. A}}+\frac{\text{sin. B}}{\text{cos. B}}+\frac{\text{sin. C}}{\text{cos. C}}=\frac{\text{sin. A}}{\text{cos. A}}\cdot\frac{\text{sin. B}}{\text{cos. B}}\cdot\frac{\text{sin. C}}{\text{cos. C}};$$

that is, tan. A + tan. B + tan. C = tan. A tan. B tan. C; a remarkable property of the angles of a plane triangle.

Again, let the sum of the three arcs be 90°, or any multiple thereof, then the cosine of this sum will be 0, so that the second general equation above becomes cos. A cos. B cos. C = [illegible] A [illegible] B cos. C +

$$\text{sin. A cos. B sin. C}+\text{cos. A sin. B sin. C};$$

dividing both sides by sin. A sin. B sin. C, we have

$$\text{cot. A cot. B cot. C}=\text{cot. A}+\text{cot. B}+\text{cot. C}.$$

(28.) To deduce formulas for multiple arcs we have only to put nA for $A + B$ in the preceding expressions. We thus get from (1)

$$\sin. nA = \sin. A \cos. (n-1) A + \sin. (n-1) A \cos. A$$
$$\cos. nA = \cos. A \cos. (n-1) A - \sin. A \sin. (n-1) A;$$

so that putting for n, 1, 2, 3, &c. successively, we have

$$\left.\begin{array}{l} \sin. \ A = \sin. A \\ \sin. 2A = 2 \sin. A \cos. A \\ \sin. 3A = \sin. A \cos. 2A + \sin. 2A \cos. A \\ \sin. 4A = \sin. A \cos. 3A + \sin. 3A \cos. A \\ \quad \&c. \qquad\qquad \&c. \end{array}\right\} \quad . \ . \ . \quad (10)$$

$$\left.\begin{array}{l} \cos. \ A = \cos. A \\ \cos. 2A = \cos.^2 A - \sin.^2 A \\ \cos. 3A = \cos. A \cos. 3A - \sin. A \sin. 2A \\ \cos. 4A = \cos. A \cos. 3A - \sin. A \sin. 3A \\ \quad \&c. \qquad\qquad \&c. \end{array}\right\} \quad . \ . \ . \quad (11).$$

We may put the general expressions for sin. nA, and cos. nA, under a different form, by making use of the second equation in (1) and (2), thus putting $(n-1)A$ for A, and A for B, these become

$$\sin. (n-2) A = \sin. (n-1) A \cos. A - \sin. A \cos. (n-1) A$$
$$\cos. (n-2) A = \cos. (n-1) A \cos. A + \sin. (n-1) A \sin. A;$$

or, by transposing,

$$0 = - \sin. A \cos. (n-1) A + \sin. (n-1) A \cos. A - \sin. (n-2) A$$
$$0 = + \cos. A \cos. (n-1) A + \sin. A \sin. (n-1) A - \cos. (n-2) A;$$

adding these two equations to those above, there results

$$\left.\begin{array}{l} \sin. nA = 2 \sin. (n-1) A \cos. A - \sin. (n-2) A \\ \cos. nA = 2 \cos. (n-1) A \cos. A - \cos. (n-2) A \end{array}\right\} \quad . \ . \ . \quad (12);$$

hence,

$$\left.\begin{array}{l} \sin. \ A = \sin. A \\ \sin. 2A = 2 \sin. A \cos. A \\ \sin. 3A = 2 \sin. 2A \cos. A - \sin. A \\ \sin. 4A = 2 \sin. 3A \cos. A - \sin. 2A \\ \quad \&c. \qquad\qquad \&c. \end{array}\right\} \quad . \ . \ . \quad (13)$$

$$\left.\begin{array}{l} \cos. \ A = \cos. A \\ \cos. 2A = 2 \cos. A \cos. A - 1 \\ \cos. 3A = 2 \cos. 2A \cos. A - \cos. A \\ \cos. 4A = 2 \cos. 3A \cos. 2A - \cos. 2A \\ \quad \&c. \qquad\qquad \&c. \end{array}\right\} \quad . \ . \ . \quad (14).$$

(29.) The sines and cosines of multiple arcs may also be developed in terms of the powers of the sine and cosine of the simple arc, by help of a remarkable formula, known by the name of De Moivre's formula, which may be easily established, as follows.

Multiply together the two expressions,

$$\cos. A + \sin. A . \sqrt{-1} \text{ and } \cos. A_1 + \sin. A_1 . \sqrt{-1};$$

and we shall have the product, $\cos. A \cos. A_1 - \sin. A \sin. A_1 +$ $(\cos. A \sin. A_1 + \sin. A \cos. A_1) \sqrt{-1}$; which, by the equations (1), (2), is the same as, $\cos. (A + A_1) + \sin. (A + A_1) \sqrt{-1}$; which is of the same form as the original factors, consequently, multiplying this by the new factor, $\cos. A_2 + \sin A_2 . \sqrt{-1}$, we must have for the product $\cos. (A + A_1 + A_2) + \sin. (A + A_1 + A_2) \sqrt{-1}$, and thus by continually introducing a new factor, we must have generally

$$(\cos. A + \sin. A . \sqrt{-1}) (\cos. A_1 + \sin. A_1 . \sqrt{-1}) (\cos. A_2 + \sin. A_2 . \sqrt{-1}) \&c. =$$
$$\cos. (A + A_1 + A_2 + \&c.) + \sin. (A + A_1 + A_2 + \&c.) \sqrt{-1}.$$

Suppose now that $A = A_1 = A_2 = \&c.$ then this equation will become

$$(\cos. A + \sin. A . \sqrt{-1})^n = \cos. nA + \sin. nA . \sqrt{-1}$$

or, writing the radical with the double sign,

$$(\cos. A \pm \sin. A . \sqrt{-1})^n = \cos. nA \pm \sin. nA . \sqrt{-1} \quad . . (15);$$

n is here a whole number, but, in order to show that the formula holds when the exponent is a fraction, put $a = \frac{n}{m} A$; then by this formula,

$$(\cos. a \pm \sin. a . \sqrt{-1})^m = \cos. ma \pm \sin. ma . \sqrt{-1} =$$

$\cos. nA \pm \sin. nA . \sqrt{-1} = (\cos. A \pm \sin. A . \sqrt{-1})^n$; therefore, extracting the mth root of the first and last members, restoring the value of a, we have, $\cos. \frac{n}{m} A \pm \sin. \frac{n}{m} A . \sqrt{-1} = (\cos. A \pm \sin. A . \sqrt{-1})^{\frac{n}{m}}$ . . (16); which is the formula of *De Moivre*.

a

If we take the reciprocal of each side of this question we shall have

$$\frac{1}{\cos. \frac{n}{m} A \pm \sin. \frac{n}{m} A . \sqrt{-1}} = (\cos. A \pm \sin. A . \sqrt{-1})^{-\frac{n}{m}}$$

and if we multiply both numerator and denominator of the first member of this by $\cos. \frac{n}{m} A \mp \sin. \frac{n}{m} A . \sqrt{-1}$, the denominator will then become $\cos.^2 \frac{n}{m} A + \sin.^2 \frac{n}{m} A = 1$; hence

$$\cos. \frac{n}{m} A \mp \sin. \frac{n}{m} A . \sqrt{-1} = (\cos. A \pm \sin. A . \sqrt{-1})^{-\frac{n}{m}} \quad (17);$$

so that the formula (16) remains true, whether $\frac{n}{m}$ be positive or negative. If in (16) we make $\frac{n}{m}$ negative, the signs $\pm$, in the first member, will be inverted as here, because the sign of the sine is the same as that of the arc.

It may seem to the student that there is a want of generality in the first members of (16) and (17), which ought to contain m values, seeing that the mth root appears in the second members. But this defect is only apparent; for it must be remembered that while the lines sin. A, cos. A, in the second member have each a certain fixed value, the arcs A, to which these lines indifferently belong are innumerable. The first member involves a proposed fractional part, not of any particular one of these arcs, but of any one of them indifferently; it is easy to see, therefore, that the first member involves a variety of values, and they may be shown to be in number m.

We are to show here that in formula to De Moivre, viz.

$$\cos. \frac{n}{m} A \pm \sin. \frac{n}{m} A \cdot \sqrt{-1} = (\cos. A \pm \sin. A . \sqrt{-1})^{\frac{n}{m}}$$

the first member has m values as well as the second. This fact we shall easily establish, by means of the property adverted to in the text, viz. that to any given values of the lines sin. A, cos. A, there correspond innumerable different arcs, viz. every arc in the infinite series,

$$A,\ 2\pi + A,\ 4\pi + A\ 6\pi + A, \text{ \&c.}$$

so that the first member of the above formula involves in it the following values, viz. $\cos. \frac{n}{m} A \pm \sin. \frac{n}{m} A . \sqrt{-1}$

$$\cos. \frac{n}{m} (2\pi + A) \pm \sin. \frac{n}{m} (2\pi + A) . \sqrt{-1}$$

[illegible] here 2π is supposed to be incommensurable,

$$\cos.\ \frac{n}{m}(4\pi + \text{A}) \pm \sin.\ \frac{n}{m}(2\pi + \text{A})\ .\ \sqrt{-1}$$

$$\cos.\ \frac{n}{m}(6\pi + \text{A}) \pm \sin.\ \frac{n}{m}(6\pi + \text{A})\ .\ \sqrt{-1}$$

&c. &c.

These values will continue different till we arrive at such a value, N, for one of the numerical coefficients, 0, 2, 4, 6, &c. as will render $\frac{n}{m}\text{N}\pi$ a multiple of 2π, when the first of the foregoing values will obviously recur, so that by continuing the series we shall merely obtain a repetition of the former values. Now $\frac{n}{m}\text{N}\pi$ cannot become a multiple of 2π till N become equal to $2m$; hence we shall have expressed all the different values involved in the first member of De Moivre's formula, when we have continued the above series of values as far as that in which the numeral coefficient is $2m - 2$; that is when we have written m values. Hence each member of the formula involves m different values.

(30.) Let the first side of (15) be developed by the binomial theorem and the equation will become $\cos.^{n}\text{A} \pm n \cos.^{n-1}\text{A}p + \frac{n(n-1)}{2}\cos.^{n-2}\text{A}p^{2} \pm \text{\&c.} = \cos.\ n\text{A} \pm \sin.\ n\text{A}\ .\ \sqrt{-1}$; p being put for the imaginary $\sin.\ \text{A}\ .\ \sqrt{-1}$.

Now as in any equation the imaginaries on one side are together equal to those on the other, (Alg. p. 88,) we have by expunging all the imaginaries on both sides, the following expression for cos. n A, viz.

$$\cos.\ n\text{A} = \cos.^{n}\text{A} - \frac{n(n-1)}{2}\cos.^{n-2}\text{A}\sin.^{2}\text{A} + \frac{n(n-1)(n-2)(n-3)}{2\cdot 3\cdot 4}\cos.^{n-4}\text{A}\sin.^{4}\text{A} - \text{\&c.}$$

In the like manner by expunging all the rational terms on each side of the same equation, and then dividing by $\sqrt{-1}$, there results for [illegible]

$$\sin.\ n\text{A} = n\cos.^{n-1}\text{A}\sin.\ \text{A} - \frac{n(n-1)(n-2)}{2\cdot 3.}\cos.^{n-2}\text{A}\sin.^{3}\text{A} + \text{\&c.}$$

From these two expressions may be obtained series for the value of the sine and cosine of an arc in terms of the arc itself.

For let $n = \frac{1}{0}$, and $\sin.\ \text{A} = 0 = \text{A}$, then $n\text{A} = \frac{0}{0}$ = any finite quantity x; hence by these substitutions the foregoing series become

$$\cos.\ x = 1 - \frac{x^{2}}{1\cdot 2} + \frac{x^{4}}{1\cdot 2\cdot 3\cdot 4} - \text{\&c.}$$

$$\sin.\ x = x - \frac{x^{3}}{1\cdot 2\cdot 3} + \frac{x^{5}}{1\cdot 2\cdot 3\cdot 4\cdot 5} - \text{\&c.}$$

by means of which we may calculate the values of the sine and cosine of any arc x, in parts of the radius or linear unit, when we know the length of x itself, according to the same scale. The length of any arc in parts of the radius is easily ascertained from the known value of 180° or of a semicircle, in those parts, which by putting π for the semicircumference to radius 1, is (see Geom. p. 139) $\pi = 3{\cdot}14159265358979$, &c. so that the length of an arc x degrees is $\frac{x}{180}\ .\ \pi = \frac{x}{90}\ .\ \frac{\pi}{2}$.

As in calculating the sines and cosines x may be always taken less than

90, it follows that $\frac{x}{90}$ will be a decimal fraction; if we call this m we may write the foregoing series thus,

$$\sin. (m \cdot 90^\circ) = m\frac{\pi}{2} - \frac{(\frac{1}{2}\pi)^3}{1\cdot2\cdot3}m^3 + \frac{(\frac{1}{2}\pi)^5}{1\cdot2\cdot3\cdot4\cdot5}m^5 - \&c.$$

$$\cos. (m \cdot 90^\circ) = 1 - \frac{(\frac{1}{2}\pi)^2}{1\cdot2}m^2 + \frac{(\frac{1}{2}\pi)^4}{1\cdot2\cdot3\cdot4}m^4 - \&c.$$

which series are now in a form suited to immediate calculation.

Suppose, for example, the sine and cosine of 1′ are required, then,

$$m = \frac{1}{90 \times 60} \therefore m\frac{\pi}{2} = \cdot0002908882, \&c, \therefore \sin. 1' = \cdot0002908882,$$

$$\&c. - \frac{1}{1\cdot2\cdot3}(\cdot0002908882, \&c.)^3 + \&c. = \cdot0002908882, \&c.$$

$$\cos. 1' = 1 - \frac{1}{1\cdot2}(\cdot0002908882, \&c.)^2 \&c. = \cdot9999999577, \&c.$$

and from knowing the value of sin. 1′ and cos. 1′ we might compute the sines and cosines for every minute in the quadrant, by means of the formula (3), which when B = 1′, becomes

$$\sin. (A + 1') = 2 \sin. A \cos. 1' - \sin. (A - 1'),$$

in which A is to be made successively equal to 1′, 2′, 3′, &c. But we shall not enter into the details of this computation here, our present object being to deduce formulas for the sines, cosines, &c. of multiple arcs.

From the general expressions already given for sin. nA, and cos. nA, those for tan. nA, cot. nA, &c. may be readily obtained by help of the equations at (9); we shall not, therefore, occupy the space by writing them down, but confine ourselves throughout the remainder of this article entirely to the consideration of double arcs, as formulas for these are in much more frequent request than for any higher multiple. The formulas of which we speak, may, of course, all be deduced from the general expressions investigated in the beginning of this article, but, for the sake of simplicity, we shall go nearer the first principles, and deduce them from the expressions in art. (26).

Referring to the equations (1), (2), art. (26), we have when A = B,

$$\sin. 2\,A = 2 \sin. A \cos. A \;\ldots\ldots\; (18)$$

$$\cos. 2\,A = \cos.^2 A - \sin.^2 A, \text{ or } \cos. 2\,A = 2\cos.^2 A - 1,$$

$$\text{or } \cos. 2\,A = 1 - 2\sin.^2 A \;\ldots\ldots\; (19);$$

and from the last two of these we immediately get

$$\cos. A = \sqrt{\tfrac{1}{2} + \tfrac{1}{2}\cos. 2\,A}, \sin. A = \sqrt{\tfrac{1}{2} - \tfrac{1}{2}\cos. 2\,A} \ldots\ldots (20);$$

and, therefore, by division,

$$\tan. A = \sqrt{\frac{1 - \cos. 2\,A}{1 + \cos. 2\,A}}, \cot. A = \sqrt{\frac{1 + \cos. 2\,A}{1 - \cos. 2\,A}} \;\ldots\ldots\; (21);$$

from which we get two new expressions for cos. 2 A, viz.

$$\cos. 2\,A = \frac{1 - \tan.^2 A}{1 + \tan.^2 A} = \frac{\cot.^2 A - 1}{\cot.^2 A + 1} \;\ldots\ldots\; (22).$$

If instead of A we write 45° — A, then since cos. (90° — 2 A) = sin. 2 A, we have

$$\sin. 2\,A = \frac{1 - \tan.^2 (45^\circ - A)}{1 + \tan.^2 (45^\circ - A)} = \frac{\cot.^2 (45^\circ - A) - 1}{\cot.^2 (45^\circ - A) + 1}.$$

It may be worth while to remark that the radical in the above expressions for tan. A, cot. A, may be removed by multiplying the numerator and denominator of each fraction by its numerator: we thus have

$$\tan. A = \frac{1 - \cos. 2\,A}{\sin. 2\,A}, \cot. A = \frac{1 + \cos. 2\,A}{\sin. 2\,A}.$$

For the tangent and cotangent of a double arc we have, by division, (18), (19), $\frac{\text{sin. } 2\text{ A}}{\text{cos. } 2\text{ A}} = \frac{2 \text{ sin. A cos. A}}{\text{cos.}^2 \text{ A} - \text{sin.}^2 \text{ A}}$; that is, dividing numerator and denominator of the second member by $\text{cos.}^2\text{A}$, or by $\text{sin.}^2\text{A}$, and recollecting that $\frac{\text{sin.}}{\text{cos.}} = \text{tan.}$, and that $\frac{1}{\text{tan.}} = \text{cot.}$, we have

$$\left.\begin{array}{l} \text{tan. } 2\text{ A} = \dfrac{2 \text{ tan. A}}{1 - \text{tan.}^2 \text{ A}} = \dfrac{2 \text{ cot. A}}{\text{cot.}^2 \text{ A} - 1} = \dfrac{2}{\text{cot.A} - \text{tan. A}} \\ \text{cot. } 2\text{ A} = \dfrac{1 - \text{tan.}^2 \text{ A}}{2 \text{ tan. A}} = \dfrac{\text{cot.}^2 \text{ A} - 1}{2 \text{ cot. A}} = \tfrac{1}{2}(\text{cot. A} - \text{tan. A}); \end{array}\right\} (23);$$

which expressions also immediately come from the values of tan. (A + B), cot. (A + B), at (26), by putting A = B. Comparing the above value of tan. 2A with the expression (8), art. (26), we have, 2 tan. 2 A = tan. (45° + A) — tan. (45° — A); or which is the same thing,

$$2 \text{ tan. A} = \text{tan. }(45^\circ + \tfrac{1}{2}\text{ A}) - \text{tan. }(45^\circ - \tfrac{1}{2}\text{ A}) \;....\; (\text{V}).$$

Formulas for the secants and cosecants of double arcs are easily deduced from those for the cosine and sine, because

$\text{sec.} = \frac{1}{\text{cos.}}$, and $\text{cosec.} = \frac{1}{\text{sin.}}$, thus, from equation (22) above,

we have $\text{sec. } 2\text{ A} = \frac{1 + \text{tan.}^2 \text{ A}}{1 - \text{tan.}^2 \text{A}} = \frac{\text{sec.}^2 \text{ A}}{2 - \text{sec.}^2 \text{ A}}$; and. from equation (18),

$$\text{cosec. } 2\text{ A} = \frac{1}{2 \text{ sin. A cos. A}} = \tfrac{1}{2} \text{ sec. A cosec. A.}$$

(31.) Another useful class of formulas are those for half arcs; they may be easily deduced from the expressions for the double arcs; thus putting ½ A for A, we have from (20).

$$\text{sin. } \tfrac{1}{2}\text{ A} = \sqrt{\tfrac{1}{2} - \tfrac{1}{2} \text{ cos. A}}, \text{cos. } \tfrac{1}{2}\text{ A} = \sqrt{\tfrac{1}{2} + \tfrac{1}{2} \text{ cos. A}} \;....\; (24);$$

also from (21),

$$\left.\begin{array}{l} \text{tan. } \tfrac{1}{2}\text{ A} \sqrt{\dfrac{1 - \text{cos. A}}{1 + \text{cos. A}}} = \dfrac{1 - \text{cos. A}}{\text{sin. A}} \\ \text{cot. } \tfrac{1}{2}\text{ A} = \sqrt{\dfrac{1 + \text{cos. A}}{1 - \text{cos. A}}} = \dfrac{1 + \text{cos. A}}{\text{sin. A}} \end{array}\right\} .. (25).$$

Other useful values of sin. ½ A, and cos. ½ A, are derivable from the equation (18) last article, for when ½ A is put for A the equation is $\text{sin. A} = 2 \text{ sin. } \tfrac{1}{2}\text{ A cos. } \tfrac{1}{2}\text{ A}$ .... (26), and if this be either added to or subtracted from $1 = \text{sin. }^2 \tfrac{1}{2}\text{ A} + \text{cos. }^2 \tfrac{1}{2}\text{ A}$,
the second member will become in each case a perfect square, viz.

$$1 + \text{sin. A} = (\text{sin. } \tfrac{1}{2}\text{ A} + \text{cos. } \tfrac{1}{2}\text{ A})^2$$
$$1 - \text{sin. A} = (\text{sin. } \tfrac{1}{2}\text{ A} - \text{cos. } \tfrac{1}{2}\text{ A})^2;$$

hence,

$$\sqrt{1 + \text{sin. A}} = \text{sin. } \tfrac{1}{2}\text{ A} + \text{cos. } \tfrac{1}{2}\text{ A}$$
$$\sqrt{1 - \text{sin. A}} = \text{sin. } \tfrac{1}{2}\text{ A} - \text{cos. } \tfrac{1}{2}\text{ A}.$$

Let A be less then 90°, then the radical must be taken positive in the first, and negative in the second expression; hence, by addition and subtraction,

$$\left.\begin{array}{l} \text{sin. } \tfrac{1}{2}\text{ A} = \tfrac{1}{2}(\sqrt{1 + \text{sin. A}} - \sqrt{1 - \text{sin A}}) \\ \text{cos. } \tfrac{1}{2}\text{ A} = \tfrac{1}{2}(\sqrt{1 + \text{sin. A}} + \sqrt{1 - \text{sin. A}}) \end{array}\right\} (\text{V}).$$

By means of these two expressions the accuracy of a table of sines and cosines may be examined; that is to say, from the calculated values sin. A, in the table, we may compute, by these equations, the values of sin. ½ A, and of cos. ½ A; if these agree with the tabular values, found by other means, we may conclude that the tables are correct in the parts thus verified. Formulas employed in this manner to put the accuracy of the tables to the test are called *formulas of verification*. We have given three of these, and marked them with the letter (V).

(32.) The following formulas involving the half sums and half difference of two arcs are of frequent application: substitute ½ (A + B) for A and ½ (A — B) for B, in the equations (3), (4), at art. (26) and we have

$$\left.\begin{aligned}\sin. A + \sin. B &= 2\sin. \tfrac{1}{2}(A+B)\cos. \tfrac{1}{2}(A-B)\\ \cos. A + \cos. B &= 2\cos. \tfrac{1}{2}(A+B)\cos. \tfrac{1}{2}(A-B)\\ \sin. A - \sin. B &= 2\cos. \tfrac{1}{2}(A+B)\sin. \tfrac{1}{2}(A-B)\\ \cos. B - \cos. A &= 2\sin. \tfrac{1}{2}(A+B)\sin. \tfrac{1}{2}(A-B)\end{aligned}\right\} \quad . \; . \; . \quad (27);$$

and from these we get, by division,

$$\frac{\sin. A + \sin. B}{\cos. A + \cos. B} = \tan. \tfrac{1}{2}(A+B); \quad \frac{\sin. A - \sin. B}{\cos. B. - \cos. A} = \cot. \tfrac{1}{2}(A+B).(28)$$

$$\frac{\sin. A - \sin. B}{\cos. A + \cos. B} = \tan. \tfrac{1}{2}(A-B); \quad \frac{\sin. A + \sin. B}{\cos. B - \cos. A} = \cot. \tfrac{1}{2}(A-B).(29).$$

In each of these expressions let A = 90°, and we shall have

$$1+\sin. B = 2\sin. (45^\circ + \tfrac{1}{2}B)\cos. (45^\circ - \tfrac{1}{2}B) = 2\sin.^2(45^\circ + \tfrac{1}{2}B)$$

$$\cos. B. = 2\cos. (45^\circ + \tfrac{1}{2}B)\cos. (45^\circ - \tfrac{1}{2}B) = 2\cos.^2\tfrac{1}{2}B - 1, \text{ by eq. } 18$$

$$\begin{aligned}1-\sin. B &= 2\cos. (45^\circ + \tfrac{1}{2}B)\sin. (45^\circ - \tfrac{1}{2}B) = 2\cos.^2(45^\circ + \tfrac{1}{2}B)\\ &= 2\sin.^2(45^\circ - \tfrac{1}{2}B)\end{aligned}$$

$$1-\cos. B = 2\sin. (45^\circ + \tfrac{1}{2}B)\sin. (45^\circ - \tfrac{1}{2}B) = 2\sin.^2\tfrac{1}{2}B, \text{ by eq. } 19,$$

$$\left.\begin{aligned}\frac{1+\sin. B}{\cos. B} &= \tan. (45^\circ + \tfrac{1}{2}B)\\ &= \cancel{\cot. (45^\circ - \tfrac{1}{2}B)},\end{aligned}\right\} \left.\begin{aligned}\frac{1-\sin. B}{\cos. B} &= \cot. (45^\circ + \tfrac{1}{2}B)\\ &= \tan. (45^\circ - \tfrac{1}{2}B)\end{aligned}\right\}$$

$$\frac{1+\sin. B}{1-\sin. B} = \tan.^2 (45^\circ + \tfrac{1}{2}B), \quad \frac{1+\cos. B}{1-\cos. B} = \cot.^2 \tfrac{1}{2}B.$$

Again, dividing (28) by (29,) we have

$$\frac{\sin. A + \sin. B}{\sin. A - \sin. B} = \frac{\tan. \tfrac{1}{2}(A+B)}{\tan. \tfrac{1}{2}(A-B)}; \quad \frac{\sin. A - \sin. B}{\sin. A + \sin. B} = \frac{\cot. \tfrac{1}{2}(A+B)}{\cot. \tfrac{1}{2}(A-B)} \; (30).$$

Lastly, substituting A + B for A in (26) last article, we have $\sin. (A+B) = 2\sin. \tfrac{1}{2}(A+B)\cos. \tfrac{1}{2}(A+B)$; and dividing this by each of the formulas (27) in succession, there results

$$\frac{\sin. (A+B)}{\sin. A + \sin. B} = \frac{\cos. \tfrac{1}{2}(A+B)}{\cos. \tfrac{1}{2}(A-B)}; \quad \frac{\sin. (A+B)}{\cos. A + \cos. B} = \frac{\sin. \tfrac{1}{2}(A+B)}{\cos. \tfrac{1}{2}(A-B)}$$

$$\frac{\sin. (A+B)}{\sin. A - \sin. B} = \frac{\sin. \tfrac{1}{2}(A+B)}{\sin. \tfrac{1}{2}(A-B)}; \quad \frac{\sin. (A+B)}{\cos. B - \cos. A} = \frac{\cos. \tfrac{1}{2}(A+B)}{\sin. \tfrac{1}{2}(A-B)}.$$

(33.) We shall conclude this chapter on the theory of the trigonometrical lines, with two curious and useful propositions.

1. *To express the sine and cosine of a real arc by means of imaginary exponentials.* By the *exponential theorem*,*

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{2\cdot 3} + \frac{x^4}{2\cdot 3\cdot 4} +$ &c. where e represents the base of the Naperian logarithms, that is, $e = 2{\cdot}7182818$, &c. For x substitute $x\sqrt{-1}$, and $-x\sqrt{-1}$ successively, and we have these developments

$$e^{x\sqrt{-1}} = 1 + x\sqrt{-1} - \frac{x^2}{2} - \frac{x^3\sqrt{-1}}{2\cdot 3} + \frac{x^4}{2\cdot 3\cdot 4} + \text{\&c. } (1);$$

$$e^{-x\sqrt{-1}} = 1 - x\sqrt{-1} - \frac{x^2}{2} + \frac{x^3\sqrt{-1}}{2\cdot 3} + \frac{x^4}{2\cdot 3\cdot 4} - \text{\&c. } (2);$$

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* See the "Elementary Essay on the construction of Logarithms," p. 68; or Young's Algebra, just published by Carey, Lea, & Co. Philadelphia.

hence, by addition,

$$e^{x\sqrt{-1}}+e^{-x\sqrt{-1}}=2\,(1-\frac{x^2}{2}+\frac{x^4}{2\cdot3\cdot4}-\text{\&c.})$$

But by art. (30) the series on the right is the development of cos. x,

hence $\cos. x=\dfrac{e^{x\sqrt{-1}}+e^{-x\sqrt{-1}}}{2}$

By subtracting (2) from (1) we have

$$e^{x\sqrt{-1}}-e^{-x\sqrt{-1}}=2\sqrt{-1}\,(x-\frac{x^3}{2\cdot3}+\frac{x^5}{2\cdot3\cdot4\cdot5}-\text{\&c.})$$

But by art. (30) the series on the right is the development of sin. x;

hence, $\sin. x=\dfrac{e^{x\sqrt{-1}}-e^{-x\sqrt{-1}}}{2\sqrt{-1}}$.

2. *To develop* $\sin.^n x$, $\cos.^n x$, *in terms of the sine and cosine of the multiples of* x.

Put $\left.\begin{aligned}\cos. x+\sin. x.\sqrt{-1}&=u\\ \cos. x-\sin. x.\sqrt{-1}&=v\end{aligned}\right\}$ . . . , ... (1),

then art. (29), $\left.\begin{aligned}\cos. nx+\sin. nx.\sqrt{-1}&=u^n\\ \cos. nx-\sin. nx.\sqrt{-1}&=v^n\end{aligned}\right\}$ . . (2);

from which, by addition and multiplication, we get $u^n+v^n=2\cos. nx$, $u^n v^n=1\ldots(3)$. Add together the equations (1); there will result $\cos. x=\frac{1}{2}(u+v)$; and, therefore,

$\cos.^n x=\frac{1}{2^n}(u+v)^n=\frac{1}{2^n}(v+u)^n$; hence, by the binomial theorem,

$$\cos.^n x=\frac{1}{2^n}\{u^n+nu^{n-1}v+\frac{n(n-1)}{2}u^{n-2}v^2+\text{\&c.}\},$$

or, $\cos.^n x=\dfrac{1}{2^n}\{v^n+nv^{n-1}u+\dfrac{n(n-1)}{2}v^{n-2}u^2+\text{\&c.}\}$;

adding these equations together, and dividing by 2, we have

$$\cos.^n x=\frac{1}{2^{n+1}}\{u^n+v^n+nuv\,(u^{n-2}+v^{n-2})+$$
$$\frac{n(n-1)}{2}u^2v^2\,(u^{n-4}+v^{-4})+\text{\&c.}\}$$

But from (3) $u^n+v^n=2\cos. nx$ | $uv=1$
$u^{n-2}+v^{n-2}=2\cos. (n-2)\,x$ | $u^2v^2=1$
$u^{n-4}+v^{n-4}=2\cos. (n-4)\,x$ | $u^4v^4=1$
&c. &c. | &c. &c.

hence, by substitution, the developement of $\cos.^n x$ becomes

$$\cos.^n x=\frac{1}{2^n}\{\cos. nx+n\cos. (n-2)\,x+$$
$$\frac{n(n-1)}{2}\cos. (n-4)\,x+\text{\&c.}\}\quad . . . \quad (4).$$

Again, subtract the second of (1) from the first, and we have

$$2\sin. x\,.\sqrt{-1}=u-v\ \therefore \sin. x=\frac{u-v}{2\sqrt{-1}};$$

and, consequently, $\sin.^n x=\dfrac{(u-v)^n}{(2\sqrt{-1})^n}$.

1. Let n be even, then (Algebra, p. 149,) $(u-v)^n = (v-u)^n$; hence,

$$\sin.^n x = \frac{1}{(2\sqrt{-1})^n}(u-v)^n, \text{ or } \sin.^n x = \frac{1}{(2\sqrt{-1})^n}(v-u)^n;$$

and by adding these equations together after having developed $(u-v)^n$, and $(v-u)^n$, we have

$$2\sin.^n x = \frac{1}{(2\sqrt{-1})^n}\{u^n+v^n-nuv(u^{n-2}+v^{n-2})+ \frac{n(n-1)}{2}u^2v^2(u^{n-4}+v^{n-4})-\&c.\};$$

and making the same substitution as before in virtue of (3), and recollecting that, because n is even, $(\sqrt{-1})^n = \mp 1$, the upper sign having place when n is either of the numbers 2, 6, 10, &c. and the lower sign when n is either of the numbers 4, 8, 12, &c. we have for the development of $\sin.^n x$

$$\sin.^n x = \mp\frac{1}{2^n}\{\cos. nx - n\cos.(n-2)x + \frac{n(n-1)}{2}\cos.(n-4)x - \&c.\} \quad . \;.\; . \quad (5).$$

2. Let n be odd, then $(u-v)^n = (-1)^n(v-u)^n = -(v-u)^n$; #
therefore,

$$\sin.^n x = \frac{1}{(2\sqrt{-1})^n}(u-v)^n, \text{ or } \sin.^n x = -\frac{1}{(2\sqrt{-1})^n}(v-u)^n;$$

and developing $(u-v)^n$, $(v-u)^n$ as before, and taking the sum of these equations, we have

$$2\sin.^n x = \frac{1}{(2\sqrt{-1})^n}\{u^n-v^n-nuv(u^{n-2}-v^{n-2})+ \frac{n(n-1)}{2}u^2v^2(u^{n-4}-v^{n-4})-\&c.\}$$

But from the equations (2), $u^n - v^n = 2\sin. nx\sqrt{-1}$, $u^nv^n = 1$; consequently, since $(\sqrt{-1})^{n-1} = \mp 1$, the foregoing development becomes

$$\sin.^n x = \mp\frac{1}{2^n}\{\sin. nx - n\sin.(n-2)x + \frac{n(n-1)}{2}\sin.(n-4)x - \&c.\};$$

the upper sign having place when $n-1$ is either of the numbers 2, 6, 10, &c. and the lower sign having place when $n-1$ is either of the numbers, 4, 8, 12, &c. The general term of the first series of numbers is $4m+2$, that of the second series $4m$. these |||

[illegible]

[illegible]

for if u or v be negative the terms involving their odd powers must be negative

PART II.

ELEMENTS OF SPHERICAL TRIGONOMETRY.

CHAPTER I.

ON THE SPHERE.

(34.) A **Sphere** is a solid whose surface is every where equally distant from a certain point within it, called the centre. It may be generated by the revolution of a semicircle about the diameter.

Any line drawn from the centre to the surface of the sphere is called the radius; and the line through the centre having both its extremities in the surface, is the diameter.

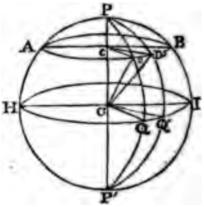
A plane surface, or simply a plane, is that in which if any two points whatever be taken, the straight line which joins them shall lie wholly in that surface.

A plane may be drawn through any three points, taken at random in space, but not through more than three; for having joined two of the proposed points by a straight line, we may pass a plane through this line in any direction, and we may turn it round upon this line till it arrives at the other point. Three points, therefore, not in the same straight line, fix the position of a plane.

It follows from this, that the common intersection of two planes must be a straight line; for, if among the points in the intersection there be three which are not in the same straight line, the two planes passing through them must coincide and form but one.

A straight line is said to be perpendicular to a plane when it is perpendicular to every straight line in that plane, drawn through its *foot*, or the point where the perpendicular meets the plane. These definitions will suffice for the purpose of establishing the necessary preliminary theorems of spherical Geometry.

(35.) *If a sphere be any how cut by a plane, the section must be a circle.*



Let C be the centre of the sphere, and ADB the plane section; draw Cc perpendicular to this plane, and from c draw any line cD in the section and terminating at the surface; then the angle CcD must be a right angle. Join CD, then wherever the point D may be, CD will always be of the same constant length, being the radius of the sphere; and in consequence of the right angle c, $c\text{D} = \sqrt{\text{CD}^2 - \text{C}c^2}$; hence CD must have the same constant length in whatever direction it be drawn; that is, the bounding line ADB is the circumference of a circle of which c is the centre.

The circle is, obviously, the larger, as it is nearer to the centre C of the sphere, or as its perpendicular distance Cc is less, because CD being constant, cD increases as Cc diminishes, and becomes the greatest possible when Cc is 0, that is, when the section passes through the centre of the sphere; hence every circle whose plane passes through the centre of the sphere is called a *great circle* of the sphere, and every other a *small circle*.

It is obvious that the circumference of a great circle may be drawn through any two points on the surface of a sphere, because a plane may be drawn through these two points and through the centre also, but a great circle cannot be drawn through three points on the surface, taken at random, because then a plane might be drawn through four points taken at random; a circle of some kind, however, may always be drawn through three points on the surface of the sphere, since a plane may be drawn through them.

The line C*c* from the centre of the sphere perpendicular to the plane of the circle passes, as we have seen, through its centre *c*; if this line be produced both ways to the surface of the sphere, the opposite points P, P′, are called the *poles* of the circle. Thus every circle on the sphere has two poles diametrically opposite, the diameter which joins them being perpendicular to the plane of the circle. The poles of a small circle are unequally distant from its plane, the inequality of distance amounting to twice C*c*; but in a great circle this inequality vanishes, and the poles are equidistant from the circle.

As the poles of any circle are at the extremities of a diameter of the sphere, an infinite number of great circles may be drawn through them; indeed, every circle passing through them will necessarily be a great circle, because the entire diameter joining them must be comprised in every plane drawn through them. The distance of any circle from either of its poles, measured upon any of these infinite number of great circles, is constantly the same, that is, the distances or arcs PB, PD′, PD, PA, &c. are equal, because the constant line P*c* is the common versed sine of all these arcs to the common radius CP; hence the other distances P′B, P′E, &c. must be equal. Every arc of a great circle is thus distant from either pole by a quadrant or 90°.

(36.) *Two great circles always intersect in two points at the distance of a semicircle from each other, that is, the circumferences bisect each other.* For as the plane of each circle passes through the centre of the sphere their intersection must be a diameter common to both circles, and it is at the extremities of this diameter that the circumferences cross each other.

From this we learn that if from any point on the sphere two quadrantal arcs can be drawn to two points in any great circle, the distance between the points being less than 180°, then the first point must be the pole of this great circle; for it is necessarily the pole of some great circle passing through the proposed points, and as only one great circle can pass through two points, which are not 180° apart, the pole must belong to the circle in question.

In spherical trigonometry, the arcs of great circles only are concerned, and the angle included between two such arcs, that is to say, a spherical angle, is measured in a manner analogous to that in which a plane angle is measured. For the measure of a plane angle we take the intercepted arc of that circle whose centre is at the vertex, and whose radius is some assumed unit: in like manner for the measure of a spherical angle we take the intercepted arc of that circle whose pole is at the vertex, and whose radius is some fixed unit, viz. the radius of the sphere on whose surface the angle is: thus, in the foregoing figure the spherical angle DPD′ is measured by the intercepted arc QQ′ of which the pole is P, and radius, CQ, that of the sphere.

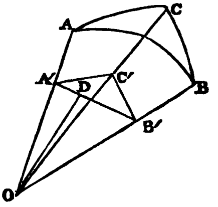
It is as easy to justify the propriety of adopting this mode of measuring spherical angles as it is to justify the method of measuring plane angles, for in both cases the intercepted arc varies as the angle; this, by the by, is true of the intercepted arc DD′ of any small circle whose pole is P, but we are compelled to refer the measure to a *great* circle, in order that all the trigonometrical lines concerned in the same inquiry may be related to a common radius, for as we have before remarked, the sides of a spherical triangle are always arcs of great circles.

From what we have just said it appears that a spherical angle DPD′ has the same measure as either of the equal plane angles QCQ′, DcD′, &c. situated in the planes of the circles whose common pole is P, and whose sides are formed by the intersection of these planes with those of the two great circles, forming the sides of the spherical angle. If at P tangents were drawn to the two great circles PD, PD′, and in their planes they would obviously include the same angle as the lines CQ, CQ′, to which they are parallel; indeed if we conceive the plane of the circle HQQ′, to move parallel to itself towards the pole, P, the path of C being along the line CP, the angle QCQ′ will successively coincide with QCQ′, DcD′, &c. till C coincides with P, when the lines CQ, CQ′, will become tangents to the circles at P, and will remain each in the plane along which it has moved; hence *the measure of the angle included between these tangents is also the measure of the spherical angle.*

(37.) If in the plane of HQI perpendiculars be drawn from C to each of the planes of the circles PQP′, PQ′P′, these will be perpendicular to the lines CQ, CQ′, and will therefore, include the same angle, which angle will be measured by the arc of HQI, which the said perpendiculars intercept; but these perpendiculars will meet the surface at the poles of the circles to whose planes they are perpendicular; hence *the great circle distance between the poles of two intersecting great circles measures their angle of intersection.*

Every great circle which passes through the poles of another is at right angles to it. Thus the great circle PDQP′, through the poles of HQQ′I, is at right angles to HQQ′I; for if a tangent were drawn to PQP′ at the point Q it would be in the same plane with and parallel to CP, and if a tangent were drawn to HQI at the point Q it would be in the same plane with and parallel to CH; hence if these two tangents were to move simultaneously to themselves, the path of their point of concourse Q being along QC, they would necessarily coincide with the perpendiculars CP, CH, when Q arrived at C: these tangents, therefore, form a right angle; hence the great circles are perpendiculars to each other, or the spherical angle at Q is a right angle.

(38.) *Any one side of a spherical triangle is less than the sum of the other two.*

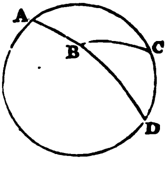


Let ABC be any spherical triangle, and O the centre of the sphere; draw the radii OA, OB, OC, then there will be about O three angles in three distinct planes respectively, measured by the arcs AB, BC, CA. Let AB be the greatest of these arcs, then it will only be necessary to show that $AB < AC + CB$, or that $AOB < AOC + BOC$. In the plane of AOB draw any line A′B′, and then draw OD, making an angle B′OD equal to BOC; make OC′ equalto OD, and join C′B′, C′A′

Then since by construction the two sides B′O, OD, and the included angle, are respectively equal to the two sides B′O, OC′, and the included angle, $B'D = B'C'$. But in the plane triangle A′B′C′, $A'B' < A'C' + B'C'$ $\therefore$ $A'D < A'C'$; hence the two sides OA′, OD, of the triangle A′OD, are equal to the two sides OA′, OC′, of the triangle A′OC′, but the third side A′D of the former is less than the third side A′C′ of the latter, and, consequently, $A'OD < A'OC'$; hence, since $B'OD$ has been made equal to $B'OC'$, it follows that

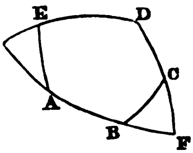
$$A'OD + B'OD = A'OB' < A'OC' + B'OC' \therefore AB < AC + CB.$$

(39.) *The sum of all the three sides of a spherical triangle is less than the circumference of a great circle.*



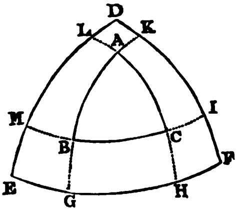
Let ABC be any spherical triangle; produce the sides AB, AC, till they meet again in D, then the arcs ABD, ACD, will be semi-circumferences, since (36,) two great circles always bisect each other. But in the triangle BCD we have $BC < BD + CD$, and, consequently, by adding $AB + AC$ to both, we shall have $AB + AC + BC < ABD + ACD$; that is to say, the sum of the three sides is less than a whole circumference.

By help of this theorem we may show that the sum of the sides of any spherical polygon whatever is less than the circumference of a great circle.



Take the spherical pentagon ABCDE for example. Produce the sides AB, DC, till they meet in F; then since $BC < BF + CF$, the perimeter of the pentagon will be less than the quadrilateral AEDF. Again, produce the sides DE, BA, till they meet in G; we shall have $EA < EG + AG$; hence the perimeter of the quadrilateral AEDF is less than that of the triangle DFG; which last is itself less than the circumference of a great circle; the perimeter of the original polygon is, therefore, less still.

(40.) *If from the three vertices of a spherical triangle, taken as poles, arcs be described, forming a new triangle, then the vertices of the new triangle will be the poles of the other triangle.*



For let ABC be any spherical triangle, and with the pole A, and circular radius AG equal to a quadrant, describe the arc EF; in like manner with the pole B and same radius describe the arc FD, meeting the former in F; and, lastly, with the pole C and same radius describe the arc ED, completing the spherical triangle DEF.

Then, because the arcs, whose poles are A and C, intersect at E, the points A, C, are each 90° distant from E; and as the arc AC is less than 180°, E must be the pole of AC (36). In like manner is it shown that F is the pole of AB, and D the pole of BC.

The triangle DEF is sometimes, from the mode of its construction, called the *polar triangle*, and the original one ABC the *primitive triangle*.

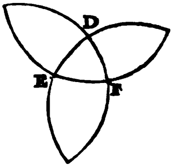
(41.) *Any angle of the primitive triangle is the supplement of the side opposite to it of the polar triangle, and any angle of the polar triangle is the supplement of the side opposite to it in the primitive triangle.*

For EH being the radius of HL is $= 90°$, and FG being the radius of GK is also $= 90°$, and the sum of these radii, namely, $EF + GH = 180°$, therefore, GH, which is the measure of the angle A, is the supplement of the side EF opposite to it. In like manner it is shown that B is the supplement of DF, and C the supplement of DE. Again, BI being the radius of ID, and CM the radius of MD, the sum of these $MI + BC = 180°$ · therefore, BC is the supplement of MI, which measures the

from any two [illegible]
[illegible] of [illegible]

angle D. On account of the property just demonstrated, the triangles ABC, DEF, are frequently called *supplemental triangles.*

It is proper to remark here, as *Legendre* has done, that besides the triangle DEF three others might be formed by the intersection of the three arcs DE, EF, DF. But the proposition immediately before us is applicable only to the central triangle, which is distinguished from the others by the circumstance that the two angles A and D (see preceding fig.) be on the same side of BC, the two B and E on the same side of AC, and the two C and F on the same side of AB.



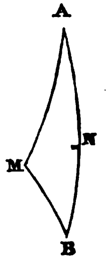
(42.) From the foregoing proposition it follows that *three angles of every spherical triangle are together greater than two right angles, and less than six.*

For the sides of the supplemental triangle DEF are together less than four right angles (39), and as these are supplements of the angles A, B, C, and therefore when added to them make six right angles, these last must together exceed two right angles. But they cannot amount to six right angles, for in that case the sum of the sides of the supplemental triangle would be 0, which is absurd. Hence, unlike plane triangles, a spherical triangle may have all its angles right angles or all obtuse angles.

(43.) The foregoing geometrical properties comprise all that we require, for the foundation of the analytical theory of spherical Trigonometry: we need not, therefore, enumerate any more. We shall, however, in conclusion, endeavour to establish the fact that *the arc of a great circle joining two points is the shortest line that can be drawn on the sphere from the one to the other.*

The following proof of this property is by *Legendre.*

Let ANB be the arc of the great circle which joins the points A and B; and without this line, if possible, let M be a point in the shortest path, between A and B. Through the point M draw MA, MB, arcs of great circles; and take $BN = MB$.

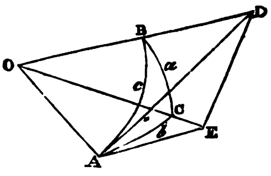


Then, by (38), the arc ANB is shorter than $AM + MB$; take $BN = BM$, respectively from both; there will remain $AN < AM$.

Now, the distance of B from M, whether it be the same with the arc BM or with any other line, is equal to the distance of B from N; for, by making the plane of the great circle BM revolve about the diameter, which passes through B, the point M may be brought into the position of the point N; and the shortest line between M and B, whatever it may be, will then be identical with that between N and B: hence the two paths from A to B, one passing through M, the other through N, have an equal part in each, the part from M to B equal to the part from N to B. The first path is the shorter by hypothesis; hence the distance from A to M must be shorter than the distance from A to N; which is absurd, the arc AM being proved greater than AN; hence no point of the shortest line from A to B can be out of the arc ANB; hence this arc is itself the shortest distance between its two extremities.

CHAPTER II.

INVESTIGATION OF FORMULAS, AND RULES FOR THE SOLUTION OF SPHERICAL TRIANGLES.



(44.) Let ABC be a triangle traced on the surface of a sphere of which the centre is O, and the radius equal to the linear unit. The angles of this triangle we shall represent by the letters at their vertices, A, B, C, and the sides opposite to them by the small letters a, b, c; so that having drawn the two tangents AD, AE, to meet the radi OB, OC, produced through the other extremities of the arcs AB, AC, we shall have

$$\left.\begin{array}{l} AD = \tan. c = \dfrac{\sin. c}{\cos. c}, AE = \tan. b = \dfrac{\sin. b}{\cos. b} \\ OD = \sec. c = \dfrac{1}{\cos. c}, OE = \sec. b = \dfrac{1}{\cos. b} \end{array}\right\} \quad . \; . \; . \quad (1).$$

Draw DE, then in the two triangles ODE, ADE, we have (17)

$$DE^2 = OE^2 + OD^2 - 2\,OE \cdot OD \cos. a$$
$$DE^2 = AE^2 + AD^2 - 2\,AE \cdot AD \cos. A;$$

recollecting that (p. 43–44) the plane angle DAE measures the spherical angle A. Substituting in these equations the values given by (1), they become $DE^2 = \sec.^2 b + \sec.^2 c - \dfrac{2 \cos. a}{\cos. b \cos. c}$

$$DE^2 = \tan.^2 b + \tan.^2 c - \frac{2 \sin. b \sin. c \cos. A}{\cos. b \cos. c} \quad \therefore \text{ by subtraction}$$

$$0 = 1 + 1 + (\sin. b \sin. c \cos. A - \cos. a) \frac{2}{\cos. b \cos. c}.$$

Hence multiplying by $\dfrac{\cos. b \cos. c}{2}$, and transposing, we have $\cos. a = \cos. b \cos. c + \sin. b \sin. c \cos A$; which is a general expression for the cosine of any side in terms of the other two sides, and their included angle. If we had taken the side b instead of a, the other two would have been a, c, and their included angle B; and if we had taken the side c the other two would have been a, b, and their included angle C; we have, therefore, the three following symmetrical equations, viz.

$$\left.\begin{array}{l} \cos. a = \cos. b \cos. c + \sin. b \sin. c \cos. A \\ \cos. b = \cos. a \cos. c + \sin. a \sin. c \cos. B \\ \cos. c = \cos. a \cos. b + \sin. a \sin. b \cos. C \end{array}\right\} \quad . \; . \; . \; . \quad (A);$$

and these equations embody the whole theory of spherical trigonometry and are sufficient to supply rules for the solution of every case.

(45.) Some interesting geometrical properties flow also from these equations.

1. Suppose two sides b, c, of the triangle are equal, that is, let it be isosceles, then it will follow from the two last of these equations that, like as in the isosceles plane triangle, *the angles opposite the equal sides will be equal.* For taking the difference of these two equations on the supposition that $b = c$; we have $0 = \sin. a \sin. b \cos. B - \sin. a \sin. b \cos. C$; and, consequently, $B = C$.

2. If $a = b = c$, then it is in a similar manner proved that $A = B = C$, that is, *every equilateral spherical triangle is equiangular.*

3. *The arc which bisects the vertical angle* A *of a spherical isosceles*

triangle also bisects the base a. For let p represent this bisecting arc, and m, m' the parts into which it divides the base, then the two spherical triangles thus formed give, by the above equations,

$$\cos. m = \cos. b \cos. p + \sin. b \sin. p \cos. \tfrac{1}{2} A$$
$$\cos. m' = \cos. a \cos. p + \sin. a \sin. p \cos. \tfrac{1}{2} A;$$

therefore, since by hypothesis $a = b$, we have $m = m'$, that is, the arc bisecting the vertical angle also bisects the base, and the student will find no difficulty in further showing that this same arc is also perpendicular to the base.

4. *If two sides and the included angle in one triangle are equal to two sides, and the included angle in another, the third side of the one must be equal to the third side of the other.* This is obvious from the first of (A), which shows that cos. a, and therefore a, becomes fixed when the other two sides b, c, and their included A, is fixed; moreover, the *remaining angles of the one triangle are equal to the remaining angles of the other;* for by the second and third of (A), cos. B, cos. C, and therefore, B, C, become fixed when a, b, and c, are fixed.

5. *If the three sides of one triangle are severally equal to the three sides of another, the three angles of the one are also severally equal to those of the other, the equal angles being opposite to the equal sides.* For with fixed values for a, b, c, the formulas (A) give fixed values for cos. A, cos. B, cos. C, and, therefore, for A, B, C. We may, in like manner, infer the equality of the sides from that of the angles, but perhaps the inference is a little more obvious from the equations (B), p. 51, following.

In these deductions the student will observe that we have abstained from saying that the triangles are *equal in all respects* as in the analogous theorems of plane geometry; because two spherical triangles may exist, of which the several *parts* of the one may be equal to the several parts of the other, and yet not admit of coincidence, as plane triangles would under like conditions. Thus, if two plane triangles, of which the sides in the one are equal to those in the other, be joined together by a corresponding side of each, and if we turn one of the triangles about this common side either above or below the plane on which they are situated till it comes to that plane again, we know that we shall thus obtain a perfect coincidence between the two; but if the sides of the triangles thus joined are the chords of two spherical triangles, these triangles will, as we have seen, have all their parts equal, each to each, because, the chords being equal, the arcs must be equal, and yet it is very plain that the corresponding parts of the two triangles cannot be brought into coincidence as in plane triangles, and only in the particular case in which the two triangles are isosceles can they coincide, by being laid the one over the other. We cannot therefore, say, as in plane triangles, that two triangles, whose corresponding parts are equal, have equal surfaces, without distinct proof. This proof will be given in Part IV.

We shall add here but one more inference from the fundamental equations (A).

6. By the first of (A) if the sides b, c, are fixed, cos. a will necessarily diminish as cos. A diminishes; that is, a will increase as A increases: hence *if two triangles have two sides in the one equal to two sides in the other, but the included angle in the first greater than the included angle in the second, then the third side of the first triangle must be greater than the third side of the second.*

Let us now proceed with the analytical discussion.

The three general equations above involve all the six parts of a triangle, the sides, and the angles; and in order to solve them, fewer than three of these parts will be insufficient; but, knowing any three, the others may be determined from them by the usual algebraical process of elimination; yet, as in the general formulas for the solution of plane triangles, so here, the result thus obtained would require considerable modification in certain cases to fit them for logarithmic computation, and on this account it is better to deduce particular formulas by a less direct process. Thus, in order to ascertain the relation between the sides and opposite angles of a spherical triangle, we proceed as follows.

(46.) From the equation (A), $\cos. A = \frac{\cos. a - \cos. b \cos. c}{\sin. b \sin. c} \ldots (1)$

$$\therefore \sin. A = \sqrt{1 - \cos.^2 A} = \frac{\sqrt{\sin.^2 b \sin.^2 c - (\cos. a - \cos. b \cos. c)^2}}{\sin. b \sin. c}$$

or, since $\sin.^2 b \sin.^2 c = (1 - \cos.^2 b)(1 - \cos.^2 c)$

$$\sin. A = \frac{\sqrt{1 - \cos.^2 a - \cos.^2 b - \cos.^2 c + 2 \cos. a \cos. b \cos. c}}{\sin. b \sin. c}. \quad (2)$$

$$\therefore \frac{\sin. A}{\sin. a} = \frac{\sqrt{1 - \cos.^2 a - \cos.^2 b - \cos.^2 c + 2 \cos. a \cos. b \cos. c}}{\sin. a \sin. b \sin. c}.$$

Now the second side of this equation is plainly of such a form, that, however we interchange the quantities a, b, c, the *value* of the expression remains unaltered; so that if we had set out with cos. B, as given by the second of (A), instead of with cos. A, we should have had the very same result for $\frac{\sin B}{\sin. b}$; hence $\frac{\sin. A}{\sin. a} = \frac{\sin. B}{\sin. b} = \frac{\sin. C}{\sin. c} \ldots . (3).$

that is, *in any spherical triangle the sines of the sides are to each other as the sines of the opposite angles;* so that when two of the three given quantities are a side and its opposite angle, the unknown, which is opposite to the third given quantity, may be determined by a simple proportion, or by an easy logarithmic process.

(47.) The equation (2) above might serve to find an angle, from knowing the three sides; it is, however, much less simple than the original expression (1), but neither of them are adapted to logarithms.

In order to obtain one that is adapted, add 1 to each member of (1) and there results (form 24, p. 38), $1 + \cos. A = 2 \cos.^2 \frac{1}{2} A$

$$= \frac{\cos. a + \sin. b \sin. c - \cos. b \cos. c}{\sin. b \sin. c} = \frac{\cos. a - \cos. (b + c)}{\sin. b \sin. c};$$

but a and $b + c$ are respectively the difference and sum of the two arcs $\frac{1}{2}(a + b + c)$, and $\frac{1}{2}(b + c - a)$; hence (form 4, p. 32),

$$\cos. a - \cos. (b + c) = 2 \sin. \tfrac{1}{2}(a + b + c) \sin. \tfrac{1}{2}(b + c - a);$$

therefore, putting S for the sum of the three sides, we have

$$\cos. \tfrac{1}{2} A = \sqrt{\frac{\sin. \frac{1}{2} S \sin. (\frac{1}{2} S - a)}{\sin. b \sin. c}} \quad . . . . (1).$$

If instead of adding 1 to each side of (1) art. 46 we subtract each side from 1, and proceed as above, we shall obtain for sin. $\frac{1}{2}$ A the value,

$$\sin. \tfrac{1}{2} A = \sqrt{\frac{\sin. (\frac{1}{2} S - b) \sin. (\frac{1}{2} S - c)}{\sin. b \sin. c}} \quad . . . . (2);$$

and, by dividing this equation by the former, we have

$$\tan. \tfrac{1}{2} A = \sqrt{\frac{\sin. (\frac{1}{2} S - b) \sin. (\frac{1}{2} S - c)}{\sin. \frac{1}{2} S \sin. (\frac{1}{2} S - a)}} \quad . . . . (3)$$

and all these expressions are adapted to logarithms.

It is unnecessary to put down the corresponding expressions for the other angles, as they may be obtained from these by simply changing the letters: thus for sin. $\frac{1}{2}$ B, we have, by changing A for B and b for a in (2), the formula $\sin. \frac{1}{2} B = \sqrt{\frac{\sin. (\frac{1}{2} S - a) \sin. (\frac{1}{2} S - c)}{\sin. a \sin. c}}$,

whence $\dfrac{\sin.\tfrac{1}{2}\,\mathrm{A}}{\sin.\tfrac{1}{2}\,\mathrm{B}} = \sqrt{\dfrac{\sin. a \sin.(\tfrac{1}{2}\,\mathrm{S} - b)}{\sin. b \sin.(\tfrac{1}{2}\,\mathrm{S} - a)}}$;
from which it appears that if $a > b$, $\sin.\tfrac{1}{2}\,\mathrm{A} > \sin.\tfrac{1}{2}\,\mathrm{B}$, and therefore $\mathrm{A} > \mathrm{B}$; also if $b > a$, $\sin.\tfrac{1}{2}\,\mathrm{B} > \sin.\tfrac{1}{2}\,\mathrm{A}$; and therefore $\mathrm{B} > \mathrm{A}$. Consequently *the greater side is always opposite to the greater angle*,

If $b = c$, the equation (2) becomes $\sin.\tfrac{1}{2}\,\mathrm{A} = \dfrac{\sin.\tfrac{1}{2}\,a}{\sin. b}$.

(48.) We have thus got convenient formulas for the determination of the unknown parts, when two sides and an opposite angle are given, when two angles and an opposite side are given, and when all the three sides are given. We shall now seek the solution to the case in which two sides and the included angle are given, or two angles and the interjacent side; that is to say, we shall proceed to deduce an equation involving only the four quantities a, b, A and C.

For $\cos. c$ in the first of equations (A) substitute its value, as given by the third, and there results, after putting $1 - \sin.^2 b$, for its equal $\cos.^2 b$, $\cos. a = \cos. a - \cos. a \sin.^2 b + \sin. a \sin. b \cos. b \cos. \mathrm{C} + \sin. b \sin. c \cos \mathrm{A}$; or cancelling $\cos. a$ on each side, dividing by $\sin. b$, and transposing, $\cos. a \sin. b = \sin. a \cos. b \cos. \mathrm{C} + \sin c \cos. \mathrm{A}$. (1). For $\sin. c$ in this equation substitute its value given by (3, p. 49), viz. $\sin. c = \dfrac{\sin. a \sin. \mathrm{C}}{\sin. \mathrm{A}}$; and it becomes $\cos. a \sin. b = \sin. a \cos. b \cos. \mathrm{C} + \dfrac{\sin. a \sin. \mathrm{C} \cos. \mathrm{A}}{\sin. \mathrm{A}}$ that is dividing by $\sin. a$,

$$\cot. a \sin. b = \cos. b \cos. \mathrm{C} + \sin. \mathrm{C} \cot. \mathrm{A};$$

which is the equation we proposed to deduce, and from which we at once get an expression for cot. A, when the two sides a, b, and their included angle C, are given, or for $\cot. a$ when the two angles A, C, and interjacent side b are given. The remaining parts of the triangle may, obviously, be found by the relation (p. 49) between the sides and opposite angles; but if the third side, in terms of the other two, and the included angle, is required in a single formula, we must then recur to the fundamental equations (A), which obviously furnish that formula. But neither this nor that which we have just deduced are calculable by a single logarithmic operation; by the introduction, however, of a subsidiary arc the solution may be conducted by logarithms, although two operations will be necessary. But we shall explain this artifice in the next chapter, which will contain the practical application of the formulas deduced in this.

(49.) It now only remains for us to furnish a formula for the side of a spherical triangle in terms of the three angles, and this we may easily do by help of the formulas already given for an angle in terms of the sides, availing ourselves of the property of the supplemental triangle, viz. that the angles and sides of this are supplements of the sides and angles of the former (41). For let the formulas (47) refer to the supplemental triangle of that in question, then, by marking the letters of the former with an accent for distinction sake we have $\mathrm{A}' = 180° - a$,

$$a' = 180° - \mathrm{A},\ b' = 180° - \mathrm{B},\ c' = 180° - \mathrm{C},\ \mathrm{S}' = 540° - \mathrm{S};$$

S′ being the sum of the sides of the triangle in (47), and S the sum of the angles of the triangle with which we are now occupied. Consequently, $\cos.\tfrac{1}{2}\,\mathrm{A}' = \cos.(90° - \tfrac{1}{2}\,a) = \sin.\tfrac{1}{2}\,a$, $\sin. b' = \sin.(180° - \mathrm{B}) = \sin. \mathrm{B}$ $\sin. c' = \sin.(180° - \mathrm{C}) = \sin. \mathrm{C}$, $\sin.\tfrac{1}{2}\,\mathrm{S}' = \sin.(270° - \tfrac{1}{2}\,\mathrm{S}) = -\cos.\tfrac{1}{2}\,\mathrm{S}$; $\sin.(\tfrac{1}{2}\,\mathrm{S}' - a') = \sin.[90° - (\tfrac{1}{2}\,\mathrm{S} - \mathrm{A})] = \cos.(\tfrac{1}{2}\,\mathrm{S} - \mathrm{A})$; therefore by substituting these values the formula (2) becomes

$$\sin.\tfrac{1}{2}\,a = \sqrt{\frac{-\cos.\tfrac{1}{2}\,\mathrm{S}\cos.(\tfrac{1}{2}\,\mathrm{S} - \mathrm{A})}{\sin. \mathrm{B} \sin. \mathrm{C}}};$$

and the other two become $\cos.\ \tfrac{1}{2}a = \sqrt{\dfrac{\cos.\ (\frac{1}{2}\mathrm{S}-\mathrm{B})\cos.\ (\frac{1}{2}\mathrm{S}-\mathrm{C})}{\sin.\ \mathrm{B}\sin.\ \mathrm{C}}}$

$$\tan.\ \tfrac{1}{2}a = \sqrt{\frac{-\cos.\ \frac{1}{2}\mathrm{S}\cos.\ (\frac{1}{2}\mathrm{S}-\mathrm{A})}{\cos.\ (\frac{1}{2}\mathrm{S}-\mathrm{B})\cos.\ (\frac{1}{2}\mathrm{S}-\mathrm{C})}}.$$

As $\frac{1}{2}$ S exceeds 90° but falls short of 270° art. (42), cos. $\frac{1}{2}$ S is always negative, and, therefore, the numerators, of the first and third of these expressions although appearing with a negative sign, are in reality positive.

(50.) By means of the polar triangle it is obvious that we may, in all cases as well as in this, convert any formula involving the sides and angles of a triangle into another, similarly involving the angles and sides; the sides in the one formula being replaced by the angles opposite to them in the other, and the angles being replaced by the opposite sides. To effect this change we need only write, instead of *sin.* and *cos* in the original formula, *sin.* and —*cos.* of the opposite arc, whether side or angle.

Thus the fundamental equations (A) become in this manner changed into the following

$$\left.\begin{array}{l}\cos.\ \mathrm{A} = \cos.\ a\sin.\ \mathrm{B}\sin.\ \mathrm{C} - \cos.\ \mathrm{B}\cos.\ \mathrm{C}\\ \cos.\ \mathrm{B} = \cos.\ b\sin.\ \mathrm{A}\sin.\ \mathrm{C} - \cos.\ \mathrm{A}\cos.\ \mathrm{C}\\ \cos.\ \mathrm{C} = \cos.\ c\sin.\ \mathrm{A}\sin.\ \mathrm{B} - \cos.\ \mathrm{A}\cos.\ \mathrm{B}\end{array}\right\}\ \ldots\ (\mathrm{B}).$$

which plainly show that *if the three angles of one triangle are equal to the three angles of another, the sides of the former must also be equal to those of the latter;* and also that *if two angles* B, C, *and interjacent side, a, of one triangle are respectively equal to two angles, and the interjacent side of another, the remaining angle* A *of the one must be equal to the remaining angle of the other;* and thus *all parts of the one triangle are equal severally to those of the other.*

(51.) The theory now delivered is sufficient for the solution of every case of spherical triangles; but we shall add two more theorems applicable to the case in which the two sides and included angles are given to find the other angles, and to that in which two angles and the interjacent side are given to find the other sides. These theorems have the advantage of being very simple, and are of a form easily retained in the memory. They were first given by *Lord Napier*, and are known by the name of *Napier's Analogies.*

By the equation (1), page 50, we have

$$\sin.\ c\cos.\ \mathrm{A} = \cos.\ a\sin.\ b - \sin.\ a\cos.\ b\cos.\ \mathrm{C}.$$

Similarly,

$$\underline{\sin.\ c\cos.\ \mathrm{B} = \cos.\ b\sin.\ a - \sin.\ b\cos.\ a\cos.\ \mathrm{C}}$$

$$\therefore \sin.\ c\,(\cos.\ \mathrm{A}+\cos.\ \mathrm{B}) = \sin.\ (a+b)\,(1-\cos.\ \mathrm{C})\ \ldots\ldots\ (1).$$

Now from the equations (3), page 49, we have

$$\sin.\ \mathrm{A}\sin.\ c = \sin.\ a\sin.\ \mathrm{C}$$

$$\underline{\sin.\ \mathrm{B}\sin.\ c = \sin.\ b\sin.\ \mathrm{C}}$$

$$\therefore (\sin.\ \mathrm{A}\pm\sin.\ \mathrm{B})\sin.\ c = (\sin.\ a\pm\sin.\ b)\sin.\ \mathrm{C}\ \ldots\ (2).$$

Dividing (2) by (1) there results

$$\frac{\sin.\ \mathrm{A}\pm\sin.\ \mathrm{B}}{\cos.\ \mathrm{A}+\cos.\ \mathrm{B}} = \frac{\sin.\ a\pm\sin.\ b}{\sin.\ (a+b)}\cdot\frac{\sin.\ \mathrm{C}}{1-\cos.\ \mathrm{C}}$$

that is, arts. (32) and (31) taking the upper and lower signs separately.

$$\tan.\ \tfrac{1}{2}(\mathrm{A}+\mathrm{B}) = \frac{\cos.\ \frac{1}{2}(a-b)}{\cos.\ \frac{1}{2}(a+b)}\cot.\ \tfrac{1}{2}\mathrm{C}$$

$$\tan.\ \tfrac{1}{2}(\mathrm{A}-\mathrm{B}) = \frac{\sin.\ \frac{1}{2}(a-b)}{\sin.\ \frac{1}{2}(a+b)}\cot.\ \tfrac{1}{2}\mathrm{C}.$$

For the supplemental triangle the corresponding formulas are

$$\tan.\ \tfrac{1}{2}(a+b) = \frac{\cos\ \frac{1}{2}(\mathrm{A}-\mathrm{B})}{\cos.\ \frac{1}{2}(\mathrm{A}+\mathrm{B})}\tan.\ \tfrac{1}{2}c$$

$\tan. \frac{1}{2}(a-b) = \frac{\sin. \frac{1}{2}(A-B)}{\sin. \frac{1}{2}(A+B)} \tan. \frac{1}{2}c$; and these are the four equations which furnish the Analogies of Napier, viz.

$$\left.\begin{array}{l}\cos. \frac{1}{2}(a+b) : \cos. \frac{1}{2}(a-b) :: \cot. \frac{1}{2}C : \tan \frac{1}{2}(A+B) \\ \sin. \frac{1}{2}(a+b) : \sin. \frac{1}{2}(a-b) :: \cot. \frac{1}{2}C : \tan. \frac{1}{2}(A-B)\end{array}\right\} \quad . \ . \quad (2)$$

$$\left.\begin{array}{l}\cos. \frac{1}{2}(A+B) : \cos. \frac{1}{2}(A-B) :: \tan. \frac{1}{2}c : \tan. \frac{1}{2}(a+b) \\ \sin. \frac{1}{2}(A+B) : \sin. \frac{1}{2}(A-B) :: \tan. \frac{1}{2}c : \tan. \frac{1}{2}(a-b)\end{array}\right\} \quad . \ . \quad (3).$$

As the arcs $\frac{1}{2}(a-b)$, and $\frac{1}{2}C$, are always less than 90°, the two means in the first of these analogies are positive, and, therefore, the two extremes must have the same signs, that is, they must either be both positive or both negative: hence $\frac{1}{2}(a+b)$, and $\frac{1}{2}(A+B)$, must either be both acute or both obtuse, and consequently the arcs $a+b$, $A+B$, must be either both less or both greater than 180°. From this circumstance we may always avoid doubtful solutions to the cases in which the given parts are two sides and an opposite angle, or two angles and an opposite side, as will be exemplified in next chapter.

CHAPTER III.

SOLUTIONS OF THE DIFFERENT CASES OF SPHERICAL TRIANGLES.

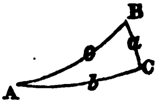
(52.) We are now to show the application of the preceding theory to the actual determination of any of the six parts of a spherical triangle when three of them are known; and as in Plane Trigonometry, so here, we shall find it convenient to begin with right-angled triangles.

Right-Angled Spherical Triangles.

The formulas for which all the rules for right-angled triangles are derived are those marked (A), (B), and 3, (p. 49), in the preceding chapter, viz. $\frac{\sin. A}{\sin. a} = \frac{\sin. B}{\sin. b} = \frac{\sin. C}{\sin. c}$ . . . (1)

$$\left.\begin{array}{l}\cos. A = \cos. a \sin. B \sin. C - \cos. B \cos. C \\ \cos. B = \cos. b \sin. A \sin. C - \cos. A \cos. C \\ \cos. C = \cos. c \sin. A \sin. B - \cos. A \cos. B\end{array}\right\} \quad . \ . \ . \quad (2)$$

$$\left.\begin{array}{l}\cos. a = \cos. b \cos. c + \sin. b \sin. c \cos. A \\ \cos. b = \cos. a \cos. c + \sin. a \sin. c \cos. B \\ \cos. c = \cos. a \cos. b + \sin. a \sin. b \cos. C\end{array}\right\} \quad . \ . \ . \quad (3).$$



Let ABC be a spherical triangle, right-angled at C; then from the first of these formulas we have, since $\sin. C = 1$, the equations $\sin. a = \sin. c \sin. A$, $\sin. b = \sin. c \sin. B$ . . (4).

Two different expressions for $\sin. a$, $\sin. b$, may also be obtained from the first and second of (2). Thus C being 90° these two equations give $\cos. A = \cos. a \sin. B$, $\cos. B = \cos. b \sin. A$. (5); substituting in these the values of $\sin. A$, $\sin. B$, as deduced from (1) they become

$$\cos. A = \frac{\cos. a}{\sin. a} \sin. A \sin. b, \cos. B = \frac{\cos. b}{\sin. b} \sin. B \sin. a$$

$$\therefore \sin. b = \tan. a \cot. A, \sin. a = \tan. b \cot. B.$$

For the hypotenuse c we get from the third of (2) the expression

$$\cos. c = \frac{\cos. A \cos. B}{\sin. A \sin. B} = \cot. A \cot. B,$$

and from the third of (3) the expression $\cos. c = \cos. a \cos. b$ . . (6). In the equations (5) substitute for $\sin. A$, $\sin. B$, their values in (4), and for $\cos. a$, $\cos. b$, their values in (6), and they then take the form

$$\cos. A = \tan. b \cot. c,\ \cos. B = \tan. a \cot. c \quad . \quad . \quad . \quad (7).$$

Collecting together all these equations, we have

$$\sin. a = \tan. b \cot. B = \sin. c \sin. A$$
$$\sin. b = \tan. a \cot. A = \sin. c \sin. B$$
$$\cos. c = \cot. A \cot. B = \cos. a \cos. b$$
$$\cos. A = \tan. b \cot. c = \cos. a \sin. B$$
$$\cos. B = \tan. a \cot. c = \cos. b \sin. A;$$

and these furnish solutions to every possible case of right-angled triangles; for it is plain that whichever two of the *five* quantities *a*. *b*, *c*, A, B, are given, any one of the others may be immediately found by one or other of these equations. Instead, however, of deducing from these five equations so many distinct rules for the solution of the various cases, the whole, by help of an ingenious contrivance, may be comprehended in two rules of very remarkable simplicity.

Before announcing these rules we shall, however, just stop to mention an inference from the first of this group of equations which will be useful hereafter, viz. that *from any point on a sphere to a given great circle the shortest great circle arc that can be drawn is the perpendicular;* for by the equation referred to sin. *a* exceeds sin. *c*, since sin. A is less than 1. If the point is the pole of the proposed great circle, then, indeed, (p. 43) sin. *a* = sin. *c*, and sin. A = 1, all great circle arcs from the point to the circle being perpendicular. From the last of the preceding equations we infer that cos. B, cos. *b*, always have the same sign, that is, either side is of the same affection as its opposite angle. From the middle equation we see that the hypotenuse is acute if the sides are of the same affection, or if the angles opposite to them are of the same affection, but otherwise the hypotenuse is obtuse.

The rules to which we have adverted above were invented by Baron Napier, the celebrated inventor of logarithms, and are called *Napier's Rules for the Circular Parts.* We shall now explain them.

In a right-angled triangle we are to recognise but five parts, viz. the three sides and the two angles A and B. If we take any one of these as a middle part, the two which lie next to it, one on each side will be *adjacent* parts: thus taking A for a middle part (last fig.), *b* and *c* will be the *adjacent* parts; if we take *c* for the middle part, A and B will be the adjacent parts; if we take B for the middle part, *c* and *a* will be the adjacent parts; but if we take *a* for the middle part, then, as the part C is not recognised we do not consider it as intervening between *a* and *b*, and, therefore, we call in this case B and *b*, the adjacent parts; and, lastly, if *b* is the middle part then the adjacent parts are A and *a*. The two parts immediately beyond the adjacent parts, one on each side, still disregarding the right-angle, are called the opposite parts; thus if A is the middle part the opposite parts are *a*, next to the adjacent part *b*, and B next to the adjacent part *c*. This being understood, Napier's two rules may be expressed as follows, carefully observing to *use the complements of the two angles and of the intervening hypotenuse instead of these parts themselves.*

I. Rad. × sin. middle part = product of tan. adjacent parts.

II. Rad. × sin. middle part = product of cos. opposite parts.

Both these rules may be comprehended in a single expression, thus

rad. sin. mid. = prod. tan. adja. = prod. cos. opp.;

and to retain this in the memory we have only to remember that the vowels in the contractions mid., adja., opp., are the same as those in the contractions sin., tan., cos., to which they are joined.

That these rules comprehend all the equations given above will be seen by taking *a*, *b*, *c*, &c. in succession for the middle part, as in the subjoined table, keeping in mind the condition just stated, that instead of A, B, and *c*, we are to use their complements.

Middle part. (sin.)	Adjacent parts. (tan.)	Opposite parts. (cos.)
a.	*b*, comp. B	comp. *c*, comp. A
b.	*a*, comp. A	comp. *c*, comp. B
comp. *c*	comp. A, comp. B	*a*, *b*
comp. A	*b*, comp. *c*	*a*, comp. B
comp. B	*a*, comp. *c*	*b*, comp. A

As in the solution of right-angled triangles two parts are given to find a third, we must in the application of Napier's rule choose for the middle of these three parts that which causes the other two to become either adjacent parts or opposite parts.

EXAMPLES.

(53.) 1. In the right-angled triangle ABC are given the two perpendicular sides, viz. $a = 48^\circ\ 24'\ 16''$, $b = 59^\circ\ 38'\ 27''$, to find the hypotenuse *c*.

Here the hypotenuse being made the middle part the other two will, obviously, be the opposite parts, being separated from the hypotenuse by the intervening angles A, B. Hence by the rule

$$\text{rad} \times \sin.\ \text{comp.}\ c = \cos.\ a \times \cos.\ b\ ;$$

that is, rad. $\cos.\ c = \cos.\ a \cos.\ b \therefore \cos.\ c = \dfrac{\cos.\ a \cos.\ b}{\text{rad.}}$; and as cos. *a*, cos. *b*, are both positive, cos. *c* is positive, and, therefore, *c* is acute.

rad.		10·0000000
cos. *a*	48° 24′ 16″	9·8220819
cos. *b*	59 38 27	9·7036515
cos. *c*	70 23 42	9·5257334.

2. In the spherical triangle ABC, right-angled at C, are given $b = 46^\circ\ 18'\ 23''$, $A = 34^\circ\ 27'\ 39''$, to find the other oblique angle B.

Making B the middle part, the other two will be the opposite parts. Consequently, by the rule, rad. $\times$ sin. comp. B = cos. *b* $\times$ cos. comp. A;

that is, rad. $\cos.\ B = \cos.\ b \sin.\ A \therefore \cos.\ B = \dfrac{\cos.\ b\ \sin.\ A}{\text{rad.}}$;

and as cos *b*, sin. A, are both positive, B is acute,

rad.		10·0000000
cos. *b*	46° 18′ 23″	9·8393535
sin. A	34 27 39	9·7526957
cos. B	66 59 25	9·5920492.

3. In the spherical triangle, right-angled at C, are given the two perpendicular sides, viz. $a = 116^\circ\ 30'\ 43''$, $b = 29^\circ\ 41'\ 32''$, to find the angle A.

Making *b* the middle part, the others will be the adjacent parts, and, therefore, by the rule rad. $\times$ sin. *b* = tan. *a* $\times$ tan. comp. A,

that is, rad. $\sin.\ b = \tan.\ a \cot.\ A \therefore \cot.\ A = \dfrac{\text{rad.} \sin.\ b}{\tan.\ a}$; and as sin. *b* is positive, and tan. *a* negative, cot. A will be negative, and, therefore, A will be obtuse, or the supplement of the angle given by the tables,

rad.		10·0000000
tan. *a*	116° 30′ 43″	10·3020371
sin. *b*	29 41 32	9·6949041
cot. A	103 52 48	9·3928670.

4. In a spherical triangle, right-angled at C, are given $b = 29^\circ\ 12'\ 50''$, and $B = 37^\circ\ 26'\ 21''$, to find the side *a*.

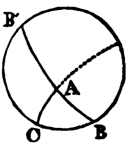
Taking a for the middle part, the other two will be adjacent parts; hence, by the rule, rad. $\times$ sin. a = tan. b $\times$ tan. comp. B

that is, rad. sin. a = tan. b cot. B $\therefore$ sin. $a = \frac{\text{tan. } b \text{ cot. B}}{\text{rad.}}$.

In this case there are two solutions, viz. a and the supplement of a, both of which have the same sine. As sin. a is necessarily positive, b and B must necessarily be always of the same species, that is, either both acute or both obtuse, so that, as observed at p. 53, the sides including the right-angle are always of the same species as the opp. angles, a circumstance which must be attended to in framing examples.

rad.	. .	. 10·0000000
tan. b	29° 12′ 50″	. 9·7475666
cot. B	37 26 21	. 10·1159745
sin. a	46° 55′ 2′ or 133° 4′ 58″	. 9·8635411.

It appears, therefore, that there exists two right-angled triangles, having an oblique angle, and the opposite side in one equal to an oblique angle and the opposite side in the other, but the remaining oblique angle in the one the supplement of the remaining oblique angle in the other. These triangles are situated, with respect to each other, on the sphere, as the triangles ABC, AB′C, in the annexed diagram, in which, with the exception of the common side, AC, and the equal angles B, B′, the parts of the one triangle are supplements of the corresponding parts of the other.



5. Given the angle A = 23° 28′, the side b = 49° 17′, to find the hypotenuse c. c = 51° 42′ 37″.

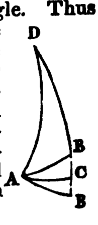
6. Given the hypotenuse c = 66° 32′, the side a = 37° 48′, to find the angle B. B = 70° 19′ 18″.

7. Given the perpendicular sides a = 59° 38′ 27″, b = 48° 24′ 16″. to find all the other parts. c = 70° 23′ 42″, A = 66° 20′ 40″, B = 52° 32′ 55″.

8. Given b = 121° 26′ 25″, and the opposite angle B = 111° 14′ 37″, to find all the other parts.

Solution of Quadrantal Triangles.

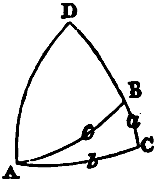
(54.) The rules for right-angled triangles will serve also for the solution of *quadrantal triangles*, or those in which one side is a quadrant. For by changing such a triangle for its supplemental triangle, we shall then have to consider a right-angled triangle, of which the hypotenuse will be the supplement of the angle opposite the quadrantal side, the two perpendicular sides supplements of the other two angles of the proposed triangle, and the two oblique angles of the new triangle supplements of the oblique sides of the primitive triangle. That is, the sides of the primitive or quadrantal triangle being a, b, and c = 90° and its angles A, B, C, the sides of the supplemental triangle will be 180° — A, 180° — B, and 180° — C, this latter being the hypotenuse; and the opposite angles will be 180° — a, 180° — b, and 90°. But the parts of a quadrantal triangle may be determined without the aid of the supplemental triangle. Thus let AD be the quadrantal side in the triangle ABD. Produce DB, if necessary, till DC becomes a quadrant, and draw the arc AC, which will, obviously, measure the angle D, since D will be the pole of the arc AC, and C will be a right angle: also the angle CAB will be the complement of the angle BAD in the proposed triangle, and the angle ABC will either be identical with ABD in the proposed, or supplemental to it, accordingly as DC exceeds, or falls short of, a quadrant; hence all the parts of the proposed triangle are easily determined from those of the right-angled triangle ABC.



If the angle DAB is less than 90°, or than the angle DAC, the side DB must, obviously, be acute; but if DAB is greater than 90°, DB will be obtuse, and conversely. Hence the angles adjacent to the quadrantal side are of the same species as the sides opposite to them. The same may be inferred from the polar triangle.

It must be remarked that the solution will be ambiguous whenever the determination of the right-angled triangle becomes ambiguous, whether we employ the polar triangle or the triangle ABC in the above diagram. This ambiguity occurs only when the given parts in the right-angled triangle are one of the perpendicular sides and the angle opposite to it. (See solution, p. 54.)

EXAMPLES.



In the triangle DAB, DA = 90°, A = 54° 43′, and D = 42° 12′, required the other parts.

As the angle DAB is less than 90°, that is, less than the angle DAC, DB is less than a quadrant, and, therefore, the right-angled triangle ABC is situated as in the figure, BC being the prolongation of DB. Of the parts of this right-angled triangle we have given A = 90° — 54° 43′ = 35° 17′, and b = 42° 12′, to find the other parts.

Let A be the middle part, then b and c will be adjacent parts, therefore, rad. × sin. comp. A = tan. b × tan. comp. c,

that is, rad. cos. A = tan. b cot. c ∴ cot. c = $\dfrac{\text{rad. cos. A}}{\text{tan. } b}$

rad.		10·0000000
cot. A	35° 17′	9·9118528
tan. b	42 12	9·9574850
cot. c	48° 0′ 9″	9·9543678.

Let B be the middle part, then A, b, will be opposite parts, and, consequently, rad. × sin. comp. B = cos. b × cos. comp. A;

that is, rad. cos. B = cos. b sin. A ∴ cos. B = $\dfrac{\text{cos. } b \text{ sin. A}}{\text{rad.}}$

rad.		10·0000000
cos. b	42° 12′	9·8697037
sin. A	35 17	9·7616424
cos. B	64° 39′ 55″	9·6313461.

hence the angle ABD is 115° 20′ 5″

It remains now to find a; let, therefore, B be the middle part, then a and c will be the adjacent parts; hence

rad. × sin. comp. B = tan. a × tan. comp. c;

that is, rad. cos. B = tan. a cot. c ∴ tan. a = $\dfrac{\text{rad. cos. B}}{\text{cot. } c}$

rad.		10·0000000
cos. B		9·6313461
cot. c		9·9543678
tan. a	25° 25′ 20″	9·6769783;

therefore, the side DB, which is the complement of this, is 64° 34′ 40″.

2. In the triangle DAB, DA = 90°, A = 112° 2′ 9″, and AB = 67° 3′ 14″, to find the other parts.

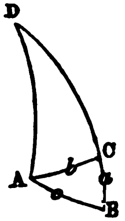
Since in this example A is obtuse, DB is obtuse.

In the right-angled triangle ABC we have $A = 22^\circ\, 2'\, 9''$ and $AB = 67^\circ\, 3'\, 14''$; let A be the middle part, then AB, AC, will be adjacent parts, and we shall have

$\text{rad.} \times \text{sin. comp. } A = \text{tan. } b \times \text{tan. comp. } c$;

that is, $\text{rad. cos. } A = \text{tan. } b \text{ cot. } c$

$\therefore \text{tan. } b = \dfrac{\text{rad. cos. } A}{\text{cot. } c}$



rad.		10·0000000
cos. A	22° 2′ 9″	9·9670560
cot. c	67 3 14	9·6267152
tan. b	65 27 9	10·3403408;

therefore, the angle $D = 65^\circ\, 27'\, 9''$.

Take now a for the middle part, then A and c will be opposite parts; hence $\text{rad.} \times \text{sin. } a = \text{cos. comp. } A \times \text{cos. comp. } c$,

that is $\text{rad. sin. } a = \text{sin. } A \text{ sin. } c \therefore \text{sin. } a = \dfrac{\text{sin. } A \text{ sin. } c}{\text{rad.}}$;

and a will be acute, because the opposite angle is acute

rad.		10·0000000
sin. A	22° 2′ 9″	9·5742471
sin. c	67 3 14	9·9641992
sin. a	22 12 44	9·5384463;

therefore $BD = 110^\circ\, 12'\, 44''$.

As we have now to find B, take a for the middle part, then b and B will be adjacent parts, therefore, $\text{rad.} \times \text{sin. } a = \text{tan. } b \text{ tan. comp. } B$;

that is, $\text{rad. sin. } a = \text{tan. } b \text{ cot. } B \therefore \text{cot. } B = \dfrac{\text{rad. sin. } a.}{\text{tan. } b}$

rad.		10·0000000
sin. a		9·5384463
tan. b		10·3403408
cot. B	81° 1′ 58″	9·1981055.

3. Given the quadrantal side and the other two sides equal to 22° 53′ 30″, and 51° 4′ 35″, to find the angle opposite to the quadrantal side.
$B = 70^\circ\, 3'\, 44''$.

4. In the quadrantal triangle ADB are given $D = 69^\circ\, 13'\, 46''$, and $A = 72^\circ\, 12'\, 4''$, to determine the other parts.
$AB = 70^\circ\, 8'\, 39''$, $BD = 73^\circ\, 17'\, 29''$, $B = 96^\circ\, 13'\, 23''$.

These examples will suffice for the present, to show the application of Napier's rules to the solution of right-angled and quadrantal triangles. We shall, therefore, now give examples of the solution of the various cases of oblique-angled triangles in general.

Solution of Oblique Angled Spherical Triangles.

(55.) The fundamental equations (A) show that in order to determine the several parts of a spherical triangle, three of those parts must be previously given. Now, three parts out of the six can be combined only in these different ways, viz.

1. The three sides.
2. The three angles.
3. Two sides and the included angle.
4. Two angles and the interjacent side.
5. Two sides and an opposite angle.
6. Two angles and an opposite side.

So that the complete solution of an oblique-angled spherical triangle presents six cases. These we shall solve in the order in which they are here enumerated.

CASE I. (56.) Given the three sides to find the angles.

For the determination of any angle A we have by (47) the three following different expressions, viz.

$$\sin. \tfrac{1}{2} A = \sqrt{\frac{\sin. (\frac{1}{2} S - b) \sin. (\frac{1}{2} S - c)}{\sin. b \sin. c}}$$

$$\cos. \tfrac{1}{2} A = \sqrt{\frac{\sin. \frac{1}{2} S \sin. (\frac{1}{2} S - a)}{\sin. b \sin. c}}$$

$$\tan. \tfrac{1}{2} A = \sqrt{\frac{\sin. (\frac{1}{2} S - b) \sin. (\frac{1}{2} S - c)}{\sin. \frac{1}{2} S \sin. (\frac{1}{2} S - a)}}$$

We may apply to these formulas the remarks made at (21) in the Plane Trigonometry. It will be sufficient to observe here that the first formula is generally the most suitable, because the angle A is rarely so large as to be very near 180°.

EXAMPLES.

1. In an oblique spherical triangle the three sides are a = 68° 46′ 2″ b = 43° 37′ 38″, c = 37° 10′; required the angle A.

a	68°	46′	2″		
sin. b	43	37	38	arith. comp.	0·1611739
sin. c	37	10	0	arith. comp.	0·2188656
	2)149	33	40		
	74	46	50		
sin. (½ S — b)	31	9	12	- -	9·7137678
sin. (½ S — c)	37	36	50	- -	9·7855698
					2)19·8793771
sin. ½ A	60	29	53	- -	9·9396885

∴ A = 120° 59′ 46″.

2. Given a = 108°, b = 52° 12′, and c = 74° 30′, to find A.

a	108°	0′		
sin. b	37	48	arith. comp.	0·2126054
sin. c	74	30	arith. comp.	0·0160895
	2)220	18		
	110	9		
sin. (½ S — b)	72	21	- -	9·9790594
sin. (½ S — c)	35	39	- -	9·7655436
				2)19·9732979
sin. ½ A	75	51	56 - -	9·9866489

∴ sin. A = 151° 43′ 52″.

3. Given a = 70° 4′ 18″, b = 63° 21′ 27″, and c = 59° 16′ 23″, to find the angles A and B. A = 81° 38′ 20″, B = 70° 9′ 38″.

4. Given a = 67° 25′ 2″, b = 80° 2′ 25″, c = 23° 27′ 46″, to find the angle A. A = 54° 55′ 19″.

5. Given a = 61° 32′ 12″, b = 83° 19′ 42″, c = 23° 27′ 46″, to find A. A = 20° 39′ 48″

Case II. (57.) Given the three angles to find the sides.

By (49) we have the following formulas for any side a in terms of the three angles, viz.

$$\sin. \tfrac{1}{2} a = \sqrt{\frac{-\cos. \tfrac{1}{2} S \cos. (\tfrac{1}{2} S - A)}{\sin. B \sin. C}}$$

$$\cos. \tfrac{1}{2} a = \sqrt{\frac{\cos. (\tfrac{1}{2} S - B) \cos. (\tfrac{1}{2} S - C)}{\sin. B \sin. C}}$$

$$\tan. \tfrac{1}{2} a = \sqrt{\frac{-\cos. \tfrac{1}{2} S \cos (\tfrac{1}{2} S - A)}{\cos. (\tfrac{1}{2} S - B) \cos. (\tfrac{1}{2} S - C)}}.$$

It may be remarked here that the first two only of the expressions in this and in the former case need be borne in the memory, as the third is an immediate consequence of them. If the expressions in the former case be recollected, these can scarcely fail to be recalled at the same time, as they differ from them only in this, viz. that the sides are replaced by their opposite angles, and, except in the denominators, cosines are written for sines, and sines for cosines.

EXAMPLES.

1. The three angles of a spherical triangle are, A = 130° 3′ 11″, B = 31° 34′ 26″, C = 30° 28′ 12″, required the side a.

A	130° 3′ 11″		
sin. B	31 34 26	arith. comp.	0·2810023
sin. C	30 28 12	arith. comp.	0·2949174
	2)192 5 49		
cos. ½ S	96 2 54½	. .	9·0227162
cos. (½ S — A)	34 0 16½	.	9·9185570
			2)19·5171929
sin. ½ a	35 1 1½	.	9·7585964

$$\therefore a = 70° \, 2' \, 3''.$$

2. The three angles of a spherical triangle are, A = 103° 59′ 57″, B = 46° 18′ 7″, C = 36° 7′ 52″; required the side a.

a = 42° 8′ 48″.

3. The three angles of a spherical triangle are 120° 43′ 37″, 109° 55′ 42″, and 116° 38′ 33″; required the three sides.

115° 13′ 26″, 98° 21′ 40″, and 109° 50′ 22.″

Case III. (58.) Given two sides a, b, and the included angle C, to find the other parts. By Napier's analogies,

$$\cos. \tfrac{1}{2} (a + b) : \cos. \tfrac{1}{2} (a \sim b) :: \cot. \tfrac{1}{2} C : \tan. \tfrac{1}{2} (A + B)$$

$$\sin. \tfrac{1}{2} (a + b) : \sin. \tfrac{1}{2} (a \sim b) :: \cot. \tfrac{1}{2} C : \tan. \tfrac{1}{2} (A - B).$$

These serve to determine the angles A, B, opposite to the given sides; after which the third side c may be determined by either of the remaining two analogies of Napier, viz.

$$\cos. \tfrac{1}{2} (A + B) : \cos. \tfrac{1}{2} (A \sim B) :: \tan. \tfrac{1}{2} c : \tan. \tfrac{1}{2} (a + b)$$

$$\sin. \tfrac{1}{2} (A + B) : \sin. \tfrac{1}{2} (A \sim B) :: \tan. \tfrac{1}{2} c : \tan. \tfrac{1}{2} (a - b).$$

EXAMPLES.

1. In a spherical triangle are given a = 38° 30′, b = 70°, and C = 31° 34′ 26″, to find the other parts.

I *To find* A *and* B.

cos. $\frac{1}{2}(a+b)$ 54° 15′ ar. comp. 0·2334015 ar. comp. sin. 0·0906719
cos. $\frac{1}{2}(a\sim b)$ 15 45 . 9·9833805 sin. 9·4336746
cot. $\frac{1}{2}$C 15 47 13 10·5486352 10·5486352

tan. $\frac{1}{2}$(A + B) 80 15 41 10·7654172, tan. $\frac{1}{2}$(A $\sim$ B) 49° 47′ 30″ 10·0729817
$\frac{1}{2}$(A + B) must be acute, because $\frac{1}{2}(a+b)$ is acute.

By taking the sum and difference of these results we have, B = 130° 3′ 11″, and A = 30° 28′ 11″.

II. *To find* c.

cos. $\frac{1}{2}$(A $\sim$ B) 49° 47′ 30″ arith. comp. 0·1900575
cos. $\frac{1}{2}$(A + B) 80 15 41 . . . 9·2282812
tan. $\frac{1}{2}(a+b)$ 54 15 0* . 10·1427296

tan. $\frac{1}{2}c$ 20 0 0 9·5610683
∴ c = 40° 0′ 0″.

When in the case we are considering, the only part required happens to be the side opposite the given angle, the finding of the other two angles then becomes merely a subsidiary operation, and the determination of the required side, by Napier's analogies, seems somewhat lengthy. But a shorter method of solution is deducible from the fundamental formula, cos. c = cos. a cos. b + sin. a sin. b cos. C . . (1).

For substituting cos. a tan. a for its equal sin. a it becomes

$$\cos. c = \cos. a\,(\cos. b + \tan. a \sin. b \cos. C).$$

$$\text{Assume } \tan. a \cos. C = \cot. \omega = \frac{\cos. \omega}{\sin. \omega};$$

$$\text{then } \cos. c = \cos. a\,\frac{\sin. \omega \cos. b + \sin. b \cos. \omega}{\sin. \omega} = \frac{\cos. a \sin. (\omega + b)}{\sin. \omega}.$$

Hence, to find the side c, we must first determine a subsidiary angle ω from the equation cot. ω = tan. a cos C . (2); after which c is found by the equation $\cos. c = \dfrac{\cos. a \sin. (\omega + b)}{\sin. \omega}$ - - - (3).

2. The same parts being given as in the last example, to determine c by these formulas,

tan. a 38° 30′ 0″ . 9·9006052, cos. a . . 9·8935444
cos. C 31 34 26 . 9·9304221, sin. ω, ar. comp. 0·0820652

cot. ω 55 52 30$\frac{1}{2}$. 9·8310273, sin. $(\omega + b)$ 9·9086437

cos. c 40° . 9·8842533.

Other formulas for the determination of c might be easily deduced from the same equation (1), but this is as short and as convenient as any. We might also introduce here a distinct formula for the determination of one of the angles A, by help of a subsidiary arc ω; but as little or nothing would be gained, in point of brevity, over the process by Napier's analogies, we shall not stop to investigate it.

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* There will be no necessity to refer to the tables for the tangent of this arc, we shall obtain it by subtracting the right-hand arithmetical complement in the preceding logarithmic process from that on the left, adding 10 to the index. For calling the right-hand complement p, and the left q, and recollecting that log. tan. = 10 + log. sin. — log. cos. = $10 + (10 - p) - (10 - q)$, we have log. tan. = $10 + q - p$.

3. In a spherical triangle are given $a = 43° 37' 38''$, $b = 37° 10'$, $C = 120° 53' 46''$, to find the side c. $c = 68° 46' 2''$.

4. In a spherical triangle are given the two sides, equal to 37° 10′ and 68° 46′ 2″, and the included angle equal to 39° 23′; required the other two angles. 33° 45′ 3″ and 120° 59′ 49″.

5. Given the two sides equal to 44° 13′ 45″ and 84° 14′ 29″, and the included angle equal to 36° 45′ 28″; to find the other parts.

The angles are 32° 26′ 6″, and 130° 5′ 22″, and the side 51° 6′ 12″.

CASE IV. (59.) Given two angles A, B, and the interjacent side c, to find the other parts.

The solution of this case as well as the former, is comprehended in Napier's analogies; the one pair, viz.

$$\cos.\tfrac{1}{2}(A+B) : \cos.\tfrac{1}{2}(A \sim B) :: \tan.\tfrac{1}{2}c : \tan.\tfrac{1}{2}(a+b)$$
$$\sin.\tfrac{1}{2}(A+B) : \sin.\tfrac{1}{2}(A \sim B) :: \tan.\tfrac{1}{2}c : \tan.\tfrac{1}{2}(a-b);$$

determining the unknown sides a, b, and either of the other pair, viz.

$$\left.\begin{array}{l}\cos.\tfrac{1}{2}(a+b) : \cos.\tfrac{1}{2}(a \sim b) :: \cot.\tfrac{1}{2}C : \tan.\tfrac{1}{2}(A+B)\\ \sin.\tfrac{1}{2}(a+b) : \sin.\tfrac{1}{2}(a \sim b) :: \cot.\tfrac{1}{2}C : \tan.\tfrac{1}{2}(A-B)\end{array}\right\};$$

enabling us to find the unknown angle C.

EXAMPLES.

1. In a spherical triangle are given two angles equal to 39° 23′ and 33° 45′ 3″, and the interjacent side equal to 68° 46′ 2″; to find the remaining parts.

I. *To find the Sides.*

$\cos.\tfrac{1}{2}(A+B)$	36° 34′ 1½″ ar. comp.	0·0951980	ar. comp. sin.	0·2249260
$\cos.\tfrac{1}{2}(A-B)$	2 48 58½ - -	9·9994752	sin.	8·6913737
$\tan.\tfrac{1}{2}c$	34 23 1 - - -	9·8352429		9·8352429
$\tan.\tfrac{1}{2}(a+b)$	40 23 49 - - -	9·9299161,	$\tan.\tfrac{1}{2}(a-b)$ 3° 13′ 48″	8·7515426

$\therefore a = 43° 37' 37''$, $b = 37° 10' 1''$.

II. *To find the Angle.*

$\sin.\tfrac{1}{2}(a-b)$	3° 13′ 48″	arith. comp.	1·2491502
$\sin.\tfrac{1}{2}(a+b)$	40 23 49		9·8116281
$\tan.\tfrac{1}{2}(A-B)$	2 48 58½*		8·6918985
$\cot.\tfrac{1}{2}C$	60 29 53		9·7526768

$\therefore C = 120° 59' 46''$.

If the angle opposite to the given side be the only part required, a more compendious method of solution may be obtained by introducing a subsidiary arc, as in last case. Thus the formula (B) art. (50) becomes when cos. A tan. A is substituted for sin. A,

$$\cos. C = \cos. A\,(\tan. A \sin. B \cos. c - \cos. B);$$

or assuming $\tan. A \cos. c = \cot. \omega = \dfrac{\cos. \omega}{\sin. \omega}$

$$\cos. C = \cos. A\,\frac{\sin. B \cos. \omega - \sin. \omega \cos. B}{\sin. \omega} = \frac{\cos. A \sin. (B-\omega)}{\sin. \omega}.$$

Hence, having found a subsidiary angle ω by the equation $\cot. \omega = \tan. A \cos. c \ldots\ldots (1)$; the sought angle is determined by the equation $\cos. C = \dfrac{\cos. A \sin. (B-\omega)}{\sin. \omega}$.

---

* The log. tangent of this arc will be equal to log. sin. — log. cos., before given, increased by 10.

2. The given quantities being the same as in last example, to determine the angle C.

tan. A	39° 23′ 0″	9·9143020	cos. A	9·8881335
cos. c	68 46 2	9·5588979	sin. ω, ar. comp.	0·0183921
cot. ω	73 26 33½	9·4731999	sin. (B — ω) 39° 41′ 30½″	9·8052682
			cos. 59° 0′ 13″	9·7117938.

As (B — ω) is negative, cos. C must be negative; hence C is the supplement of this, viz. 120° 59′ 47″.

3. Given A = 30° 28′ 11″, B = 130° 3′ 11″, and c = 40°; to determine the other parts. a = 38° 30′, b = 70°, and C = 31° 34′ 26″.

4. Given A = 31° 34′ 26″, B = 30° 28′ 12″, and C = 70° 2′ 3″; to find the angle C. C = 130° 3′ 11″.

5. Given A = 34° 15′ 3″, B = 42° 15′ 13″, and C = 76° 35′ 36″; to find a and b. a = 40° 0′ 10″, b = 50° 10′ 30″.

6. Given A = 51° 30′, B = 131° 30′, and c = 80° 19′ 12″; to find C. C = 59° 15′ 59″.

CASE V. (60.) Given two sides a, b, and the angle A opposite to a; to find the other parts B, C, c.

1. To find the angle B we have, by (46) the proportion,

$$\sin. a : \sin. b :: \sin. A : \sin. B \ldots\ldots (1).$$

2. To find C and c, we have, by Napier's analogies,

$$\left.\begin{array}{l}\cos. \tfrac{1}{2}(a \sim b) : \cos. \tfrac{1}{2}(a+b) :: \tan. \tfrac{1}{2}(A+B) : \cot. \tfrac{1}{2}C \\ \cos. \tfrac{1}{2}(A \sim B) : \cos. \tfrac{1}{2}(A+B) :: \tan. \tfrac{1}{2}(a+b) : \tan. \tfrac{1}{2}c\end{array}\right\} \ldots (2).$$

Or after either C or c is found by one of these analogies, the other part may be found by the proportion sin. A : sin. C :: sin a : sin. c (3); although we shall prefer Napier's analogy to this in order that all ambiguity may be avoided.

If only one of the parts C, c, be required, then it will be best to find first the angle B, by the proportion (1), which operation must be regarded entirely as subsidiary to the determination of the required part, by one of the analogies (2). The part determined by the proportion (1) admits of a double value, since two arcs answer to the same sine; it becomes necessary, therefore, for us to inquire under what circumstances both these values are admissible, and how we may know which to choose when but one solution exists. Referring to the fundamental formula (A), we have $\cos. B = \dfrac{\cos. b - \cos. a \cos. c}{\sin. a \sin. c}$; in which expression we may remark that if cos. b is numerically greater than either cos. a or cos. c, the second member must take the sign of cos. b, consequently, B and b must be of the same species if sin. $b <$ sin. a, or sin. $b <$ sin. c, that is, *an angle must be of the same species as its opposite side, if the sine of this side is less than the sine of either of the other sides.* But if cos. b is numerically less than cos. a, then whether the right hand member be + or — will depend upon the magnitude of cos. c, or cos. c will have two values corresponding to + cos. B, and — cos. B; hence *an angle has two values, when the sine of its opposite side is greater than the sine of the other given side.*

EXAMPLES.

1. Given the side a = 63° 50′, the side b = 80° 19′, and the angle A = 51° 30′; to determine the other parts.

I. *To find the Angle* B.

sin. a	63° 50′	arith. comp.	0·0469582
: sin. b	80 19	- -	9·9937679
:: sin. A	51 30	- -	9·8935444
: sin. B	59° 15′ 47″	-	9·9342705

The angle B admits of two values, because sin. $b >$ sin. a, so that there exist two triangles, having the data proposed.

We shall, however, take the acute value of B.

II. *To find the Angle* C.

cos. ½ $(a \sim b)$	8° 14′ 30″	arith. comp.	0·0045086
: cos. ½ $(a + b)$	72 4 30	- -	9·4882288
:: tan. ½ (A + B)	55 22 53	- -	10·1609412
: cot. ½ C	65 44 53	- -	9·6536786

∴ C = 131° 29′ 46″.

III. *To find the Side* c.

cos. ½ (A $\sim$ B)	3° 52′ 53″	arith. comp.	0·0009973
: cos. ½ (A + B)	55 22 53		9·7544333
:: tan. ½ $(a + b)$	72 4 30		10·4901618
: tan. ½ c	60 24 0		10·2455924
	2		

∴ c = 120 48 0.

2. Given $a = 40° 36′ 37″$, $b = 91° 3′ 25″$, and $\text{A} = 35° 57′ 15″$ to determine C.

I. *To find the subsidiary Angle* B.

sin. a	40° 36′ 37″	arith. comp.	0·1864788
: sin. b	91 3 25	- -	9·9999261
:: sin. A	35 57 15	-	9·7687401
: sin. B	64 24 19	- -	9·9551450.
	or 115 35 41		

The angle B admits of two values, because sin. $b >$ sin. a. We shall suppose the particular triangle under consideration to have B obtuse

II. *To find* C.

cos. ½ $(a \sim b)$	25° 13′ 24″	arith. comp.	0·0435177
: cos. ½ $(a + b)$	65 50 1	- -	9·6121350
:: tan. ½ (A + B)	75 46 28	- -	10·5959988
: cot. ½ C	29 15 28	- -	10·2516515
	2		

∴ C = 58 30 56.

3. Given $a = 40° 18′ 29″$, $b = 67° 14′ 28″$, and $\text{A} = 34° 22′ 17″$; to determine the other parts when B is acute.

B = 53° 35′ 15″, C = 119° 13′ 31″, c = 89° 47′ 6.″

4. Given $a = 84° 14′ 29″$, $b = 44° 13′ 45″$, and $\text{A} = 130° 5′ 22″$; to determine the other parts.

B = 32° 26′ 6½″, C = 36° 45′ 28″, c = 51° 6′ 12″.

5. Given $a = 97° 18′ 39″$, $b = 86° 53′ 46″$, and $\text{A} = 97° 21′ 26″$; to determine c. c = 89° 21′ 37″.

CASE VI. (61.) Given two angles A, B, and the side a opposite to one of them, to find the other parts.

1. To find b we have sin. A : sin. B :: sin. a : sin. b.

2. And to find C and c we may employ Napier's analogies, which need not be here repeated.

The nature of the arc b may be discussed, as in the preceding case. Thus the formula (B), art. (50), gives $\cos. b = \dfrac{\cos. B + \cos. A \cos. C}{\sin. A \sin. C}$; from which it follows, as in the foregoing case, that if cos. B is numerically greater than cos. A, B and b, will be of the same species. If cos. B is numerically less than cos. A, then both the values of b, given by the above proportion, will be admissible, for C may be determined so as to render cos. b positive or negative. Hence *any side will be of the same species as its opposite angle, if the sine of this angle be less than the sine of either of the other angles; and the species of the side b will be indetermined if the sine of its opposite angle* B *be greater than the sine of the other given angle* A. There cannot, therefore, be two solutions unless a and A are of the same species.

EXAMPLES.

1. In an oblique-angled spherical triangle ABC are given, A = 32° 26′ 6½″, B = 130° 5′ 22″, and the side a = 44° 13′ 42″; to determine the other parts.

I. *To find the Side b.*

As sin. A	32°	26′	6½″	arith. comp.	9·8836842
: sin. B	130	5	22	- -	0·2705556
:: sin. a	44	13	45	-	9·8435629
: sin. b	84	14	29	- -	9·9978027;

b has two values, because the sine of B is greater than that of A. We shall take the acute value.

II. *To find the Side c.*

As cos. ½ (A ∼ B)	48°	49′	37¾″	arith. comp.	0·1815543
: cos. ½ (A + B)	81	15	44¼	- -	9·1815890
:: tan. ½ $(a + b)$	64	14	7	-	10·3163591
: tan. ½ c	25	33	6	- -	9·6795024
			2		
∴ c =	51	6	12.		

III. *To find the Angle* C.

As cos. ½ $(a \sim b)$	20°	0′	22″	arith. comp.	0·0270310
: cos. ½ $(a + b)$	64	14	7	- -	9·6381663
:: tan. ½ (A + B)	81	15	44¼	-	10·8133435
: cot. ½ C	18	22	44	. .	10·4785408
			2		
∴ C =	36	45	28.		

2. Given A = 103° 59′ 57½″, B = 46° 18′ 7½″, and a = 42° 8′ 48″; to find the angle C. C = 36° 7′ 52½″.

3. Given A = 17° 46′ 16½″, B = 151° 43′ 52″, and a = 37° 48′; to find the remaining sides, b being obtuse. b = 108°, c = 74° 30′.

SCHOLIUM.

Previously to closing this second part it may be worth while to remark, that if, in the foregoing investigations, we consider the radius of the sphere, upon which the triangles concerned are described, to be infinite, then, as any finite portion of the spheric surface may be considered as a plane, the spherical triangles will become plane triangles, and the sines and tangents of their sides will become identical with the sides themselves; so that all the foregoing rules and formulas, into which cosines, cotangents, secants, or cosecants, of the sides do not enter, are applicable as well to plane as to spherical triangles.

Professor Vince, at page 43 of his Trigonometry, has the following note.

"Difficulties have frequently arisen in consequence of its being supposed that an arc of 90° has a tangent and secant, each infinite. For instance, in a right-angled spherical triangle, radius : cosine of the angle at the base :: tangent of the hypotenuse : tangent of the base; now when the base = 90°, the hypotenuse = 90°; and, therefore, these arcs being equal, if they have any tangents, of whatever value they may be, they must be equal; and, therefore, radius = cosine of the angle at the base, whatever that angle may be. This false conclusion arises from the supposition that an arc increases till it becomes 90°; the tangent and secant increase without limit; and at 90° the arc ceases to have either a tangent or secant, by their definition. As the arc, by increasing, passes through 90°, the tangent and secant increase without limit, cease to exist at 90°, and then begin again at a quantity indefinitely great. And thus in other cases where the tangent or secant of an arc enter into the computation, when the arc becomes 90°, we can draw no conclusion on which we can depend."

The foregoing reasoning is very much calculated to mislead the young student, although it does in reality tend to overturn the author's own hypothesis, and to show that the tangent of 90° must necessarily be infinite.

Taking the example chosen above, by Mr. Vince, we have for the true solution $\cos. < \text{at base} = \text{rad.} \frac{\tan. 90^\circ}{\tan. 90^\circ}$; which must necessarily involve the absurdity noticed above, except tan. 90° be either 0 or ∞; but when the proper value ∞ is put for tan. 90°, then we have $\cos. < \text{at base} = \text{rad.} \frac{\infty}{\infty} = \text{rad.} \frac{0}{0}$; and as $\frac{0}{0}$ admits not only of the particular value 1 fixed upon by Mr. Vince, but of an indefinite number of values, so does cos. < at base.

Upon the same grounds that Mr. Vince has rejected the tangent of 90°, he should have rejected the cosine of 90°, which, however, he admits to be 0.

For $\sin. < \text{at base} = \text{rad.} \frac{\cos. < \text{at vertex}}{\cos. \text{base}}$; but, when both base and hypotenuse are 90°, the angle at the vertex is 90°, and we ought, therefore, to have, according to Mr. Vince, sin. < at base = rad, which is, indeed, one solution, but by no means the only one, because the values of $\frac{0}{0}$ are innumerable.

I

$$\frac{\infty}{\infty} = \frac{\frac{1}{0}}{\frac{1}{0}} = \frac{0}{0} = \frac{\frac{0}{0}}{\frac{0}{0}}$$

PART III.

APPLICATION OF PLANE AND SPHERICAL TRIGONOMETRY TO THE PRINCIPLES OF NAVIGATION AND NAUTICAL ASTRONOMY.

(62.) Having in the two preceding parts of the present treatise pretty fully explained and illustrated the principles of plane and spherical trigonometry, we shall now, for the purpose of showing the practical utility of these principles, apply them to the solution of one of the most important mathematical problems that has ever engaged the attention of man, viz. *to determine the place of a ship at sea.*

When a ship sails from any known place, and a correct account is kept of her various directions, and rates of sailing, her situation at any time may be readily ascertained by the rules of plane trigonometry, and the solution of the problem from these data belongs to *Navigation.*

But it is impossible to measure a ship's course and the distance sailed exactly; so that after a long passage it would be unsafe to compute the place of the ship from the *ship's reckoning.* In such cases, therefore, the solution must be effected from other data, independent of the ship's account; these are furnished by astronomical observation, and the computation is performed by the rules of spherical trigonometry; the problem then becomes one of *Nautical Astronomy.* We shall devote a distinct chapter to each of these important branches.

CHAPTER I.

THE PRINCIPLES OF NAVIGATION.

Definitions.

(63.) 1. The earth is very nearly spherical. For the purposes of Navigation it may be considered as perfectly so. It revolves round one of its diameters, called its *axis,* in about twenty-four hours. This rotation is from the west towards the east, causing the heavenly bodies to have an apparent motion from the east towards the west.

2. The great circle, whose poles are the extremities of the axis, is called the *equator.* The poles of the equator are called also the poles of the earth; the one being the north pole, and the other the south pole.

3. Every great circle which passes through the poles, and which, therefore, cuts the equator at right-angles, is called a *meridian* circle. Through every place on the surface of the earth such a great circle is supposed to be drawn; it is the meridian of the place. It is expedient for the purposes of Geography and Navigation to fix upon one of these meridians as a *first meridian,* from which the meridians of other places are measured.

The English have fixed upon the meridian of Greenwich Observatory for the first meridian.

4. The longitude of any place is the arc of the equator, intercepted between the meridian of that place and the first meridian; the longitude, therefore, is the measure of the angle between the two meridians. The longitude is east or west, according as the place is situated on the right or on the left of the first meridian, when we look towards the north pole.

5. The difference of longitude between two places is the arc of the equator intercepted between the meridians of those places, or the measure of the angle which they include; hence, when the longitudes of the places are of the same denomination, that is, either both east or both west, the difference is found by *subtracting* the one from the other; but when they are of contrary denominations the difference is found by adding the one to the other.

6. The latitude of a place is its distance from the equator, measured on the meridian of the place. Latitude, therefore, is north or south, according to the pole towards which it is measured, and cannot exceed 90°.

7. The small circles drawn parallel to the equator are called *parallels* of latitude. The arc of a meridian, intercepted between two such parallels, drawn through any two places, measures the differences of latitude of those places: when the latitudes are of the same denomination the difference of latitude is found by subtraction, but when the denominations are not the same the difference of latitude is found by addition, like difference of longitude.

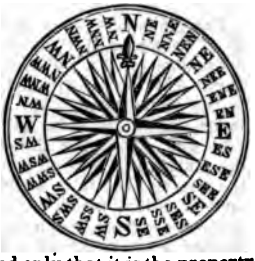
8. The horizon of any place is an imaginary plane, conceived to touch the surface of the earth at that place, and to be extended to the heavens; such a plane is called the *sensible horizon*, and one parallel to it, but passing through the earth's centre is the *rational horizon* of the place. A line drawn across the horizon and through the place, in the plane of its meridian, is the meridian of the horizon, or the north and south line; the horizontal line through the same point, and perpendicular to this, is the east and west line. Besides the North, South, East, and West, points thus marked on the boundary of the horizon, this boundary is conceived to be subdivided into other intermediate points, corresponding to the divisions in the circle below.

9. The *course* of a ship is the angle which her track makes with the meridians; so long as this angle remains the same, the ship is said to sail on the same *rhumb line* or *loxodromic curve.* The magnitude of the angle or the course is indicated by the *mariner's compass.*

10. The *Mariner's compass* consists of a circular card whose circumference is divided into thirty-two equal parts, called *points*, and each of these are subdivided into four equal parts, called quarter points; across this card is fixed a slender bar of magnetized steel, called *the needle;* the tapering extremities of which point to two diametrically opposite divisions of the card. These opposite divisions are marked N. and S., corresponding to the *north* and *south* poles, or ends, of the magnetized bar. The diameter W. E., at right angles to the diameter N. S., point out the west and east points; these four are called the cardinal points, and the others are marked as in the subjoined diagram.

Thus one point from the north towards the east is *north by east;* two points, *north, north east;* three points, *north-east by north;* and so on. (See the table of Rhumbs at the end, p. ~~300.~~)

The card thus furnished being now suspended horizontally, and so as to allow the needle to settle itself freely, will point out the four cardinal points of the horizon, as also the several intermediate points, provided only that it is the property of the magnetic needle to point due north and south. Such, however, is not strictly the case, as the needle is found from accurate observations, to deviate from this position, and at some places very considerably, and this deviation is itself subject to variation. But the true



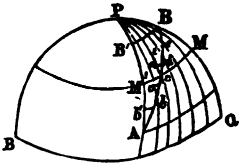
direction of the compass, or the angle it makes at any place with a line pointing duly north and south, may be ascertained at any time by astronomical observations, and thus the deviation of the compass-points, from the corresponding points of the horizon, may always be found and allowed for.

The compass is so placed on ship-board that the vertical plane, cutting the ship from stem to stern, may pass through the centre of the card, so that that point of the compass which is directed to the ship's head shows the *compass-course*, and the proper correction for variation being applied the true course will be obtained.

11. A ship's *rate* of sailing is determined by means of an instrument, called the *Log*, and an attached line called the *log-line*. The log is a piece of wood forming the sector of a circle, and its rim is so loaded with lead that when *heaved* into the sea it assumes a vertical position, with its centre barely above the water. The log line is so attached as to keep the face of the log towards the ship, that it may offer the greater resistance to be dragged after the ship by the log-line, as it unwinds from a reel on board, by the advancing motion of the ship. The length of line thus unwound in *half a minute*, gives the rate of sailing. For convenience the log-line is divided into equal parts, called *knots*, of which each measures the 120th of a nautical or geographical mile,* and as half a minute is the 120th of an hour, it follows that the number of knots, and parts of a knot, run in half a minute expresses the number of miles and parts of a mile, run in an hour, at the same rate of sailing.

On Plane Sailing.

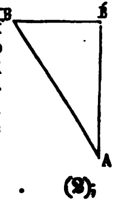
(64.) Let the annexed diagram represent a portion of the earth's surface, P being the pole, and EQ the equator. Let AB be any rhumb line, or track described by a ship in sailing on a single course from A to B. Conceive the path of the ship to be divided into portions Ab, bc, cd, &c. so small that each may differ insensibly from a straight line, and draw meridians through these several divisions, as also the parallels of latitude bb', cc', dd', &c.; we shall thus have a series of triangles described on the surface of the globe, but so small that each may be considered as a plane triangle. These triangles are all similar, for the angles at b', c', d', &c. are right angles, and the ship's path cuts all the meridians at equal angles; hence (Geom. prop. 9, Book 6,)



$Ab : Ab' :: bc : bc' :: cd : cd'$, &c. therefore, (Geom. prop. 5, Book 5,)

$$Ab : Ab' :: Ab + bc + cd + \&c. : Ab' + bc' + cd' + \&c.$$

But $Ab + bc + cd +$ &c. is the whole distance sailed, and $Ab' + bc + cd' +$ &c. $= AB'$, is the difference of latitude between A and B; consequently, if a right-angled triangle ABB′, similar to the small triangle Abb', be constructed, that is, one in which the angle A is equal to the course, and if the hypotenuse AB represent the distance sailed, the side AB′ will represent the difference of latitude. Moreover, the other side BB′, or that opposite to the course, will represent the sum $b'b + c'c + d'd +$&c. of all the minute *departures* which the ship makes from the successive meridians which it crosses; for as the triangle ABB′, in this last diagram, is similar to the small triangle Abb', in the former, we have



$$Ab : bb' :: AB : BB' \quad . \quad . \quad (1);$$

but in the first figure we have $Ab : bb' :: bc : cc' :: cd : dd'$, &c.

$$\therefore Ab : bb' :: Ab + bc' + cd + \&c. : bb' + cc' + dd' + \&c. \quad . \quad (2);$$

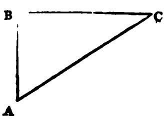
---

[*] The geographical mile is one minute of the earth's circumference. Taking the diameter at 7916 English miles, the geographical mile will be about 6079 feet.

consequently, since the three first terms of (1) are respectively equal to those of (2), the remaining terms BB′, $bb' + cc' + dd' +$ &c. must be equal. This last quantity is called the *departure* of the ship in sailing from A to B. It follows, therefore, that the *distance sailed*, the *difference of latitude* made, and the *departure*, are correctly represented by the *hypotenuse* and *sides* of a right-angled plane triangle, in which the angle opposite the departure is the *course*, so that when any two of these four things are given the others may be found simply by the resolution of a right-angled plane triangle; as far, therefore, as these particulars are concerned the results are the same as if the ship were sailing on a plane surface, the meridians being parallel straight lines, and the parallels of latitude cutting them at right-angles; and hence that part of Navigation in which only distance sailed, departure, difference of latitude, and course are considered, is called *Plane sailing*.

EXAMPLES.

1. A ship from latitude 47° 30′ N. has sailed S. W. by S. 98 miles, What latitude is she in, and what departure has she made?



Let C be the place sailed from, CB the meridian, the angle C = 3 points = 33° 45′ and CA = 98 miles, the distance sailed; then CB will be the difference of latitude, and BA the departure

As rad.	.	10·	As rad.	.	10·
: Distance	98	1·9912261	: Dist.	.	1·9912261
:: cos. course	33° 45′	9·9198464	:: sin. course		9·7447390
: Diff. of lat.	81·48	1·9110725	: Departure	54·45	1·7359651

Latitude left 47° 30′ N.
Diff. of lat. = 81·48 minutes = 1° 22′ S. Dep. = 54·45 miles W.
Latitude in 46 8 N.

2. A ship sails for 24 hours on a direct course, from lat. 38° 32′ N., till she arrives at lat. 36° 56′ N.; the course is between the S. and E., and the rate 5½ miles an hour. Required the course, distance, and departure.

Lat. left 38° 32′ N. 24 × 5½ = 132 miles, the distance.
Lat. in 36 56 N.

Diff. 1 36 = 96 miles.

As Dist.	132	2·1205739	As rad.	.	10·
: Rad.	.	10·	: Dist.	.	2·1205739
:: Diff. lat.	96	1·9822712	:: sin. course	.	9·8364771
: cos. course	43° 20′	9·8616973	: Dep.	90·58	1·9570510

Hence the course is S. 43° 20′ E., and the departure 90·58 miles E.

3. A ship sails from lat 3° 52′ S. to lat. 4° 30′ N., the course being N. W. by W. ½ W.; required the distance and departure.
Distance 1065 miles, Departure 938·9 miles W.

4. Two ports lie under the same meridian, one in latitude 52° 30′ N., and the other in latitude 47° 10′ N. A ship from the southernmost sails due east at the rate of 9 miles an hour, and two days after meets a sloop which had sailed from the northernmost port; required the sloop's direct course and distance run.
Course S. 53° 28′ E., or S. E. ¾ E.; the distance run 537·6 miles:

5. If a ship from lat. 48° 27′ S., sail S. W. by W., 7 miles an hour, in what time will she arrive at the parallel of 50° S.? In 23·914 hours.

6. If after a ship has sailed from lat. 40° 21′ N. to lat. 46° 18′ N., she be found 216 miles to the eastward of the port left; required her course and distance sailed. Course N. 31° 11′ E., dist. 417·3 miles.

Traverse Sailing.

(65.) When a ship in going from one place to another, sails on different courses, it is called *traverse sailing;* and the determination of the single course and distance from the one place to the other is called *working* or *compounding* the traverse. To effect this, it is obviously merely necessary to find the difference of latitude, and departure, due to each distinct course, to take the aggregate of these for the whole difference of latitude and departure, and from these to find, as in last article, the single course and distance. It is usual in thus compounding courses to form a table consisting of six columns, called a *traverse table*, and in the first column to register the several component courses, and against them, in the second column, the proper distances; the next two columns, marked N. and S., are to receive the several differences of latitude, whether N. or S., due to each course, and distance, and the two remaining columns marked E. and W. are to receive, in like manner, the corresponding *eastings* and *westings*, that is, the departures. When these several particulars are all inserted, the columns are added up, and the difference of the results of the N. and S. columns will be the required difference of latitude, and the difference of the results of the E. and W. columns will be the corresponding departure.

The columns appropriated to the differences of latitude and departures are usually filled up from a table already computed to every quarter point of the compass, and to all distances from one mile up to a hundred or 120; so that, by entering this table with any given course and distance, the proper difference of latitude and departure is found by inspection. Most books on navigation contain also a second and more enlarged traverse table, being computed to every course from a quarter of a degree up to forty-five degrees. This latter table we have not thought it necessary to insert in our collection, but the former we have given (Table IV.), and its use is fully explained in the introduction prefixed.

But there is another mode of finding the direct course and distance, much practised by seamen, viz. by construction. To facilitate this construction the *mariner's scale* is employed, which is a two-foot flat rule exhibiting several scales on each side, by help of which and a pair of compasses the usual problems in sailing may be all solved. One of these scales is a scale of chords, commonly called a scale of rhumbs, being confined to every quarter point of the compass; and another is a more enlarged scale of chords, being to every single degree. Both these scales are constructed in reference to the same common radius, so that the chords on the scale of rhumbs belong to that circle whose radius equals the chord of 60° on the scale of chords; and the method of laying down a traverse from these scales, and one of equal parts, and of thence measuring the equivalent single course, and distance made good, will be at once understood from the following examples.

EXAMPLES.

1. A ship sails from a place in lat. 24° 32′ N., and has run the following courses and distances, viz.

1st, S. W. by W., distance 45 miles; 2d, E. S. E., distance 50 miles; 3d, S. W., distance 30 miles; 4th, S. E. by E., distance 60 miles; 5th, S. W. by S. ½ W., distance 63 miles: required her present latitude, with the direct course and distance from the place left to the place arrived at.

Traverse Table.

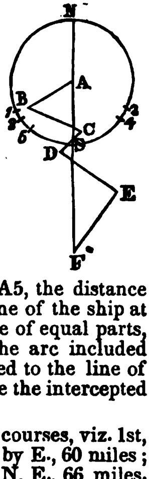
Courses.	Dist.	Difference of Lat.		Departure.	
		N.	S.	E.	W.
S. W. by W.	45		25·0		37·4
E. S. E.	50		19·1	46·2	
S. W.	30		21·2		21·2
S. E. by E.	60		33·3	49·9	
S. W. by S. ¼ W.	63		50·6		37·5
			149·2	96·1	96·1

It appears from the results of this table that the difference of latitude made by the ship during the traverse is 149·2 S. = 2° 29′ S.

Lat. left	- - - - -	24° 32′ N.
Diff. lat.	- - - - -	2 29 S.
Lat. in	- - - - -	22 3 N.

It appears also that the departures east are equal to the departures west, so that the ship has returned to the meridian she sailed from, consequently the direct course from the place left to that come to is due south, and the distance is equal to the difference of latitude, which is 149·2 miles.

The construction of this traverse is as follows.



With the chord of 60°, taken from the line of chords on the mariner's scale, describe the horizon circle, and draw the north and south line N. S. From the line of rhumbs take the chords of the several courses, and as these are all southerly, they must be laid off from the south point S, those which are westerly to the left, and those which are easterly to the right, their extremities being marked 1, 2, 3, &c. in the order of the courses. This done, lay off from any convenient scale of equal parts, and in the direction A1 the distance AB sailed on the first course; then in the direction parallel to A2, the distance BC sailed on the second course; in the direction parallel to A3, the distance CD on the third course; in the direction parallel to A4, the distance DE on the fourth course; and, lastly, in the direction parallel to A5, the distance EF on the third course; then F will represent the plane of the ship at the end of the traverse; FA, being applied to the scale of equal parts, will show the distance made good, and the chord of the arc included between this distance, and the meridian, being applied to the line of rhumbs, will show the direct course. In the present case the intercepted arc will be 0, showing that F is on the meridian of A.

2. A ship from lat. 28° 32′ N., has run the following courses, viz. 1st, N. W. by N., 20 miles; 2d, S. W., 40 miles; 3d, N. E. by E., 60 miles; 4th, S. E., 55 miles; 5th, W. by S., 41 miles; 6th, E. N. E., 66 miles. Required her present latitude, the distance made good, and the direct course from the place left to that come to.

The direct course is due east, and distance 70·2 miles, the ship being in the same latitude at the end as at the beginning of the traverse.

3. A ship from lat. 41° 12′ N., sails S. W. by W., 21 miles; S. W. ½ S. 31 miles; W.S.W. ½ S., 16 miles; S. ¾ E., 18 miles; S. W. ¼ W., 14 miles; and W. ½ N., 30 miles: required the latitude of the place arrived at, and the direct course and distance to it.

Lat. 40° 5′ N.; course S. 52° 49′ W.; distance 111·7 miles.

4. A ship from Cape Clear, in lat. 51° 25′ N., sails 1st, S. S. E. ¼ E., 16 miles; 2d, E.S.E., 23 miles; 3d, S.W. by W. ½ W., 36 miles; 4th, W. ¾ N., 12 miles; 5th, S.E. by E. ¼ E., 41 miles: required the distance made good, the direct course, and the latitude in?

Traverse Table.

Courses.	Dist.	Difference of Lat.		Departure.	
		N.	S.	E.	W.
S. S. E. ¼ E.	16		14·5	6·3	
E. S. E.	23		8·8	21·3	
S. W. by W. ½ W.	36		17·0		31·8
W. ¾ N.	12	1·8			11·9
S. E. by E. ¼ E.	41		21·1	35·2	
		1·8	61·4	63·3	43·7
			1·8	43·7	
			59·6	19·6	

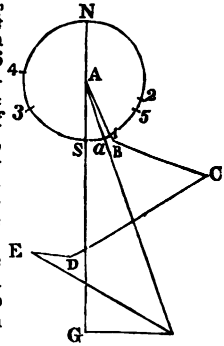
Lat. left . . . . . 51° 25′ N.
Diff. lat. 59·6*m* . . . . . 1 0 S.

Lat. in 50 25 N.

As diff. lat. 59·6 .	1·7752463	As sin. course .	9·4946205
: rad. . .	10·	: departure . .	1·2922561
:: departure 19·6 .	1·2922561	:: rad . .	10·
: tan. course 18° 12′	9·5170098	: distance 62·74 .	1·7976356

therefore, as the difference of latitude is south, and the departure east, the direct course is S. 18° 12′ E., and the distance made good 62·74 miles.

To construct this traverse, describe, as before, the horizon circle, with a radius equal to the chord of 60°, and taking from the line of rhumbs the chord of the first course, 2¼ points, apply it from S. to 1, to the right of S.N., as this course is south-easterly; apply, in like manner, the chord of the second course, 6 points from S. to 2, also to the right of the meridian line; apply the chord of the third course, 5½ points from S. to 3, to the left of the meridian, the chord of the fourth course, 7¼ from N. to 4, to the left of N. S., this course being north-westerly, and, lastly, apply the chord of the fifth course, 5¼ points, from S. to 5, to the right of S. N. In the direction A1, lay off the distance AB = 16 miles from a scale of equal parts; in the direction parallel to AQ, lay off the distance BC = 23 miles; in the direction parallel to A3, lay off CD = 36 miles; in the direction parallel to A4, lay off DE = 12 miles; and, lastly, in the direction parallel to A5, lay off EF = 41, then F will be the place of the ship at the end of the traverse; consequently, AF will be the distance made good, and the angle FAS the direct course; applying, therefore, the distance AF to the scale of equal parts, we shall find it reach from 0 to 62¾; and applying the distance S*a* to the line of chords, we shall find it reach from 0 to 18°.



5. A ship runs the following courses, viz.
1st, S. E., 40 miles; 2d, N.E., 28 miles; 3d, S.W. by W., 52 miles;

4th, N.W. by W., 30 miles; 5th, S.S.E., 36 miles; 6th, S.E. by E., 58 miles: required the direct course and distance made good.

Direct course S. 25° 59′ E., or S.S.E. ¼ E. nearly; distance 95·87 miles.

6. A ship in latitude 37° 10′ N. is bound to a port in the latitude of 33° 0′ N. which lies 180 miles west of the meridian of the ship; but by reason of contrary winds, she sails the following courses, viz. S. W. by W. 27 miles, W. S. W. ½ W. 30 miles, W. by S. 25 miles, W. by N. 18 miles, S. S. E. 32 miles, S. S. E. ¼ E. 27 miles, S. E. 25 miles, S. 31 miles, and S. S. E. 39 miles. Required the latitude the ship is in, and her departure from the meridian, with the course and distance to her intended port?

The difference of latitude and departure made on each course, will be seen by sketching a traverse table; hence it appears that the difference of latitude made good is 169·4 miles, the departure 47·4 miles, and by plane sailing, the course S. 15° 38′ W. and distance 175·9 miles; and the course to the intended port S. 58° 42′ W., distance 155·2 miles; the latitude being 34° 21′ N.

These examples will, perhaps, suffice to illustrate the principles of plane sailing, in which, course, distance, difference of latitude, and departure, are the only things concerned. The determination of the difference of longitude made on any course cannot be effected by these principles, for this element is not the same as if the meridians were all parallel to each other, as is the case with the other elements. The finding of the difference of longitude is the easiest when the ship sails due east or due west, that is, upon a parallel of latitude; this is called *parallel sailing.*

Parallel Sailing.

(66.) The theory of parallel sailing is comprehended in the following proposition, viz.

The cosine of the latitude of the parallel is to the distance run as the radius to the difference of longitude. This may be demonstrated as follows.

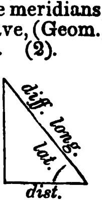
In the figure, at page 42, let IQH represent the equator, and BDA any parallel of latitude; CI will be the radius of the equator, and cB the radius of the parallel. Let BD be the distance sailed, then the difference of longitude will be measured by the arc IQ of the equator, and since (Geom., prop. 12, Cor. 2, B. 7) similar arcs are to each other as the radii of the circles to which they belong, we have

$$cB : CI :: \text{dist. } BD : \text{diff. long. } IQ.$$

But cB is the cosine of the latitude IB to the radius CI, that is, cB is CI times the trigonometrical cosine of the latitude; hence the above proportion is CI × cos. lat. : CI :: distance : diff. long.

$$\therefore \text{cos. lat.} : \text{Rad.} \; (=1) :: \text{distance} : \text{diff. long.} \quad . \quad . \quad . \quad (1).$$

Corollary: hence if the distance between any two meridians, measured in a parallel in latitude L be D, and the distance of the same meridians, measured on a parallel, in latitude L′ be D′, we shall have, (Geom., prop. 15, Cor. 2, Book 5,) cos. L : D :: cos. L′ : D′ . . . (2).



Hence if one of the legs of a right-angled triangle represent the distance run on any parallel, and the adjacent acute angle be equal to the degrees of lat. of that parallel, then the hypotenuse will represent the difference of longitude, since this hypotenuse will be determined by the foregoing proportion (1). It follows, therefore, that any problem in parallel sailing may be solved by the traverse table, computed to degrees, as a simple case of plane sailing; for by considering the latitude as the course, and the distance as the difference of latitude, the corresponding distance in the table will express the difference of longitude.

EXAMPLES.

1. A ship from latitude 53° 56′ N., longitude 10° 18′ E., has sailed due west, 236 miles: required her present longitude.

By the rule;	As cos. lat. 53° 56′	9·7699134
	: radius	10·
	:: distance 236	2·3729120
	: diff. long. 408·87	2·6029986
	Long. left	10° 18′ E.
	Diff. long. $= \frac{400}{60}$ degrees =	6 40 W.
	Long. in	3 38 E.

2. If a ship sail E. 126 miles, from the North Cape, in lat. 71° 10′ N., and then due N., till she reaches lat. 73° 26′ N.; how far must she sail W. to reach the meridian of the North Cape?

Here the ship sails on two parallels of latitude, first on the parallel of 71° 10′, and then on the parallel of 73° 26′, and makes the same difference of longitude on each parallel. Hence, by the corollary,

As cos. lat. 71° 10′ arith. comp.	0·4910444
: distance 126	2·1003705
:: cos. lat. 73 26	9·4550441
: distance 111·3	2·0464590.

3. A ship in latitude 32° N., sails due east; till her difference of longitude is 384 miles; required the distance run. 325·6 miles.

4. If two ships in latitude 44° 30′ N., distant from each other 216 miles, should both sail directly south till their distance is 256 miles, what latitude would they arrive at? 32° 17′ S.

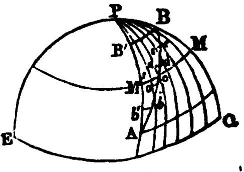
5. Two ships in the parallel of 47° 54′ N., have 9° 35′ difference of longitude, and they both sail directly south, a distance of 836 miles: required their distance from each other at the parallel left, and at that reached. 385·5 miles, and 479·9 miles.

Middle Latitude Sailing.

(67.) Having seen how the longitude which a ship makes when sailing on a parallel of latitude may be determined, we come now to examine the more general problem, viz. to find the longitude a ship makes when sailing upon any oblique rhumb.

There are two methods of solving this problem, the one by what is called *middle latitude sailing*, and the other by *Mercator's sailing*. The first of these methods is confined in its application, and is moreover somewhat inaccurate even where applicable; the second is perfectly general, and rigorously true; but still there are cases in which it is advisable to employ the method of middle latitude sailing, in preference to that of Mercator's sailing; it is, therefore, proper that middle latitude sailing should be explained, especially since, by means of a correction to be hereafter noticed, the usual inaccuracy of this method may be rectified.

Middle latitude sailing proceeds on the supposition that the departure or sum of all the meridional distances $b'b$, $c'c$, $d'd$, &c. from A to B, is equal to the distance M′M of the meridians of A and B, measured on the middle parallel of latitude between A and B.



This supposition becomes very inaccurate when the course is small, and the distance run great; for it is plain that the middle latitude

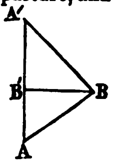
[illegible] departure

distance will receive a much greater accession than the departure, if the track of B cuts the successive meridians at a very small angle.

The principle approaches nearer to accuracy as the angle A of the course increases, because then as but little advance is made in latitude, the several component departures lie more in the immediate vicinity of the middle latitude parallel. But still, as in very high latitudes, a small advance in latitude makes a considerable difference in meridional distances, this principle is not to be recommended in such latitudes if much accuracy is required.

By means, however, of a small table of corrections, recently constructed by *Mr. Workman*, and judiciously introduced by *Mr. Riddle* in the second edition of his valuable Treatise on Navigation, the imperfections of the middle latitude method may be removed, and the results of it rendered in all cases accurate. This table we have given at the end of the present volume, and have explained its construction in the introductory explanation to the Tables.

The rules for middle latitude sailing may be thus deduced.



It has been seen at (64) that the difference of latitude, departure, and distance, sailed on any oblique rhumb, will be all accurately represented by the sides AB′, B′B, AB, of a plane triangle. Now, by the present hypothesis, the departure B′B is equal to the middle latitude distance between the meridians of the places sailed from, and arrived at, so that the difference of longitude of the two places of the ship is the same as if it had sailed the distance B′B, on the middle latitude parallel; the determination of this difference of longitude is, therefore, reduced to a case of parallel sailing, for BB′, now representing the distance on the parallel, and an angle A′BB′ being made equal to the latitude of that parallel, we shall have the difference of longitude, represented by the hypotenuse A′B. We thus have the following theorems, viz. in the triangle A′B′B,

cos. A′BB′ : BB′ : : radius : BA′;

that is, I. Cos. mid. lat. : departure : : radius : diff. of long.

In the triangle A′BA, sin. A′ : AB : : sin. A : A′B;

that is, II. Cos. mid. lat. : distance : : sin. course : diff. long.

In the triangles ABB′, A′BB′,

AB′ tan. A = B′B; A′B cos. A′BB′ = B′B; therefore,

AB′ : A′B : : cos. A′BB′ : tan. A; that is,

III. Diff. lat. : diff. long. : : cos. mid. lat. : tan. course.

These three proportions comprise the theory of middle latitude sailing, and when to the middle latitude the proper correction, taken from Mr. Workman's table is added, these theorems will be rendered strictly accurate.

EXAMPLES.

1. A ship, in latitude 51° 18′ N., longitude 22° 6′ W., is bound to a place in the S.E. quarter, 1024 miles distant, and in lat 37° N.: what is her direct course and distance, as also the difference of longitude between the two places.

Lat. from 51° 18′ N. }	Sum of latitudes	88° 18′
Lat. to 37 0 N. }		
	Mid. lat.	44 9

Diff. lat. 14 18 = 858 miles.

For the course.			For the diff. long.		
As distance	1024	3·0103000	cos. mid. lat.	44° 9′ ar. com.	0·1441668
: radius		10·	: tan. course	33 5	9·8138993
: : diff. lat.	858	2·9334873	: : diff. lat.	858	2·9334873
: cos. course	33° 5′	9·9231873	: diff. long.	779	2·8915534

In this operation the middle latitude has not been corrected, so that the difference of longitude here determined is not without error. To find the proper correction look for the given middle latitude, viz. 44° 9′ in the table of corrections, the nearest to which we find to be 45°; against this and under 14° diff. of lat. we find 27′, also under 15°, we find 31′, the difference between the two being 4′: hence corresponding to 14° 18′ the correction will be about 28′. Hence the corrected middle latitude is 44° 37′, therefore,

cos. corrected mid. lat. 44° 37′ ar. comp. 0.1483780
: tan. course 33 5 9·8138993
: : diff. lat. 858 2·9334873

: diff. long. 786·6 2·8957646;

therefore, the error in the former result is about 7½ miles.

2. A ship sails in the N. W. quarter, 248 miles, till her departure is 135 miles, and her difference of longitude 310 miles: required her course, the latitude left, and the latitude come to.

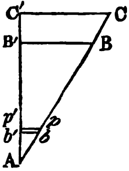
Course N. 32° 59′ W.; lat. left 62° 27′ N.; lat. in 65° 55′ N.

3. A ship, from latitude 37° N., longitude 9° 2′ W., having sailed between the N. and W., 1027 miles, reckons that she has made 564 miles of departure; what was her direct course, and the latitude and longitude reached?

Course N. 33° 19′ W. or N. W. by N. nearly: lat. 51° 18′ N.; long 22° 8′ W.

4. Required the course and distance from the east point of St. Michael's, lat. 37° 48′ N., long. 25° 13′ W., to the Start Point, lat. 50° 13′ N., long. 3° 38′, the middle latitude being corrected by Workman's Table. Course N. 51° 11′ E.; distance 1189 miles.

Mercator's Sailing.



(68.) It has been already seen that when a ship sails on any oblique rhumb, the difference of latitude, the departure, and the distance run, are truly represented by the sides of a right-angled plane triangle. The departure B′B represents the sum of all the very small meridian distances, or elementary departures, $b'b$, $c'c$, &c. in the diagram, at page 74, the difference of latitude AB represents the sum of all the corresponding small difference in the figure referred to; and the distance AB, the sum of all the distances to which these several departures and differences belong, and each of these elements are supposed to be taken so excessively small as to form on the sphere a series of triangles, differing insensibly from plane triangles.

Let $Ab'b$ in the annexed diagram represent one of these elementary triangles, $b'b$ will be one of the elements of the departure, and Ab', the corresponding difference of latitude; and as $b'b$ is a small portion of a parallel of latitude, it will be to a similar portion of the equator, or of the meridian, as the cosine of its latitude to radius (66). This similar portion of the equator, or of the meridian, being the difference of longitude between b' and b. Suppose now the distance Ab prolonged to p, till the departure $p'p$ is equal to the difference of longitude of b', and b, then $b'b$ will be to $p'p$ as the cosine of the latitude of $b'b$ to the radius; but $b'b : p'p :: Ab' : Ap'$; hence the proper difference of latitude Ab' is to the increased difference Ap' as the cosine of the latitude of $b'b$ to the radius. Calling, therefore, the proper difference of latitude d, the increased difference D, the latitude of $b'b$, C, and the radius R, we have $D = \frac{Rd}{\cos. l} = Rd \sec. l$; the ship, therefore, having made the small departure $b'b$, and the difference of latitude Ab', must continue her course till the difference of latitude becomes D, in order that her departure may become equal to the difference of longitude corresponding to $b'b$. Conceiving all the elementary distances to be in this manner increased, the sum of all the corresponding increased departures will necessarily be the whole difference of longitude made by the ship during the course; to represent, therefore, the difference of longitude due to the departure B′B, and difference of latitude AB′, we must prolong AB′ till AC′ is equal to the sum of all the elementary differences increased as above, and the departure C′C, due to this difference of latitude, will represent the difference of longitude actually made in sailing from A to B. The determination of AC′ requires the previous determination of all its elementary parts; if d be taken equal to 1′, each of these parts will be expressed by $D = 1' \sec. l$, from which equation the values of D, corresponding to every minute of l, from the equator to the pole, may be calculated; and by the continued addition of these there will be obtained, in succession, the values of the increased latitude corresponding to 1′, 2′, 3′, &c. of proper latitude; these values are called the *meridional parts*, corresponding to the several proper latitudes, and when registered in a table, form a table of meridional parts, given in all books on Navigation.

The following may serve as a specimen of the manner in which such a table may be constructed, and, indeed, of the manner in which the first table of meridional parts was actually formed by *Mr. Wright*, the proposer of this ingenious and valuable method.

Mer. pts. of 1′ = nat. sec. 1′.
Mer. pts. of 2′ = nat. sec. 1′ + nat. sec. 2′.
Mer. pts. of 3′ = nat. sec. 1′ + nat. sec. 2′ + nat. sec. 3′.
Mer. pts. of 4′ = nat. sec. 1′ + nat. sec. 2′ + nat. sec. 3′ + nat. sec. 4′.
&c. &c.

Hence, by means of a table of natural secants, we have

		Nat. secs.	Mer. parts.
Mer. pts. of 1′ =	. .	1·0000000	= 1·0000000
Mer. pts. of 2′ =	1·0000000 +	1·0000002	= 2·0000002
Mer. pts. of 3′ =	2·0000002 +	1·0000004	= 3·0000006
Mer. pts. of 4′ =	3·0000006 +	1·0000007	= 4·0000013
	&c.	&c.	

There are other methods of construction, but this is the most simple and obvious; we shall, however, presently have to advert to another process of computation, by which the meridional parts for any latitude may be found independently of previous calculations. The meridional parts, thus determined, are all expressed in geographical miles, because in the general expression $D = 1' \sec. l$, 1′ is a geographical mile.

Having thus formed a table of meridional parts, (see Riddle's Navigation, or Robertson's Treatise,) if we enter it with the latitudes sailed from, and come to, and take the difference of the corresponding parts in the table, the remainder will be the meridional difference of latitude, or the line AC′ in the preceding diagram, and the difference of longitude C′C will then be obtained by this proportion, viz.

1. *As radius is to the tangent of the course, so is the meridional difference of latitude to the difference of longitude*; or if the departure be given instead of the course then the proportion will be

2. *As the proper difference of latitude is to the departure, so is the meridional difference of latitude to the tangent of the course.* Other proportions immediately suggest themselves from the preceding figure.

(69.) As an example of Mercator's, or more properly of Wright's, sailing, let us take the following.

1. Required the course and distance from the east point of St. Michael's to the Start Point.

Start	lat. 50° 13′ N.	Mer. pts.	3494·8	long.	3° 38′ W.
St. Michael's	lat. 37 48 N.	Mer. pts.	2453·1	long.	25 13 W.
	12 25	Mer. diff. lat.	0141·7	diff. long.	21 35 W.
	60				60
Proper diff. lat.	745 miles.				1295 miles.

For the course.			For the distance.		
As Mer. diff. lat.	1041·7	3·0177427	As cos. course	.	9·7971501
: radius . .		10·	: prop. diff. lat.	.	2·8721563
:: diff. long.	1295	3·1122698	:: rad. . .		10·
: tan. course 51° 11′ E.		10·0945271	: distance	1189	3·0750062

(70.) In the absence of a table of meridional parts, a table of logarithmic tangents may be employed for the same purposes; and, indeed, the meridional parts corresponding to any given latitude may be expeditiously computed by help of such a table, and independently of any previous computations.

It was shown, at page 76, that if a ship in latitude x, vary her latitude by a very small portion Δx, and that she continue her course till her departure equals the difference of longitude due to the difference of latitude Δx, then the enlarged difference of latitude (Δy), due to this departure, will be $\Delta y = \text{sec. } x\,\Delta x \therefore \frac{\Delta y}{\Delta x} = \text{sec. } x$. This expression, it must be remembered, is nearer the truth the smaller we suppose Δx to be, and is, therefore, accurately true only when $\Delta x = 0$; in other words, sec. x is the value to which the ratio $\frac{\Delta y}{\Delta x}$ continually approaches, as we continually diminish Δx, (and in consequence Δy,) and which value it actually becomes only when the terms of the ratio vanish, and the fractions take the form $\frac{0}{0}$. By adopting the language of the *Differential Calculus* we have, in this case, $\frac{dy}{dx} = \text{sec. } x \therefore dy = \text{sec. } x dx = \frac{dx}{\text{cos. } x} \therefore y = \text{log. tan. } (45° + \frac{1}{2} x)$, *see Int. Calculus*, p. 69; the logarithm here used is the *Naperian*. To change it into a common logarithm we must multiply by the modulus 2·302585, &c.; it must be observed, however, that it is the logarithm of the *natural* tangent which is here expressed, and not the tabular *logarithmic* tangent; it is, therefore, equal to the tabular logarithmic tangent minus 10. Hence, employing the table of logarithmic tangents, we may compute y from the formula $y = 2{\cdot}302585 \{\text{log. tan. } (45° + \frac{1}{2} x) - 10\} \times \text{Rad.}$ and thus, as stated above, the meridional parts, y, corresponding to any given latitude x, may be expeditiously computed, independently of any previous computations.

The tables of meridional parts are usually expressed in nautical miles, and we shall have the number of miles in y, if, instead of multiplying by the radius of the earth, we multiply by the number of miles or minutes in it. Now in every circle the radius is equal to 34·3774679 minutes of that circle, because 3·14159, &c. : 180° :: 1 : 343·774679 minutes; hence for the number of miles in y the expression is 7915·7044679 $\{\text{log. tan. } (45° + \frac{1}{2} x) - 10\}$; or, since tan. $45° + \frac{1}{2} x = \text{cot. } 45° - \frac{1}{2} x$, and, since, moreover, log. cot. $= \text{log. } \frac{R^2}{\text{tan.}} = 20 -$ tan.; this expression 34

R [illegible] but is

1 or 57·3 [illegible] for [illegible] must also

may be written thus, 7915·7044679 $\{10 - \log. \tan. (45^\circ - \frac{1}{2}x)\}$, which gives the rule in the text. We had intended to have introduced here some other particulars relating to Mr. Wright's projection of the meridian line, but we are precluded from doing so, as this treatise has already exceeded the limits assigned to it. We must, therefore, content ourselves with referring the student to Robertson's Nav. vol. II. p.135—146.

The practical rule is as follows, viz. if the log. tangent of half the complement of any latitude be subtracted from 10, and the remainder be multiplied by 7915·7044679, &c. the product will give the meridional parts in miles, corresponding to that latitude.

From this rule the method of operating with logarithmic tangents, instead of with meridional parts, may be easily derived. Call t, t', the logarithmic tangents of the half complements of the latitudes left and reached, and put a for the constant multiplier 7915·7044, &c. Then, by the rule just given, the meridional difference of latitude will be

$$a\{(10-t')-(10-t)\} = a(t-t') = (t-t')\, 10000 \div \frac{10000}{a}.$$

Now $\log. \frac{10000}{a} = \cdot 1015104$, therefore, the logarithm of the meridional difference of latitude is found by removing the decimal point in the difference $t-t'$ four places to the right, and then subtracting the constant number ·1015103. Hence, if instead of the logarithm of the radius 10, we use 10·1015104, and instead of the meridional parts the logarithmic tangents t, t', of the complements of the half latitudes, taking care in setting down the difference of these to remove the decimal point four places to the right, the proportion (1), at page 77, may be still employed. Thus, taking the foregoing example, the operation by this method will be as follows.

$$\tfrac{1}{2}\text{ Co. lat.} \begin{cases} \text{St. Michael's } 26^\circ\ 6' & \therefore t = 9{\cdot}6901030 \\ \text{Start } 19\ 53\tfrac{1}{2} & \therefore t' = 9{\cdot}5585051 \end{cases}$$

$t - t' =$ 1315·979

As $t-t'$	1315·979	arith. comp. -	6·8807448
: Const. log.	- -	- - -	10·1015104
:: diff. long.	1295	- - -	3·1122698
: tan. course N. 51° 11′ E.		- -	10·0945250

The reason why the resulting logarithm here does not exactly coincide with that obtained by using the meridional parts, is that the meridional parts have been computed to but one place of decimals; if they had been computed to two or three places, the two results would have been exactly the same.

2. Given the Lizard in lat. 49° 55′ N. Barbadoes in lat. 13° 10′ N. and their difference of longitude 53°, or 3180′ W.; to determine the course and distance. Course S. 49° 59′ W.; distance 3429 miles.

3. A ship sails from lat. 37° N. long. 22° 56′ W., on the course N., 33° 19′ E., till she arrives at lat. 51° 18′ N.: required the distance sailed, and the longitude arrived at.

Distance 1027 miles; longitude in 9° 45′ W.

We shall here terminate the present chapter on the principles of Navigation, having now discussed the several cases of sailing which actually occur in practice. But the student who is desirous of prosecuting his inquiries on this very important branch of practical science to greater extent, will, of course, consult works expressly devoted to the subject. Of these, the most elaborate in our language is the valuable "Elements" of Robertson, in two octavo volumes. The Treatise of Mr. Riddle is also an excellent work abounding with practical examples very accu-

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1 10 [illegible]

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rately solved, and upon the whole, better adapted to modern practice, as well as more compendious, than Robertson's. Mr. Norie's Navigation is also a good practical book, and so is that of Dr. Bowditch.

CHAPTER II.

APPLICATION OF SPHERICAL TRIGONOMETRY TO ASTRONOMICAL PROBLEMS.

(71.) THE solution of Astronomical Problems forms one of the most useful and agreeable applications of the theory of spherical Trigonometry. To such inquiries the theory itself, no doubt, owes its origin, as well as many of the successive improvements which it has gradually received, so that a specimen of its use in the solution of astronomical problems may reasonably be looked for in a book on Trigonometry.

For the purpose of measuring the *angular* distances of the heavenly bodies from each other, and from the horizon, it is convenient to suppose them all situated as they really appear to an observer on the earth, viz. in a spherical concave surrounding our earth and concentric with it. This imaginary concave is called the celestial sphere, or the apparent heavens; in it all the apparent motions of the heavenly bodies are, for the convenience of trigonometrical application, supposed actually to take place; and the entire celestial sphere to revolve daily round the earth, as if this were at rest in its centre. All this is allowable, because the applications of which we speak are not affected by the inquiry, whether the motions which the heavenly bodies present to an observer on the earth are really as they appear or not.

At the opening of last chapter we defined several lines which geographers had found it convenient to consider as described on the surface of the earth; most of these, astronomers extend to the heavens. Thus the plane of the earth's equator, when extended to the heavens, marks on the celestial sphere the great circle called the *equinoctial*, and in like manner, the meridians being extended to the heavens, mark out the celestial meridians; also the axis of the earth, about which its real motion takes place, when extended to the heavens, is the axis about which the apparent motion of the celestial sphere takes place: this axis marks out the north and south poles of the heavens.

As the sun performs its apparent revolution about the earth in 24 hours it passes over 15° in an hour; if then we consider, as astronomers do, that the day at any place commences at noon, or when the sun is on the meridian of that place, the time shown by the sun in any position will be expressed in degrees by the arc of the equinoctial, intercepted between the fixed meridian of the place, and that passing through the sun, or it will be expressed by the angle included by these meridians. Celestial meridians are, therefore, also called *hour circles*, and the angle between the meridian of the place and that through the sun is called the hour angle, or the *horary angle*. That meridian which is at right-angles to the meridian of the place is the six o'clock hour circle, since the sun obviously reaches it when half way between noon and midnight.

Besides these lines, thus transferred from the earth to the heavens, there are others peculiar to the celestial sphere, which must be mentioned; these are, 1st, *ecliptic*, which is the great circle path described by the sun among the fixed stars in its apparent annual motion about the earth: in reality it is the path of the earth moving in a contrary direction about the sun. This circle crosses the equinoctial at an angle subject to an exceedingly small variation, determinable by observation and computation; its inclination to the equinoctial is about 23° 28′, but it is always given with the minutest attainable accuracy in the *Nautical*

Almanack. The points where the ecliptic crosses the equinoctial are called the *equinoctial points:* the sun enters these points about the 21st of March and the 23d of September; the former being called the *vernal equinox*, and the latter the *autumnal equinox.* These names are given because at such times the nights are equal in length to the days all over the world; for as the two poles of the earth are at these times symmetrically situated with respect to the sun, the circular boundary, which separates the enlightened hemisphere from the darkened, must pass through both poles; and hence any point on the earth will be as long in being carried, by the earth's uniform rotation, through the enlightened part as through the dark part. The meridian through the equinoctial points is called the *equinoctial colure.*

The position of any point on the celestial sphere, like the position of a point on the terrestrial sphere, is marked out by its latitude and longitude. On the celestial sphere the circle of longitude is the ecliptic; and perpendiculars to this, passing, therefore, through the poles of the ecliptic, are the circles of celestial latitude; the point from which longitude is measured is the vernal equinoctial point. Commencing at this point, too, the ecliptic is divided into twelve parts, called signs; a sign is therefore 30°. The twelve signs are named, and symbolically expressed, as follow:

1. ♈ Aries.
2. ♉ Taurus.
3. ♊ Gemini.
4. ♋ Cancer.
5. ♌ Leo.
6. ♍ Virgo.
7. ♎ Libra.
8. ♏ Scorpio.
9. ♐ Sagittarius.
10. ♑ Capricornus.
11. ♒ Aquarius.
12. ♓ Pisces.

The first six of these signs are on the north of the equinoctial, the others on the south, and the vernal equinoctial point is called the first point of Aries. The longitude is measured from this point in but one direction, viz. in the order of the signs.

Besides the above method of marking out the position of a celestial body, by means of its latitude and longitude, there is another way, viz. by means of its *Right Ascension* and *Declination.* The right ascension is measured on the equinoctial from the first of Aries, in the order of the signs, and the declination is measured on the perpendicular to this, or circle of declination passing through the object. We see, therefore, that what on the terrestrial sphere is latitude and longitude, is on the celestial sphere declination and right ascension; and parallels of latitude on the one correspond to parallels of declination on the other. Of these the two which are 23° 28′ from the equinoctial, one on each side, and which therefore touch the ecliptic in the first points of Cancer and Capricorn, are called the *tropics* of Cancer and of Capricorn. These first points of Cancer and Capricorn are respectively called the summer and winter *solstice;* because for a day or two before and after the sun enters them he appears to be stationary, and the days to be of equal length, so slowly does his declination at those times change, for his motion is obviously very nearly parallel to the equinoctial. The meridian, through the solstitial points, is called the *solstitial colure*, and that through the equinoctial points, the *equinoctial colure.*

Having described the principal circles and points of the celestial sphere which are considered as permanent, or which do not alter with the situation of the observer on the earth, we come now to describe those which change with his place. The principal of these is the *horizon*, which has been defined already (63), and *vertical circles* which are perpendicular to the horizon, and on which the altitudes of celestial objects are measured.

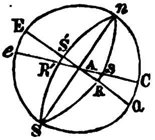
These vertical circles all meet in two points diametrically opposite, viz., the poles of the horizon; that one which is directly over the head of the observer is called his *zenith*, and the opposite one his *nadir.* That vertical which passes through the east and west points of the horizon is called the *prime vertical;* it necessarily intersects the meridian of the place (which passes through the north and south points) at right-angles.

The *azimuth* of a celestial object is an arc of the horizon, comprised between the meridian of the observer and the vertical circle through the object, and hence vertical circles are sometimes called azimuth circles.

The *amplitude* of a celestial object is the arc of the horizon, comprised between the east point and the point where the object rises, or between the west point and that where it sets; the one is called the rising amplitude, the other the setting amplitude. These definitions and remarks will suffice to render the following problems intelligible.

PROBLEM I.

(72.) Given the sun's right ascension and declination to determine his longitude and the obliquity of the ecliptic.



Let *n* E*s*Q represent the celestial meridian through the first of Cancer and Capricorn, that is, let it be the solstitial colure, *ns* the axis of the sphere, EQ the equator, *e*C the ecliptic, and *n*S*s* the declination circle, passing through the sun S; then ARS is a right angle, and in the right-angled spherical triangle ARS there are given the right ascension AR, and the declination RS to find the longitude AS, and the obliquity SAR, which is an easy operation in right-angled spherics. It is necessary, however, to remark that as celestial longitude and right ascension are measured from A, the first point of Aries in the direction AS of the signs quite round the celestial sphere, when, of the four quantities in the problem, the obliquity and the declination are given to find the others, we must know on what side of the equinoxial the sun is, that is, whether the declination is north or south, for if the sun have the north declination RS, the longitude will be AS; but if it have the equal south declination R′S′, the longitude being measured in the direction ASC round the globe to S′, will be, instead of A′S′, $360^\circ - A'S'$.

It is moreover necessary to know not only on which side of the equinoctial the sun is, but also on which side of the tropic; for the sun, in passing from a tropic to the equinox, descends through the same gradations of declination as it ascended through in passing from the preceding equinox to the tropic, although its longitude and right ascension goes on increasing; in addition, therefore, to knowing whether the declination is north or south, we must also know whether it be increasing or decreasing, in order to determine the longitude and right ascension without ambiguity; and these particulars will be known from knowing the time of the year when the proposed declination is observed; thus from the 21st of March to the 21st June, during which time the sun is in the first quadrant of the ecliptic, the sun's declination is north and increasing; it afterwards continues to decrease, still remaining north, during the second quadrant, that is till the 23d of September, from which, till the 21st of December, that is, during the third quadrant, the declination is south and increasing, after which or during the fourth quadrant, the declination still south, decreases till the 21st of March.

EXAMPLES.

1. Given the sun's right ascension on the 17th of May, $53^\circ\ 38'$, and its declination $19^\circ\ 15'\ 57''$; required his longitude and the obliquity of the ecliptic.

Applying Napier's rule to the right-angled triangle **ARS**, we have

$$\text{Rad.} \times \cos. \text{AS} = \cos. \text{AR} \cos. \text{RS}.$$

$$\text{Rad.} \sin. \text{AR} = \tan. \text{RS} \cot. \text{A} \therefore \cot. \text{A} = \frac{\text{Rad.} \sin. \text{AR}}{\tan. \text{RS}}.$$

Hence the computation for AS and A is as follows.

For the longitude AS.		For the obliquity A.	
cos. AR 53° 38′ 0″	9·7730185	sin. AR	9·9059247
cos. RS 19 15 57	9·9749710	tan. RS arith. comp.	0·4565209
cos. AS 55 57 43	9·7479895	cot. A 23° 27′ 50½″	10·3624456

2. On the 31st of March, 1816, the sun's declination was observed at Greenwich to be 4° 13′ 31½″: required his right ascension, the obliquity of the ecliptic being 23° 27′ 51″. The right ascension was 9° 47′ 59″.

3. Required the sun's longitude on the 28th of November, 1810, when his declination was 21° 16′ 4″, and his right ascension, in time, 16ʰ 14ᵐ 58·4ˢ, or in degrees 243° 44′ 36″.

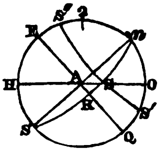
The longitude was 245° 39′ 10″, or 8 signs 5° 39′ 10.″

4. The sun's longitude being 8ˢ 7° 40′ 56″, and the obliquity of the ecliptic 23° 27′ 42½″: required his right ascension in time.

The right ascension is 16ʰ 23ᵐ 34ˢ.

PROBLEM II.

Giving the sun's declination to find the time of his rising and setting at any place whose latitude is known.



Let *n*E*s*Q represent the meridian of the place, Z being the zenith, and HO the horizon, and let *s′ s″* be the apparent path of the sun on the proposed day, cutting the horizon in S. Then the arc EZ will be the latitude of the place, and consequently EH, or its equal QO, will be the colatitude, and this measures the angle OAQ; also RS will be the sun's declination, and AR, expressed in time, will express the time of sunrise from 6 o'clock, for *n*A*s* is the 6 o'clock hour circle.

Hence, in the right-angled triangle ARS, we have given RS and the opposite angle A to find AR, the time from 6 o'clock.

EXAMPLE I.

Required the time of sunrise at latitude 52° 13′ N., when the sun's declination is 23° 28′.

By Napier's rule, Rad. sin. AR = cot. A tan. RS = tan. lat. tan. dec.

tan. 23° 28′ - - 9·6376106

tan. 52 13 - - 10·1105786

sin. 34 3 21¼″ - 9·7481892

4*

AR in time 2ʰ 16′ 13″ 25‴

6

3 43 46 35 = time of rising.

SCHOLIUM.

It should be here remarked that the time thus determined is *apparent time*, which is that which would be shown by a clock so adjusted as to pass over 24 hours during one apparent revolution of the sun, or from its leaving the meridian to its return to it again, the index pointing to 12, when the sun is on the meridian. But it is impossible that any clock can be so adjusted, because the interval between the successive return of the sun to the meridian is continually varying, on account of the unequal motion of the sun in its orbit, and of the obliquity of the ecliptic; each of these varying intervals is called a *true solar*

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[*] Degrees are converted into hours by multiplying by 4 and dividing by 60.

day, and it is the mean of these during the year which is measured by the 24 hours of a well regulated clock, this period of time being a mean solar day; hence, at certain periods of the year, the sun will arrive at the meridian before the clock points to 12, and at other periods the clock will precede the sun; the small interval between the arrival of the index of the clock to 12 and of the sun to the meridian is called the *equation of time*, and it is given in page ii. of the Nautical Almanack for every day in the year; this correction, therefore, must always be applied to the apparent time determined by trigonometrical calculation to obtain the *true time*, or that shown by a well regulated clock or chronometer.

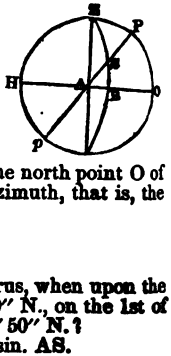
Another circumstance too must be taken into account, in order to determine the *apparent* time with rigorous accuracy, viz. the change in the declination of the sun from sunrise to noon. In the Nautical Almanack the declination of the sun is given for every day at noon, and if this be used in the computation, we shall assume that the declination has not varied from sunrise to noon, which is not the case; hence it will be necessary to compute the declination for the time of sunrise, as determined above, and then to resolve the problem with this corrected declination. The correction is obtained by taking from the Nautical Almanack the variation of declination in 24 hours, and then finding by proportion the variation for the time required.

2. Required the time of sunrise at latitude 57° 2′ 54″, when the sun's declination is 23° 28′? $3^h\ 11^m\ 49^s$.

3. How long is the sun above the horizon in latitude 58° 12′ N., when his declination is 18° 40′ S.? $7^h\ 35^m\ 52^s$.

PROBLEM III.

Given the latitude of the place, and the declination of a heavenly body, to determine its altitude and azimuth when on the six o'clock hour circle.



Let HZPO be the meridian of the place, Z the zenith, HO the horizon, S the place of the object on the six o'clock hour circle PS*p*, which of course passes through the east and west points of the horizon, and ZSB the vertical circle passing through the sun. Then in the right-angled triangle SBA, the given quantities are AS, the declination, and the arc OP, or angle SAB, the latitude of the place, to find the altitude BS, and the azimuth BO from the north point O of the horizon; or to find the complement AB of this azimuth, that is, the sun's bearing from the east.

EXAMPLES.

1. What was the altitude and azimuth of Arcturus, when upon the six o'clock hour circle of Greenwich, lat. 51° 28′ 40″ N., on the 1st of April, 1822; its declination on that day being 20° 6′ 50″ N.?

By Napier's rule we have Rad. sin. BS = sin. A sin. AS.

$$\text{Rad. cos.A} = \text{tan.AB cot.AS} \therefore \text{cot.BO} = \frac{\text{Rad. cos. A}}{\text{cot. AS}}.$$

	For the altitude			For the azimuth.	
sin. A	51° 28′ 40″	9·8934103	cos. A	- -	9·7943612
sin. AS	20 6 50	9·5364162	cot. AS	-	10·4362545
sin. BS	15 36 27	9·4298265	cot. BO	77° 9′ 4″	9·3581067

Hence the altitude is 15° 36′ 27″, and the azimuth 77° 9′ 4″ N.

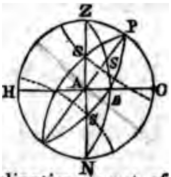
2. At latitude 62° 12′ N. the altitude of the sun at 6 o'clock in the morning was found to be 18° 20′ 23″; required his declination and azimuth. Declination 20° 50′ 12″ N., Azimuth 79° 56′ 11″ from N.

3. On the 20th of November, 1822, the declination of Aldebaran was 16° 8′ 36″ N., what was its altitude and azimuth when on the six o'clock hour circle of Greenwich, lat. 51° 28′ 40″ N.?

Altitude 12° 32′ 3″, Azimuth 79° 46′ 50″ from N.

PROBLEM IV.

The latitude of the place and the declination of the sun being given to find the time when it is due east, or upon the prime vertical, and the altitude at that time.



Having drawn the meridian of the plane as before, the vertical circle ZAN, at right-angles to it, will be the prime vertical, A being the east point of the horizon HAO: also P being the elevated pole, and S the place of the sun, ZP will be the colatitude, PS the codeclination, ZS the coaltitude, and ZPS the hour angle, or time from noon; hence, in the right-angled spherical triangle SZP, there are given SP and PZ, to find SZ and the angle P. If the declination is not of the same name as the latitude, the sun will arrive at the prime vertical at S′ before it rises: in this case the declination is to be considered as negative.

By Napier's rule Rad. cos. P = cot. SP tan. PZ = tan. dec. cot. lat.

$$\text{Rad. cos. SP} = \text{cos. SZ cos. PZ} \therefore \text{sin. alt.} = \frac{\text{Rad. sin. dec.}}{\text{sin. lat.}}$$

EXAMPLES.

1. On the 1st of August, 1831, the sun's declination was 18° 10′ 22″ N., at what hour was he due east at Greenwich; and what was his altitude at that time?

For the hour angle.			For the altitude.		
tan. dec.	18° 10′ 22″	9·5162138	sin. dec.	-	9·4939924
cot. lat.	51 28 40	9·9009509	sin. lat.	-	9·8934103
cos. hor. angle	74 51 7	9·4171647	sin. alt.	23° 29′ 37″	9·6005821

4

$4^h\ 59^m\ 24^s\ 28^t$

12

7 0 35 32. Hence the time is 35 seconds and a half past 7 o'clock, and the altitude 23° 29′ 37″.

2. Given the sun's declination 5° 8′ 26″ N., and his altitude when due east 16° 53′ 10″; required the latitude of the place.

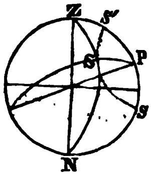
Latitude 17° 58′ N.

3. If the declination of a celestial object be 18° 4′ S., what is its altitude when on the prime vertical of latitude 27° 42′ S., and its distance from the meridian in time?

Altitude 41° 51′; Merid. distance in time $3^h\ 26^m\ 20^s$.

PROBLEM V.

To find the time when the apparent motion of a celestial object is perpendicular to the horizon, from having its declination and the latitude of the place given.

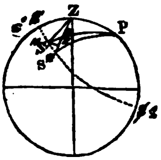


Let $s\,s'$ represent the parallel of declination, or the apparent diurnal path of the body, and let the vertical ZSN be drawn to touch it in S; then S will be the place of the body when its apparent motion is in the direction SZ of the vertical, and therefore perpendicular to the horizon: through S draw the hour circle PS, which being the shortest distance from P to ZN, is perpendicular to it (p. 53); hence the triangle PSZ is right-angled at S, and in which we have given the colatitude ZP, and the codeclination PS, to find the hour angle ZPS.

It is obvious that this problem will be impossible when Ps' exceeds PZ; that is when the declination is less than the latitude.

PROBLEM VI.

To determine upon what vertical a celestial object must be, in order that a small error, committed in taking its altitude, may have the least possible effect upon the hour angle.



Let S be the place of the sun or other body, but by an error in taking its altitude let it be referred to S′. Draw S′ S″ parallel to the horizon, and meeting the parallel of declination $s\,s'$ in S″, then when the body is at S″ it will really have the coaltitude ZS″ = ZS′, which it was erroneously supposed to have at S, so that in the determination of the hour angle P, from the colatitude, the coaltitude, and the codeclination, the small angle S′ PS″ will be the amount of the error.

As the triangle SS′ S″ is, of course, exceedingly small, it may be regarded as a rectilinear triangle, right-angled at S′; therefore SS′ = SS″ sin. S″ and SS″ = sin. PS. ∠SPS″, (see page 73,) consequently,

$$SS' = \sin. S'' \sin. PS \cdot \angle SPS'' \therefore SPS'' = \frac{SS'}{\sin. S'' \sin. PS} \quad . \quad (1).$$

Now the angle S″ is equal to the angle ZSP, because S′ SS″ is the complement of each, and therefore, by the relation between the sides and angles of a spherical triangle, we have

$$\sin. S'' : \sin PZ :: \sin. SZP : \sin. PS$$

$$\therefore \sin. S'' \sin. PS = \sin. PZ \sin. SZP.$$

Substituting the second member of this equation in (1), we have

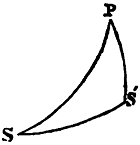
$$SPS'' = \frac{SS'}{\sin. PZ \sin. SZP} = \frac{\text{error in alt.}}{\text{cos. lat. sin. azimuth}}.$$

This expression will obviously be the least possible when the sine of the azimuth is the greatest possible, or when the azimuth is 90°; that is, when the body is on the prime vertical.

Hence, indeducing the time from an altitude of any celestial body, it will be best to make the observation when the body is either exactly, or nearly due east or due west.

PROBLEM VII.

The latitudes and longitudes of two celestial objects being given to determine their distance apart.



Let P represent the pole of the ecliptic, and PS, PS′, two arcs of celestial latitude, drawn to the two objects S S′; then will these arcs represent the colatitudes, the angle P will be the difference of longitude, and the arc SS′ will be the distance sought, so that we have two sides and their included angle given to find the third side. In order to this, we must first determine agreeably to the method explained at page 60, a subsidiary angle, ω, by the equation cot. ω = tan. PS cos. P; after which the side SS′ is found by the equation $\cos. SS' = \frac{\cos. PS \sin. (\omega + PS')}{\sin. \omega}$

EXAMPLES.

1. Required the distance between Procyon and Capella, the latitude of Procyon being 15° 58′ 14″ S., and its longitude 3^s 22° 55′ 42″; also the latitude of Capella being 22° 51′ 57″ N., and longitude 2^s 18° 57′ 57″?

Taking the difference of the longitudes, we have for the angle P, P = 33° 57′ 45″; and for the polar distances we have PS = 105° 58′ 14″, PS′ = 67° 8′ 3″; hence the logarithmic process will be as follows:

tan. PS	105° 58′ 14″	-	10·5433466	cos. PS	9·4395590
cos. P	33 57 45	-	9·9187658	sin. ω, ar. comp.	0·4865396
cot. ω	160 57 46	-	10·4621124	sin. (ω + PS′)	9·8717340
PS′ =	67 8 3			cos. SS′ 51° 6′ 39″	9·7978326
ω + PS′ =	228 5 49.				

In this example cot. ω is negative, because tan. PS is negative, and cos. P positive; also cos. SS′ is positive, because cos. PS is negative, sin. (ω + PS′) negative, and sin. ω positive. The operation will obviously be similar, when, instead of the latitudes and longitudes, the right ascensions and declinations of the two bodies are given to find their distance apart.

2. The latitude and longitude of a star S. are 38° 40′ 26″ N., and 3^s 2° 4′ 40′; and of a star S′., 13° 26′ 11″ N., and 9^s 11° 41′ 26″; required their angular distance apart. Distance 127° 7′ 11″.

3. What is the distance between Sirius and Procyon, the right ascension of Sirius being 99° 0′ 21″, and its declination 16° 26′ 35″ S.; and the right ascension of Procyon 112° 6′ 47″, and its declination 5° 45′ 3″ N. Distance 25° 42′ 10″.

PROBLEM VIII.

Given the latitude of the place and the sun's declination to find the beginning and end of twilight.

Twilight commences in the morning and ends in the evening, when the sun is about 18° below the horizon. Hence, if PZ (see the diagram to next problem) represent that portion of the meridian which is intercepted between the elevated pole and the zenith, and S′ be that point in the sun's apparent path on any day which is 108° from Z, S′ will be the place of the sun at the commencement of morning twilight, or at the termination of evening twilight; also PS′ will be the codeclination, and PZ the colatitude; we thus have the three sides of the triangle PS′Z, to find the angle P. Hence, calling the sum of the three sides S, the formula for computing the hour angle P will be

$$\sin. \tfrac{1}{2} P = \sqrt{\frac{\sin. (\frac{1}{2} S - ZP) \sin. (\frac{1}{2} S - PS')}{\sin. ZP \sin. PS'}}\text{; which is the same as}$$

$$\sin. \tfrac{1}{2} P = \sqrt{\frac{\sin. \frac{1}{2} (lat. + 18° + codec.) \sin. \frac{1}{2} (dec. + 18° + colat.)}{\cos. lat. \cos. dec}}\text{;}$$

a very convenient form for computation.

EXAMPLES.

1. At what time did twilight commence at Edinburgh, lat. 55° 57′ 20″ N., on the 20th of August, 1831, when the sun's declination was 12° 38′ 9″ N.?

lat. + 18°	73° 57′ 20″		
codec.	77 21 51	ar. comp. sin.	0·0106480
	2)151 19 11		
	75 39 35½	sin. .	9·9862531
dec. + 18°	30 38 9		
colat.	34 2 40	ar. comp. sin.	0·2519393
	2) 64 40 49		
	32 20 24½	sin. .	9·7283086
			2)19·9771490
sin. ½ P	76° 54′ 54½″	.	9·9885745.

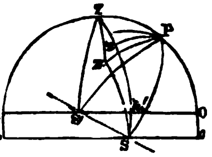
Hence P = 153° 49′ 49″ = (in time) $10^h\ 15^m\ 19\frac{1}{2}^s$, so that twilight commenced in the morning at $1^h\ 44^m\ 40\frac{1}{2}^s$, and ended in the evening at $10^h\ 15^m\ 19\frac{1}{2}^s$.

2. At what time does the twilight begin at latitude 48° 38′ 56″ N., when the sun's declination is 8° 28′ 54″ N.? Twilight begins at $3^h\ 20^m$.

3. At what time does twilight end at latitude 52° 12′ 35″ N., when the sun's declination is 15° 55′ 25″ N.? Twilight ends at $10^h\ 12\frac{1}{2}^m$.

PROBLEM IX.

Given the latitude of the place to determine on what day of the year the twilight is the shortest, and its duration on that day.



Let HO represent the horizon, and *ho* the parallel to it, 18° below it; also let PS be the declination circle, passing through the sun at sunset, and PZ, that passing through the zenith. Conceiving these two circles to revolve with S, PS will come to PS′ when S comes to S′, and PZ will take some determinate position PZ′. Now, since the angles ZPS, Z′ PS′, are equal, we have, by taking from each the common part Z′PS, ZPZ′ = SPS′; but SPS′, converted into time, expresses the duration of twilight, ZPZ′ is therefore the least possible when the twilight is the shortest possible. Now since the sides PZ, PZ′, are both given, the side ZZ′ will be the shortest when the opposite angle, P, is the least; (see equa. (A) p. 47,) hence when ZZ′ is the shortest, the twilight is the shortest; but as the two sides Z′S′, ZS, of the triangle ZZ′S′, are given, the third side will be shortest when the angle S′ is the least possible, and this is the case when Z′ falls on ZS′, for then the angle is 0. Hence the twilight is shortest when the angle PSZ is equal to the angle PS′Z.

Let then z be the proper position of Z′; we shall have $Zz = Zh' - zh' = zS' - zh' = h'S' = 18°$, and because $PZ = Pz$, the arc Pn, bisecting the angle ZPz, will also bisect the base Zz, and be perpendicular to it (54); consequently, $\sin. ZPn = \sin. \frac{1}{2} SPS' = \frac{\sin. Zn}{\sin. PZ} = \frac{\sin. 9°}{\cos. \text{lat.}}$ (1); also $\cos. Pn = \frac{\cos. PZ}{\cos. Zn}$; and, in the right-angled triangle PnS',

$$\cos. PS' = \cos. n S' \cos. Pn = \frac{\cos. n S' \cos. PZ}{\cos. Zn};\ \text{that is,}$$

$$\sin. \text{dec.} = \frac{\sin. 99° \sin. \text{lat.}}{\cos. 9°} = -\frac{\sin. 9° \sin. \text{lat.}}{\cos. 9°} = -\tan. 9° \sin. \text{lat.}\ (2).$$

The declination being known by this equation, the day of shortest twilight is also known, (*Naut. Alm.*) The declination will be of a contrary name with the latitude as its sine is negative. Equation (1) expresses the duration of the twilight. Since the angles ZPz, SPS′, are equal, the hour angles for the beginning and ending of the morning twilight, or for the ending and beginning of the evening twilight, are ZPS′, zPS′. Now, in the right-angled triangle PnS′, we have

$$\sin.\ n\mathrm{PS}' = \frac{\sin.\ \mathrm{S}'n}{\sin.\ \mathrm{PS}'} = \frac{\sin.\ 99^\circ}{\cos.\ \mathrm{dec.}} = \frac{\cos.\ 9^\circ}{\cos.\ \mathrm{dec.}} \quad . \quad . \quad (3).$$

The sum of (1) and (3) gives the angles ZPS′, and their difference the angle zPS′ = ZPS, and thus we have the hour angles for the beginning and end of the twilight.

EXAMPLES.

1. Required the time and duration of the shortest twilight at Greenwich, lat 51° 28′ 40″, in the year 1832.

For the day.				For the duration.			
tan.	9°	.	9·1997125	sin	9°	.	9·1943324
sin.	51° 28′ 40″		9·8934103	cos.	51° 28′ 40″		9·7943612
sin. dec.	7 7 5		9·0931228	sin.	14 32 49		9·3999712

The declination is therefore, 7° 7′ 5″ south, which (*Naut. Alm.*) corresponds to March the 1st and to October 12.

Also the hour angle SPS′ is 29° 5′ 38″, which, in time, is 1^h 56^m 22½^s, the duration sought.

To find the times of beginning and ending of the twilight, we have, from the equation (3)

cos.	9°	.	9·9946199
cos.	7° 7′ 5″	.	9·9966399
sin.	95 31 19		9·9979800.

The angle nPS′, thus determined, is obtuse, because its opposite side is greater than PS, and this is opposite to a right-angle. This angle, converted into time, is 6^h 22^m 5¼^s. Adding therefore, to this the angle ZPn, in time, that is half the duration, or 58^m 11¼^s, we have 7^h 20^m 16½^s, the time when the evening twilight ends. Also, by subtracting the same quantity, we have 5^h 23^m 54^s for the time when the evening twilight commences. These results respectively taken from 12^h leave the time when the morning twilight begins and ends.

CHAPTER III.

ON THE PRINCIPLES OF NAUTICAL ASTRONOMY.

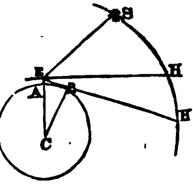
(73.) In our chapter on Navigation we have laid down several methods of determining the place of a ship at sea, by help of the account kept on board of its progress through the water, that is, of the course and distance sailed; and, if confidence could be placed in this account, even when kept with the utmost care, the art of Navigation would be perfect. Such perfection, however, it is hopeless to expect; for it does not seem possible to measure, with strict accuracy, either a ship's rate or the direction in which she moves, both of which may indeed be continually varying. In order, therefore, to determine the place of a ship at sea, with that accuracy which the safety of navigation requires, it is absolutely necessary that we be furnished with methods entirely independent of the *dead reckoning*, and these methods it is the business of Nautical Astronomy to teach.

"It must not, however, be understood that the dead reckoning is without its value; on the contrary, when combined with astronomical observations, it is of considerable utility in detecting the existence and velocity of currents, and is indispensably necessary to fill up the short intervals which may occur in unfavourable weather between celestial observations. But the too general practice of relying exclusively upon it cannot be sufficiently deprecated, and numerous instances might be adduced of the fatal consequences of this reliance, in the loss of vessels, from errors of such magnitude that they might have been detected by the most superficial knowledge of nautical astronomy, and the aid of even a good common watch." (*Capt. Kater's* Nautical Astronomy in the Ency. Met.)

On the Corrections to be applied to the observed Altitudes of Celestial Objects.

(74.) The *true altitude* of a celestial object is always understood to mean its angular distance from the rational horizon of the observer. This is not obtained directly by observation; but is the result of certain corrections applied to the observed altitude. These we shall now enumerate and explain.

Of the Dip or the Depression of the Horizon.



(75.) Let E represent the place of the observer's eye, and S the situation of any celestial body; the first object is to obtain its apparent altitude above the horizontal line EH; that is, the angular distance SEH. Now, as to the observer, the *visible* horizon is EBH′, the altitude given by the instrument is the angle SEH′; hence we must subtract from this observed altitude the angle HEH′, called the *Dip or Depression of the Horizon*, in order to obtain the apparent altitude SEH.

The angle HEH′, or its equal C, is calculated for various elevations, AE of the eye above the surface of the sea from the proportion,

$$CE : EB = \sqrt{EC^2 - CB^2} :: \text{rad.} : \text{sin. } C;$$

and the results are registered in a table.

Of the Semidiameter.

(76.) When the foregoing correction for dip has been applied, the result will be the apparent altitude of the point observed above the horizontal plane through the observer's eye. If this point be the uppermost or lowermost point of the disc of the sun or moon, a further correction will be necessary to obtain the apparent altitude of the centre; that is, we must apply the angular distance due to the semidiameter. This quantity, both for the sun and moon, is given in the Nautical Almanac. But in the case of the moon the semidiameter itself requires a small correction depending upon the observed altitude. For the semidiameter, furnished by the Nautical Almanac, is the apparent *horizontal* semidiameter, or the angle it subtends when in the horizon; but as the moon approaches the zenith, her distance from the observer diminishes, and therefore her semidiameter is viewed under a greater angle. As she is nearer to the observer when in the zenith than when in the horizon, by one semidiameter of the earth, and as her distance from the earth's centre is about 60 semidiameters of the earth, the horizontal semidiameter will in the zenith become increased by about $\frac{1}{60}$th part, and at intermediate elevations the increase will be as the sine of the altitude. On this principle is formed the Table at the end, entitled *Augmentation of the Moon's Semidiameter*, and containing the proper correction to be *added* to the given horizontal semidiameter to obtain the true semidiameter.

On account of the great distance of the sun, no such correction of his semidiameter is necessary.

The corrections for dip and semidiameter being thus applied, the result is called *the apparent altitude of the centre.* In the case of the stars the only correction for the apparent altitude is the dip. It must, however, be here remarked, that if the centre of the object were visible, and its altitude, instead of that of the limb, were to be taken, we should not, after applying the correction for dip, obtain precisely the same result as that which we have just called the apparent altitude of the centre, but should get a value somewhat less. The reason of this is, that every vertical arc in the heavens is shortened by *refraction*, as we shall shortly explain, so that the centre would not exceed the observed altitude of the lower limb, or fall short of that of the upper, by so great a quantity as the *true* semidiameter. Hence, from the apparent altitude of the centre, as found from applying the *true* semidiameter to the apparent altitude of the limb, a small quantity should in strictness be subtracted, and this small correction becomes necessary when the longitude is to be determined with accuracy. This correction was first proposed by Dr. Thomas Young. A table for it is given at the end.

To obtain the true altitude requires two other corrections, viz. for *refraction* and for *parallax.* The former of these has indeed an effect upon the two preceding corrections, dip, and semidiameter, which require certain modifications in consequence. One of these we have adverted to above, and the other will be noticed more particularly in the following article.

Of Refraction.

(77.) It is a universal fact in optics, that if a ray of light pass obliquely out of one medium into another of greater density, it will be bent out of its original direction at the point when it enters the new medium, and proceed through it in a direction more nearly perpendicular to its surface at that point. Hence the rays of light, proceeding from the celestial bodies, become bent downwards as soon as they enter the atmosphere, their course being directed more nearly towards the centre of the earth, so that the rays which enter the eye of an observer, and by which any celestial object becomes visible to him, would, if not thus bent down, pass over his head; the object is therefore seen by him above its true place; the angle between this apparent direction, and the true direction of the object, measures the refraction; and, like the correction for dip, it is always *subtractive;* it increases from the horizon, where it is greatest, to the zenith, where it vanishes, as the rays from objects in the zenith enter the atmosphere perpendicularly.

It is the refraction which causes the sun and moon, when near the horizon, to present sometimes an elliptical appearance, the vertical diameter (and, indeed, every oblique diameter) seeming to be shorter than the horizontal, because the lower *limb*, or edge, being more elevated by refraction than the upper, the two are brought, in appearance, more nearly together.

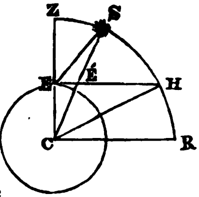
At the end of the volume we have given a table of refractions for different altitudes, from the horizon to the zenith, and adapted to the mean state of the atmosphere; but, as the actual state of the atmosphere generally differs from this, it becomes necessary, where the true altitude of the body is required with the utmost accuracy, to apply a correction to the numbers in this table, so as to adapt them to the existing temperature and density of the atmosphere at the time of observation, as is indicated by the thermometer and barometer. The table of corrections is annexed to the table of mean refractions. It should, however, be observed that below 4° the refraction is very variable and uncertain, and such low altitudes should be avoided as much as possible at sea.

It will be unnecessary to use this annexed table for correcting the altitude of a celestial object when the latitude of the ship is the only object of the observation, as such a correction could seldom make a difference so great as half a mile in the resulting latitude; but in determining the longitude by the *Lunar observations*, the neglect of these small corrections would sometimes introduce an error in the resulting longitude of more than 30 miles.

It should be remarked here, that the dip, as determined in article (75), is on the supposition that refraction has not elevated the apparent horizon, but as such is not the case, the dip requires a correction; the amount of this correction is very uncertain, on account of the irregularity of the horizontal refractions although it is unquestionable that some correction is requisite. It is usual to allow about $\frac{1}{8}$ or $\frac{1}{10}$ of the computed dip for the correction. In our table $\frac{1}{10}$ is allowed, which is according to Dr. Maskelyne, but Lambert and Legendre make it $\frac{1}{14}$.

When the foregoing corrections have been applied to the observed altitude, the result will be the true altitude of the centre above the visible horizon, and it remains now to apply the correction necessary to reduce this to the true altitude of the centre above the rational horizon; that is, to the altitude which the body would have if the observer were situated at the centre of the earth instead of on its surface.

Of the Parallax.



In order to explain the nature and effect of parallax, let S represent the place of the object observed from the surface of the earth, at E; then the angle SEH, that is, the observed angle when corrected for dip semidiameter, and refraction, will be the true altitude of the object, in reference to the observer's sensible horizon EH; and the angle SCR will be the true altitude, in reference to the rational horizon CR; and the difference of these angles is the parallax in altitude. If the body be at H, in the sensible horizon, then the difference of which we speak is the entire angle HCR; this is called the horizontal parallax.

Since the angle SE′H is equal to the angle SCR, we have for the parallax in alt., SE′H — SEH = ESC; that is, the parallax is the angle which the semidiameter of the earth subtends at the object; it is obviously greatest in the horizon, and nothing in the zenith, and is the quantity which must be *added* to the true altitude above the sensible horizon, to obtain the true altitude above the rational horizon.

The sun's parallax in altitude is given in a Table at the end; and the moon's horizontal parallax is given for the noon and midnight at Greenwich, of every day of the year, in the Nautical Almanack: and from the horizontal parallax thus obtained the parallax in altitude must be calculated. This is easy; for since in the triangle SEC, we have the proportion SC : EC :: sin. SEC = sin. SEZ = cos. SEH : sin ESC; it follows that the sine of the parallax in altitude varies as the cosine of the altitude, so that, as rad. is to the cosine of the altitude, so is the sine of the horizontal parallax, to the sine of the parallax in altitude. In other words, the log. sine of the horizontal parallax, added to the log. cosine of the altitude, abating 10 from the index, will give the log. sine

of the parallax in altitude; but as the parallax is always a very small angle it is usual to substitute the arc for its sine, so that
log. hor. par. in seconds + log. cos. alt. — 10 = log. par. in alt. in seconds.

We must observe here that the horizontal parallax, given in the Nautical Almanack, is calculated to the equatorial radius of the earth; and, therefore, except at the equator, a small subtractive correction will be necessary, on account of the spheroidal figure of the earth. A table of such corrections is given at the end, and explained in the introduction to the tables.

Such are the corrections necessary to be applied to the observed altitudes of celestial objects in order to obtain their true altitudes. A few other preliminary, but very simple, and obvious operations must also be performed upon the several quantities taken out of the Nautical Almanack, in order to reduce them to their proper value at the time and place of observation; for the elements furnished by the Nautical Almanack are computed for certain stated epochs, and their values for any intermediate epoch must be found by proportion. But ample directions for these preparatory operations are contained in the "Explanation of the Articles in the Nautical Almanack," by the late Dr. Maskelyne, which accompanies every edition of that work.

Example of the Corrections.

1. On the 14th of January, 1833, suppose the observed altitude of the sun's lower limb to be 16° 24′, the observer's eye to be 18 feet above the level of the sea, the barometer to stand at 29 inches, and the thermometer at 58°: required the true altitude of the sun's centre.

Observed alt. ⊙'s L. L. -	16°	36′	4″
Depression of the horizon -	—	4	4
App. alt. of L. L. - - - - -	16	32	0
Refraction - - - - - -	—	3	14
Correction for Barometer -		—	6·5
Correction for Thermometer - -		—	3·2
True alt. of L. L. above visible horizon	16	28	36·3
Sun's semidiameter (Naut. Alm.)		+16	17·3
Parallax in altitude - - -		+	8·4
True altitude of Sun's centre -	16	45	2.

2. On the 20th of May, 1833, suppose that in longitude about 77° 30 west, and lat. about 48° north, at 3^h apparent time, the altitude of the moon's lower limb is observed to be 18° 8′ 34″, the height of the eye being 20 feet, the barometer 28·5 inches, and the thermometer 46°: required the true altitude of the sun's centre. Here the object being the moon, it will be necessary to compute the parallax in altitude, from having the horizontal parallax corresponding to the time at Greenwich.

The horizontal parallax is given in the Nautical Almanack for every noon and midnight; and, therefore, to find it for any other intermediate time, we must say as 12^h is to its variation in 12^h, so is the proposed time to the variation due to that time.

In like manner must the moon's semidiameter be reduced, by proportion to the time of observation, since it sensibly varies in the course of a few hours. We shall begin, therefore, with finding in this way the true horizontal parallax and semidiameter for the time of the observation reduced to the meridian of Greenwich.

Longitude of the ship in time	5^h 10^m	after Greenwich time.
Apparent time at ship	3 0	
Apparent time at Greenwich	8 10	

Hor. par. at noon (Naut. Alm.)	58′ 17″	Semidiam. at noon	15′ 53″
Hor. par. at midnight .	58 31	Semidiam. at midnight	15 57
Variation in 12^h .	0 14	Variation in 12^h	0 4
∴ 12^h : 8^h 10^m :: 14″:	9·5	∴ 12^h : 8^h 10^m :: 4″:	2·7″
Hor. par. at noon	58′ 17	Semidiameter at noon	15′ 53
Hor. par at reduced time	58 26·5	Hor. semidia. at reduced time	15 55·7″
	60	Augmentation for 18° alt.	5·2
Ditto in seconds	3506·5		
Dim. of par. for lat. 48°	—6·3	True semidiameter	16 0·9
True hor. parallax	3500·2		

For the Appdrent Altitude.

Observed altitude of ☽s' L. L. .	18° 8′ 34″
Depression	— 4 17
Semidiameter *minus* contraction	15 57·9
Apparent alt. ☽'s centre	18 20 14·9.

For the Parallax in Altitude.

cos. ☽'s app. alt. 18° 20′ 15″ .	9·9773668
hor. parallax 3500·2″ log. .	3·5440929
Par. in altitude 3322·5″ .	3·5214597

For the true Altitude.

Apparent alt. of ☽'s centre	18° 20′ 14·9″
Refraction . . .	— 2 54·2
Barometer . . .	— 8·8
Thermometer . .	+ 1·4
True alt. above sensible horizon	18 17 13·3
Parallax in altitude 3322·5″ =	+ 55 22·5
True alt. of ☽'s centre	19 12 35·8.

These two examples will serve for specimens of the corrections to be applied to an observed altitude, in order to deduce from it the true altitude of the body's centre. In the case of the moon, the corrections, when the utmost accuracy is sought, are rather numerous, as the last example shows. But in finding the latitude at sea, it is usual to dispense with some of these, more especially with the corrections for temperature, for the contraction of the moon's semidiameter, and for the spheroidal figure of the earth; because an error of a few seconds in the true altitude will introduce no error worth noticing in the resulting latitude. When, however, the object of the observer is to deduce the longitude of the ship, all the data, furnished by observation, should be as accurate as possible; for the problem is one of such delicacy that by neglecting to allow for the influence of temperature would alone introduce in some cases an error of from 30 to 40 miles in the longitude.

When the object observed is a star, several of the foregoing corrections vanish: the only corrections, in this case requisite, are those for dip and refraction, modified as usual for the temperature.

(78.) *To determine the latitude at sea from the meridian altitude of any celestial object whose declination is known.*

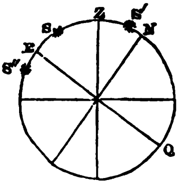
The determination of the latitude, by a meridian altitude, is the easiest, and in general the safest, method of finding the ship's place on the meridian; for both the observations and the subsequent calculations being few, they are readily performed, and with but little liability to error in the result; this method, therefore, is always to be used at sea, unless foggy or cloudy weather render it impracticable.

The declination of the object observed is supposed to be given in the Nautical Almanack for the meridian of Greenwich; it may therefore be reduced to the meridian of the ship by means of the longitude by account, which will always be sufficiently accurate for this purpose, although it should differ very considerably from the true longitude, because declination changes so slowly that even an error of an hour in the longitude would cause an error in the declination too small to deserve notice.

Having then thus found the distance of the object from the equinoctial, and having, by means of the observed altitude properly corrected, obtained the distance of the same object from the ship's zenith, the distance of the zenith from the equinoctial, that is, the latitude, immediately becomes known.

1. Let S be the object observed, the zenith Z being to the north of it, and the object itself north of the equinoctial EQ, then the latitude EZ is equal to the zenith distance, or coaltitude ZS + the declination, and it is north.

2. Let S′ be the object, still north of the equinoctial, but so posited that the zenith is south of it, then the latitude EZ is equal to the difference between the zenith distance S′Z, and declination S′E, and is still north.



3. Let now the object be at S″, south of the equinoctial, and the zenith to the north of the object, then the latitude EZ is equal to the difference between the zenith distance S″Z and declination S″E, and it is north.

We have here assumed the north to be the elevated pole, but if the south be the elevated pole, then we must write south for north, and north for south. Hence the following rule for all cases. Call the zenith distance north or south, according as the zenith is north or south of the object. If the zenith distance and declination be of the same name, that is, both north or both south, their sum will be the latitude; but, if of different names, their difference will be the latitude, of the same name as the greater.

EXAMPLES.

1. If on the 2d of May, 1833, the meridian altitude of the sun's lower limb be 47° 18′, height of the eye 20 feet, and longitude by account 32° E.: required the latitude, the sun being south at the time of observation.

Observed alt. of ⊙'s L. L. .	47° 18′ 0″
Dip. of the horizon . .	— 4 17
App. alt. of ⊙'s L. L. .	46 13 43.

The longitude in time is $2^h\ 8^m$ east, so that time at Greenwich is $2^h\ 8^m$ before the noon of the 2d of May; hence, to find the corresponding declination, we have, by the Nautical Almanack, $24^h : 2^h\ 8^m :: 18'\ 1'' : 1'\ 38''$; so that, 1′ 38″, the variation in $2^h\ 8^m$, must be subtracted from 15° 23′ 21″ N., the declination of the sun on May 2, at noon; hence the proper declination is 15° 21′ 43″ N.

Observed alt. of ⊙'s L. L.	47° 18′ 0″
Dip.	— 4 17
App. alt. of ⊙'s L. L.	46 13 43
Refraction	— 56
Parallax	+ 6
Semidiam. (Naut. Alm.)	15 53
True alt. of ⊙'s centre	46 28 46
Zenith distance	43 31 14 N.
⊙'s declination	15 21 43 N.
Latitude	58 52 57 N.

2. On the first of January, 1820, the meridian altitude of Capella was 27° 35′, the zenith being south of the star, and the height of the eye 22 feet; required the latitude,

Observed altitude	27° 35′ 0″
Dip	— 4 30
Apparent altitude	27 30 30
Refraction	— 1 51
True altitude	27 28 39
Zenith distance	62 31 21 S.
Star's dec. (Naut. Alm.)	45 48 39 N.
Latitude	16 42 42 S.

3. On the 19th of February, 1823, the ship being in longitude 40° W., the observed meridian altitude of the moon's lower limb was 55° 6′; the zenith north of the moon; and the height of the eye 16 feet: required the latitude.

Here the time of observation at the ship is not given, it must therefore be calculated, and we have these data for this purpose, viz. that the ship is 40° W. of Greenwich, and that the moon is on its meridian. The following process therefore immediately suggests itself.

The moon passed the merid. of Greenwich Feb. 19 (Naut. Alm.)	$6^h\ 56^m$ 0
Feb. 20	7 59 0
Interval between the two passages	24 + 1 3 0.

Hence $1^h\ 3^m$ is the moon's retardation in $25^h\ 3^m$, and by proportion using for the longitude 40° W., its value in time $2^h\ 40^m$, we have,

$$25^h\ 3^m : 1^h\ 3^m :: 2^h\ 40^m : 0^h\ 6^m\ 42^s;$$

that is, the moon is retarded $6^m\ 42^s$ in passing from the meridian of Greenwich to that of the ship, and, therefore, instead of the apparent time at the ship being $6^h\ 56^m$, as it necessarily would be if there were no retardation, it will be $6^m\ 42^s$ later. Hence

Apparent time at the ship	$7^h\ 2^m\ 42^s$
Ship's longitude W.	2 40 0
Time at Greenwich	9 42 42.

Having thus got the apparent time at Greenwich when the observation was made, we may, by a reference to the Nautical Almanack and a subsequent proportion, find the moon's declination at that time: thus

Moon's declination at Greenwich, Feb. 19 at noon	26° 38′ 17″
Feb. 19 at midn.	26 54 39
Change of declination in 12 hours	16 22.

$$\therefore\ 12^h : 9^h\ 42^m\ 42^s :: 16'\ 22'' : 13'\ 15'';$$

hence 13′ 15″ is the amount of the change of declination, from noon to 9ʰ 43ᵐ, on the supposition, however, that the motion of the moon in declination may be considered as equable during the twelve hours. But on account of the irregular motion of the moon, this supposition introduces a sensible error, which may however be corrected by means of the table of "Equation of second Differences," given in the Nautical Almanack, and explained in Dr. Maskelyne's accompanying "Explanation.' The correct change of declination is thus found to be 14′ 16″. But from the year 1833, the declination of the moon will be given in the Nautical Almanack to every three hours, and the change for any shorter interval may then be obtained with the requisite accuracy by proportion, as above. Taking in the present case 14′ 16″ for the correct change, we have

Declination for preceding noon	26° 38′ 17″ N.
Increase of Declination -	14 16
Declin. at the time of observation	26 52 33 N.

Before we can find the proper correction for parallax, we must deduce the apparent altitude of the centre,

Observed altitude of ☽'s L. L. -	55° 6′ 0″
Dip. - - - - - -	— 3 50
Semidiameter (Naut. Alm.) - -	16 13
Augmentation for 55° of alt. - -	13
Apparent alt. of ☽'s centre - -	55 18 36 cos. 9·7552161
Hor. par. in seconds at 9ʰ 43ᵐ (Naut. Alm.)	3572 log. 3·5529115
Parallax in altitude in seconds -	2033 log. 3·3081276

therefore the correction for parallax is 33′ 53″.

Having thus reduced all the corrections to the time of observation, we readily obtain the true altitude, and thence the latitude as follows,

Apparent alt. of ☽'s centre	55° 48′ 36″
Refraction - - -	— 40
Parallax in altitude - - -	33 53
True altitude - -	55 51 49
Zenith distance - -	34 8 11 N.
Declination - - -	26 52 33 N.
Latitude - - -	61 0 44 N.

SCHOLIUM.

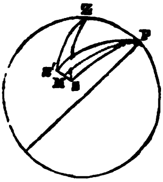
These examples will, no doubt, be found sufficient to put the student in possession of the method of applying the various corrections to the observed meridian altitude of a celestial object, in order to deduce from it the latitude of the ship. But it should be remarked, that in most works on Nautical Astronomy, subsidiary tables are inserted for the purpose of abridging some of the foregoing corrective operations; such tables, therefore, offer very acceptable aid to the practical navigator. The most esteemed works of this kind are Dr. Mackay's "Treatise on the Theory and Practice of finding the Longitude at Sea;" the "Nautical Tables" of J. De Mendoza Rios, and Mr. Riddle's book on Navigation and Nautical Astronomy.

It should also be observed here, that in the preceding examples the celestial object is supposed to be on the meridian *above* the pole; that is, to be higher than the elevated pole. But, if a meridian altitude be taken below the pole, which may be done if the object is *circumpolar*, or so near to the elevated pole as to perform its apparent daily revolution about it without passing below the horizon, then the latitude of the place will be equal to the sum of the true altitude, and the codeclination or polar distance of the object; for this sum will obviously measure the elevation of the pole above the horizon, which is equal to the latitude.

(79.) *To determine the latitude at sea, by means of two altitudes of the sun, and the time between the observations.*

In the preceding article we have shown how to determine the latitude of the ship by the meridian altitude of the sun, or of any other heavenly body, whose declination may be found. But, as already remarked, the object we wish to observe may be obscured when it comes to the meridian, and this may happen for many days together, although it may be frequently visible at other times of the day. As therefore the opportunity for a meridian observation cannot be depended upon, it becomes an important problem to determine the latitude at sea, by observations made out of the meridian; and considerable attention has accordingly been paid, by scientific persons, to the method of finding the latitude by "double altitudes," and various tables have been computed to facilitate the operation. But the direct method, by spherical trigonometry, though rather long, involving three spherical triangles, will be more readily remembered, and more easily applied by persons familiar with the rules and formulas of Trigonometry, than any indirect or approximative process; we shall therefore explain the direct method.

Let P be the elevated pole, Z the zenith of the ship, and S, S′ the two places of the sun when the altitudes are taken. Then, drawing the great circle arcs as in the figure, we shall have these given quantities, viz. the codeclinations PS, PS′; the coaltitudes ZS, ZS′, and the hour angle SPS′, which measures the interval between the observations; and the quantity sought is the coaltitude ZP. Now, in the triangle PSS′, we have given two sides and the included angle to find the third side SS′, and one of the remaining angles, say the angle PSS′. In the triangle ZSS′ we have given the three sides to find the angle S′SZ; having then the angles PSS′, S′SZ, the angle ZSP becomes known, so that we have lastly, two sides and the included angle in the triangle ZSP, to find the third side ZP.



Before the application of the trigonometrical process, the observed altitudes must, of course, be reduced to the true altitudes, as in the preceding examples. Moreover, as the ship most probably sails during the interval of the observation, an additional reduction becomes necessary; the first altitude must be reduced to what it would have been if taken at the place where the second was taken: this correction will be known if we know the number of minutes the ship has sailed directly towards or directly from the sun, upon leaving the place where the first observation was made. To find this, take the angle included between the ship's course and the sun's bearing, at the first observation; and considering this angle as a course, and the distance sailed as the corresponding distance, find by the traverse table, or by the operation of plane sailing, the difference of latitude, which will be the amount of the approach to, or departure from, the sun. This must be added to the first altitude if the angle is less than 90°, because the ship will have approached towards the sun; but it must be subtracted when the angle exceeds 90°. If the angle is 90° no correction for the ship's change of place will be necessary.

The truth of this correction will be immediately seen by considering that if the sun's centre were the elevated pole, what is in reality the coaltitude would then be the colatitude, and, therefore, that, by whatever quantity this latter is increased or diminished by the ship's motion, on the one hypothesis, by the same quantity will the former be increased or diminished on the other hypothesis.

Where great accuracy is aimed at, account should be taken of the ship's change of longitude during the interval of the observations; when converted into time it must be added to the interval of time between the observations when the ship has sailed eastward, and subtracted when she has sailed westward. This correction is very easily applied.

Having thus mentioned the necessary preparative corrections, we shall now give an example of the trigonometrical operation.

EXAMPLES.

Let the two zenith distances corrected be (see last fig.) ZS = 73° 54′ 13″, ZS′ = 47° 42′ 51″, the corresponding declinations 8° 18′ and 8° 15′ north, and the interval of time three hours; to determine the latitude.

Considering SS′ to be the base of an isoceles spherical triangle, of which one of the equal sides is ½(PS + PS′) = 81° 43′ 30″, and the vertical angle equal to 3^h or 45°, let the perpendicular PM be drawn, then we have in the triangle PMS right-angled at M, PS = 81° 43′ 30″ and $P = \frac{45°}{2} = 22° 30′$; given to find SM = ½ SS′ as follows.

I. *To find* SS′ *from the triangle* PMS.

sin. PS 81° 43′ 30″ . . . 9·9954547
sin. P 22 30 0 . . . 9·5828397

sin. SM 22 15 11·3 . . 9·5782944
2

SS′ = 44 30 22·6.

II. *To find* PSS′ *from the triangle* PSS′.

sin. SS′ 44° 30′ 22·6″ - arith. comp. 0·1542898
sin. PS′ 81 45 0 - - - 9·9954822
sin. SPS′ 45 0 0 - - - 9·8494850

sin. PSS′ 86 38 58 - - - 9·9992570.

This angle is acute like its opposite side, (see art. 60.)

III. *To find* ZSS′ *in the triangle* ZSS′

ZS′ 47° 45′ 51″
sin. ZS 73 54 13 - arith. comp. 0·0173686
sin. SS′ 44 30 22·6 - arith. comp. 0·1542898

2)166 10 26·6

½ Sum = 83 5 13·3
sin. (½ Sum — ZS) 9 11 0·3 - - - 9·2030206
sin. (½ Sum — SS′) 38 34 50·7 - - - 9.7949179

2)19·1695969

sin. ½ ZSS′ 22 36 26·4 - - 9·5847985
∴ ZSS′ = 45° 12′ 52·8″
PSS′ = 86 38 58

PSZ = 41 26 5·2.

iv. *To find* ZP *in the triangle* ZSP.

tan. PS	81°	42′	0″	-	10·8359917,	cos. PS -	9·1594364
cos. PSZ	41	26	5·2	-	9·8748930,	sin. ω, ar. comp.	·7189551
cot. ω	11	0	41·2	-	10·7108847,	sin. (ω + ZS)	9·9989874
	73	54	54·2			sin. 48° 49′ 59·7″	9·8766779

ω + ZS = 84 54 54·2.

Hence the latitude is 48° 50′.

2. The two corrected altitudes are 42° 14′ and 16° 5′ 47″, the corresponding declinations 8° 16′ 30″ and 8° 15′, and the time between the observations 3 hours; required the latitude of the place.

The latitude is 48° 54′ 27″ N.

Upon the same principle may the latitude be determined from the altitudes of two fixed stars, taken at the same time; in this case S, S′, in the preceding figure, will represent the two stars; PS, PS′, their known polar distances, and the angles SPS′ the difference of their right ascensions; the same quantities are therefore given as in the case of the sun, but as in the case of two stars PS, PS′, may differ very considerably; SS′ cannot be considered as the base of an isosceles triangle, but must be computed from the other two sides and their included angle. In the Nautical Almanack for 1825 Dr. Brinkley has computed for 1822, and tabulated, the distances SS′ for certain pairs of stars, conveniently situated for observation, and has annexed the change of distance corresponding to 10 years. The same table shows also the difference of right ascension for each pair of stars, with the change in 10 years; so that by help of this table the computation for finding the latitude from the simultaneous altitudes of two fixed stars becomes considerably abridged.

For other methods of determining the latitude, the student may consult "Mackay on the Longitude," vol. i., and Captain Kater's Nautical Astronomy, in the Ency. Metropolitan, &c.

On finding the Longitude by the Lunar Observations.

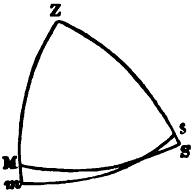
(80.) There are several astronomical methods of determining the longitude of a place, which cannot be accurately employed at sea, on account of the great difficulty of managing a telescope on shipboard; we shall not, therefore, enter here into any explanation of these methods, but shall confine ourselves to the *lunar method* of determining the longitude, which is justly regarded as the principal problem in Nautical Astronomy. Before entering upon the solution of this problem it will be necessary to make a few introductory remarks.

The determination of the longitude of a place always requires the solution of these two problems, viz. 1st, to determine the time at the place at any instant; and, 2d, to determine the time at the first meridian, at the same instant; for the difference of the times converted into degrees, at the rate of 15° to an hour, will obviously give the longitude.

When the latitude of the place is known, (and it may be found by the methods already explained,) the time may be computed from the altitude of any celestial object whose declination is known; for the coaltitude, codeclination, and colatitude, will be three sides of a spherical triangle given to find the hour angle, comprised between the codeclination and the colatitude. But to find the time at Greenwich requires the aid of additional data, besides those furnished by observations made at the place. The Greenwich time may, indeed, be obtained at once, independently of any observations at the place, by means of a chronometer, carefully regulated to Greenwich time, provided it be subject to no irregularities after having been once properly adjusted. A ship furnished with such a timepiece always carries the Greenwich time with her,* and the longitude then becomes reduced to the problem of finding the time at the place. Chronometers are now brought to such a state of perfection that very great dependence can be placed on them, and they are accordingly always taken out on long voyages for the purpose of showing the Greenwich time, and are thus of great use to the mariner. Still, however, as the most perfect contrivance of human art is subject to accident, and the more delicate the machine the more liable is it to disarrangement, from causes which we may not be able to control, it becomes highly desirable, in so important a matter as finding the place of a ship at sea, to be possessed of methods altogether beyond the influence of terrestrial vicissitudes, and such methods the celestial motions alone can supply. The angular motion of the moon in her orbit is more rapid than that of any other celestial body, and sufficiently great to render the portion of its path passed over in so short a time as two or three seconds, a measurable quantity even with a small portable instrument (the sextant).

It is obvious, therefore, that if the distance of the moon's centre from any celestial body, in or near her path, be computed for any Greenwich time, and this distance be found the same as that given by actual observation at any place, then the difference between the time of observing the phenomenon and the time at Greenwich, when it was predicted to happen, will give the longitude of the place of observation. Now in the Nautical Almanack the distances of the moon from the sun, and from several of the fixed stars near her path, are given for every three hours of apparent Greenwich time, and for several years to come; and the Greenwich time, corresponding to any intermediate distance, is obtained by simple proportion, with all requisite accuracy; so that by means of the Nautical Almanack we may always determine the time at Greenwich when any distance observed at sea was taken.

The distances inserted in the Nautical Almanack are the true angular distances between the centres of the bodies, the observer being considered as at the centre of the earth, and to the true distance therefore every observed distance must be reduced; it is this reduction which constitutes the trigonometrical difficulties of the problem; and it consists in *clearing the lunar distance from the effects of parallax and refraction;* how to do this it is now our business to explain.



Let m, s, be the observed places of the moon and sun, or of the moon and a fixed star, and let M, S, be their true places. M will be above m, because the moon is depressed by parallax more than it is elevated by refraction; but S will be below s, because the sun is more elevated by refraction than it is depressed by parallax. Observation gives the apparent distance ms, and the apparent zenith distances Zm, Zs: by applying the proper corrections to these latter we also deduce the true zenith distances ZM, ZS, and with these data we are to determine the true distance, MS, by computation.

Put d for the apparent distance.
D true distance.
a, a' apparent altitudes.
A, A′ true altitudes.

---

* As chronometers show mean time, the equation of time must be applied to obtain the *apparent* time at Greenwich.

Then in the triangle MZS, we have $\cos. Z = \dfrac{\cos. D - \sin. A \sin. A'}{\cos. A \cos. A'}$;

and in the triangle mZs, $\cos. Z = \dfrac{\cos. d - \sin. a \sin. a'}{\cos. a \cos. a'}$;

hence, for the determination of D, we have this equation, viz.

$$\frac{\cos. D - \sin. A \sin. A'}{\cos. A \cos. A'} = \frac{\cos. d - \sin. a \sin. a'}{\cos. a \cos. a'};$$

from which we immediately get

$$\cos. D = (\cos. d - \sin. a \sin. a') \frac{\cos. A \cos. A'}{\cos. a \cos. a'} + \sin. A \sin. A'$$

$$= \frac{\cos. d + \cos. (a + a') - \cos. a \cos. a'}{\cos. a \cos. a'} \cos. A \cos. A' + \sin. A \sin. A'$$

$$= \frac{2 \cos. \tfrac{1}{2}(a + a' + d) \cos. \tfrac{1}{2}(\overline{a + a'} \sim d) \cos. A \cos. A'}{\cos. a \cos. a'} - \cos. (A + A'). (1)$$

$$= \left\{ \frac{2 \cos. \tfrac{1}{2}(a + a' + d) \cos. \tfrac{1}{2}(\overline{a + a'} \sim d) \cos. A \cos. A'}{\cos. a \cos. a' \cos. (A + A')} - 1 \right\} \cos. (A + A');$$

or calling the first term within the brackets $2 \cos.^2 F$,

$$\cos. D = (2 \cos.^2 F - 1) \cos. (A + A') = \cos. 2 F \cos. (A + A') . (2).$$

The formulas marked (1) and (2) are both of them convenient for the computation of D; a third formula may be obtained from (1), as follows. Subtract each side of (1) from 1; then since (p. 37,)

$$1 - \cos. D = 2 \sin.^2 \tfrac{1}{2} D,\ 1 + \cos. (A + A') = 2 \cos.^2 \tfrac{1}{2} (A + A'),$$

we have, after dividing by 2,

$$\sin.^2 \tfrac{1}{2} D = \cos.^2 \tfrac{1}{2} (A + A') - \frac{\cos. \tfrac{1}{2}(a + a' + d) \cos. \tfrac{1}{2}(\overline{a + a'} \sim d) \cos. A \cos. A'}{\cos. a \cos. a'}$$

$$= \cos.^2 \tfrac{1}{2} (A + A') \left\{ 1 - \frac{\cos. \tfrac{1}{2}(a + a' + d) \cos. \tfrac{1}{2}(\overline{a + a'} \sim d) \cos. A \cos. A'}{\cos. a \cos. a' \cos.^2 \tfrac{1}{2} (A + A')} \right\};$$

or, calling the second term within the brackets $\sin.^2 \theta$,

$$\sin.^2 \tfrac{1}{2} D = \cos.^2 \tfrac{1}{2} (A + A') \cos.^2 \theta$$

$$\therefore \sin. \tfrac{1}{2} D = \cos. \tfrac{1}{2} (A + A') \cos. \theta \ldots . (3).$$

This latter is *Borda's* formula.

We shall solve an example by each of these formulas.

EXAMPLES.

1. Suppose the apparent distance between the centres of the sun and moon to be 83° 57′ 33″, the apparent altitude of the moon's centre 27° 34′ 5″, the apparent altitude of the sun's centre 48° 27′ 32″, the true altitude of the moon's centre 28° 20′ 48″, and the true altitude of the sun's centre 48° 26′ 49″; then we have $d = 83° 57' 33''$, $a = 27° 34' 5''$, $a' = 48° 27' 32''$; $A = 28° 20' 48''$, $A' = 48° 26' 49''$;

and the computation for D, by the first formula is as follows:

	°	′	″		
d	83	55	33		
a	27	34	5	comp. cos.	0523390
a'	48	27	32	comp. cos.	1783835
2)	159	59	10	log. 2	3010300
½ sum	79	59	35	cos.	9·2399686
½ sum ∼ d	3	57	58	cos.	9·9989587
A	28	20	48	cos.	9·9445275
A′	48	26	49	cos.	9·8217187

(Reject 40 from index) $\bar{1}$·5369260 = log. ·3442921 +

A + A′ 76 47 37 - - - nat. cos. ·2284595 −

True distance 83° 20′ 54″ nat. cos. ·1158326.

By glancing at the formula (1), we see that 30 must be rejected from the sum of the above column of logarithms, so that the logarithmic line resulting from the process is 9·5369260. Now, as in the table of log. sines, log. cosines, &c., the radius is supposed to be 10^{10}, of which the log. is 10, and in the table of natural sines, cosines, &c., the rad. is 1, of which the log. is 0; it follows that when we wish to find, by help of a table of the logarithms of numbers, the natural trigonometrical line corresponding to any logarithmic one, we must diminish this latter by 10, and enter the table with the remainder. Hence the sum of the foregoing columns of logarithms must be diminished by 40, and the remainder will be truly the logarithm of the natural number represented by the first term in the second member of the equation (1). If this natural number be less than nat. cos. (A + A′), which is to be subtracted from it, the remainder will be negative, in which case D will be obtuse.

By the second formula the process is as follows:

d	83°	57′	33″	
a	27	34	5	comp. cos. 0·0523390
a′	48	27	32	comp. cos. 0·1783835
2)	159	59	10	
½ sum	79	59	35	cos. 9·2399686
½ sum ∼ *d*	3	57	58	cos. 9·9989587
A	28	20	48	cos. 9·9445275
A′	48	26	49	cos. 9·8217187
A + A′	76	47	37	comp. cos. 0·6411909 —
				2)19·8770869
F	29	46	3	cos. 9·9385434
2 F	59	32	6	cos. 9·7050182 +

True distance 83° 20′ 54″ cos. 9·0638273.

In adding up the logarithms to find cos. ^{2}F, 20 must be rejected from the index; and the logarithm marked —, is to be subtracted from that marked +. Moreover, if A + A′ and 2 F are both acute or both obtuse, D will be acute, otherwise it will be obtuse.

We shall now exhibit the process by *Borda's formula.*

d	83°	57′	33″	
a	27	34	5	comp. cos. 0·0523390
a′	48	27	32	comp. cos. 0·1783835
2)	159	59	10	
½ sum	79	59	35	cos. 9·2399686
½ sum ∼ *d*	3	57	58	cos. 9·9989587
A	28	20	48	cos. 9·9445275
A′	48	26	49	cos. 9·8217187
A + A′	76	47	37	2)39·2358960
				19·6179480
½ (A + A′)	38	23	48½	cos. 9·8941654 +
θ	31	57	53½	sin. 9·7237826

θ - - cos. 9·9285870 +

½ D 41° 40′ 27″ sin. 9·8227524
∴ D = 83 20 54, the true distance.

An estimate may now be formed of the relative advantages of these three methods, as regards practical facility. We are inclined to prefer the first method, which we believe is new, as fewer references to the tables are requisite, and as, moreover, there are no arithmetical operations required, besides those which are actually exhibited. The second and third methods seem to offer nearly equal advantages; in the first of these, however, it may be observed that the trigonometrical lines involved are all of one name, viz. cosines, and that the final reference to the tables gives the true distance instead of its half, as in the last method.

Each of the foregoing processes may be shortened by using a subsidiary table, containing the various values of the expression $\frac{\cos. A \cos. A'}{\cos. a \cos. a'}$. Such a table computed to every degree of the moon's apparent altitude, and to every 10 seconds of her horizontal parallax, forms Table IX. of the *Requisite Tables*, published by order of the Commissioners of Longitude. But a more complete table of this kind is given in the second volume of Dr. Mackay's work, on the Longitude. If each number in this table were increased by the constant number ·3010300, the table itself would become somewhat simplified, and the process of clearing the distance by our first method would be rendered remarkably short and convenient.

The preceding example is taken from Woodhouse's Astronomy, part II., p. 859, where the day of observation is stated to be June 5, 1793. Now by the Nautical Almanack, for that year, we have

Distance at 15^h 83° 6′ 1″, Also at time of observation D = 83° 20′ 55″
at 18^h 84 28 26 at 15^h D = 83 6 1

Increase in 3^h 1 22 25, Incs. between 15^h and time of obs. 0 14 54
∴ 1° 22′ 25″ : 14′ 54″ :: 3^h : 32^m 33^s.

Hence, when the observation was made, the apparent time at Greenwich was 15^h 32^m 33^s.

To find the time at the ship, requires that we know the latitude of the place and the sun's declination. The former, therefore, must have been previously ascertained, and the latter may now be found by means of the apparent Greenwich time just deduced, and the Nautical Almanack. We shall suppose the latitude to be 10° 16′ 40″ S.; the sun's declination will be 23° 22′ 28″, and taking the true altitude of the sun = 48° 46′ 49″, we shall thus have, in order to find the time, three sides of a spherical triangle to find an angle. The computation is as follows.

coalt. 41° 33′ 11″
sin. colat. 79 43 20 arith. comp. 0·0070251
sin. sun's polar dist. 113 22 48 arith. comp. 0·0372078

2)234 39 19

117 19 39·5
sin. 37 36 19·5 - - 9·7854864
sin. 3 56 51·5 - - 8·8378712

2)18·6675905

sin. 12° 27′ 17½″ - 9·3337902

Hour angle = 24 54 35 = $1^h\,39^m\,38{\cdot}3^s$ in time.

Time at Greenwich 15 32 33

L. in time, *reckoning westward.* 13 52 54·7.

Or, subtracting this from 24 hours, we have $10^h\,7^m\,4{\cdot}3^s$, for the longitude *east*, in time, and therefore the longitude in degrees is 151° 46 4½″ E.

2. Given the apparent altitude of the moon's centre 8° 26′ 13″, the true altitude 9° 20′ 45″, the apparent altitude of a star 35° 40′, the true altitude 35° 38′ 49″, and their apparent distance 31° 13′ 26″; to determine the true distance. The true distance is 30° 23′ 56″.

Those who are desirous of entering more at large into the problem of the Longitude, and of becoming acquainted with the best methods of shortening the computation by the aid of subsidiary tables, may advantageously consult, besides the works already referred to, the Quarto Tables of J. De Mendoza Rioz, Lynn's Navigation Tables, Captain Kater's Treatise on Nautical Astronomy, in the Encyclopædia Metropolitana, Kerrigan's Navigator's Guide and Nautical Tables, and Dr. Myers's translation of Rossel on the Longitude.

Variation of the Compass.

(81.) We shall conclude this part of our subject by briefly considering the methods of finding the variation of the compass, or the quantity by which the north point, as shown by the compass, varies easterly or westerly from the true north point of the horizon

The solution of this problem merely requires that we find by computation, or by some means independent of the compass, the *bearing* of a celestial object, that we observe the bearing by the compass, and then take the difference of the two. The problem resolves itself, therefore, into two cases, the object whose bearing is sought being either in the horizon or above it: in the one case we have to compute its *amplitude*, and in the other its *azimuth*.

The computation of the amplitude is simply determining the hypotenuse of a right-angled triangle, of which one side is given, viz. the declination of the object, as also the angle opposite to it, viz. the colatitude. The computation of the azimuth requires the solution of an oblique spherical triangle, the three sides being given to find an angle; the three given sides are the colatitude; the zenith distance of the object and its polar distance: and the azimuth being measured by the angle at the zenith opposite the polar distance, this is the angle sought. We shall give an example in each of these cases of the problem.

EXAMPLES.

1. In January 1830, at latitude 27° 36′ N., the rising amplitude of Aldebaran was, by the compass* E. 23° 30′ N., required the variation.

By the Nautical Almanack the declination of Aldebaran is 16° 9′ 37″ N., therefore since Rad. × sin. dec. = sin. amp. × cos. lat., the computation is as follows.

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* The compass amplitude must be taken when the apparent altitude of the object is equal to the depression of the horizon.

sin. declination 16° 9′ 37″ - - 9·4445527
cos. latitude 27 36 - - 9·9475335
sin. Amptitude E. 18 18 17 N. - - 9·4970292

Magnetic Amptitude E. 23 30 0 N.

Variation 5 11 43.

As the object is farther from the magnetic east than from the true east, the magnetic east has therefore advanced towards the south, and therefore the magnetic north towards the east; hence the variation is 5° 11′ 43″ E.

2. In latitude 48° 50′ north, the true altitude of the sun's centre was 22° 2′, the declination at the time was 10° 12′ S., and its magnetic bearing 161° 32′ east. Required the variation.

⊙'s polar distance 100° 12′
sin. zenith distance 67 58 arith. comp. 0·0329363
sin. colatitude 41 10 arith. comp. 0·1816080

2)209 20
sin. ½ S 104 40 - 9·9856129
sin. (½ S — pol. dist.) 4 28 - 8·8914209

2)19·0915781
cos. 69° 25′ 40″ - 9·5457895
2 -

⊙'s true azimuth N. 138 51 20 E.
Observed azimuth N. 161 32 0 E.

22 40 40 West.

The variation is west, because the sun's observed distance from the north, measured easterly, being greater than its true distance, intimates that the north point of the compass has approached towards the west.

3. In latitude 48° 20′ north, the star Rigel was observed to set 9° 50′ to the northward of the west point of the compass; required the variation, the declination of Rigel being 8° 25′ S. Variation 22° 33′ West.

4. In latitude 50° 12′ north, when the sun's declination was 11° 28′ 53″ N., its true altitude was found to be 37° 0′ 16″, and the observed azimuth S. 31° E.; required the variation of the compass.

Variation 28° 2′ West.

PART IV.

MISCELLANEOUS TRIGONOMETRICAL INQUIRIES.

(82.) We now come to the final *part* of our subject, in which we propose to bring together several miscellaneous particulars which properly come under consideration in a treatise on Trigonometry. One or two of these, especially those which relate to certain compendious solutions of plane triangles, and to the trigonometrical lines of small arcs, might have been introduced much earlier, although we have preferred to postpone their consideration for a supplementary chapter, agreeing with *Woodhouse*, that it is better for the student first "to attend solely to the general solutions, and to postpone to a time of leisure and of acquired knowledge the consideration of the methods that are either more expeditious or are adapted to particular exigencies.

CHAPTER I.

ON THE SOLUTIONS OF CERTAIN CASES OF PLANE TRIANGLES, AND ON DETERMINING THE TRIGONOMETRICAL LINES OF SMALL ARCS.

PROBLEM I.

(83.) Given two sides and the included angle of a plane triangle, to determine the third side, without finding the remaining angles.

The general expression for the side c, in terms of the two sides a, b, and the included angle C, is (17),

$$c^2 = a^2 + b^2 - 2ab\cos.\mathrm{C} = (a-b)^2 + 2ab\,(1 - \cos.\mathrm{C})$$

$$= (a-b)^2 + 2ab\cdot 2\sin.^2\tfrac{1}{2}\mathrm{C} = (a-b)^2\left\{1 + \frac{4ab}{(a-b)^2}\sin.^2\tfrac{1}{2}\mathrm{C}\right\}.$$

Assume the second term within the brackets equal to $\tan.^2\theta$ then, since $1 + \tan.^2\theta = \sec.^2\theta = \dfrac{\mathrm{rad.}^2}{\cos.^2\theta}$, we have $c = (a-b)\dfrac{\mathrm{rad.}}{\cos.\theta}$.

Hence c is determined by these two formulas, viz.

$$\log.\tan.\theta = \log.2 + \tfrac{1}{2}\log.a + \tfrac{1}{2}\log.b + \log.\sin.\tfrac{1}{2}\mathrm{C} - \log.(a-b)$$

$$\log.c = \log.(a-b) + 10 - \log.\cos.\theta.$$

EXAMPLE.

Given $a = 562$, $b = 320$, and $\mathrm{C} = 128°\,4'$, to find c.

$$\begin{array}{rlr}
\log.2 & 0{\cdot}3010300 & \\
\tfrac{1}{2}\log.562 & 1{\cdot}3748681 & \\
\tfrac{1}{2}\log.320 & 1{\cdot}2525750 & \\
\log.\sin.64°\,2' & 9{\cdot}9537833 & \\
\text{ar. comp. }\log.242 & 7{\cdot}6161846,\ \log.242+10\ .\ . & 12{\cdot}3838154 \\
\hline
\log.\tan.\theta & 10{\cdot}4984410\ \therefore\ \log.\cos.\theta = & 9{\cdot}4807177 \\
\hline
\log.c & 800{\cdot}01\qquad .\qquad . & 2{\cdot}9030977.
\end{array}$$

PROBLEM II.

Given the logarithms of two sides of a plane triangle, as also the included angle, to determine the remaining angles.

Let log. a, log. b, and C, be given. Suppose a greater than b, and assume $r\,\frac{a}{b} = \tan.\,\theta$; then tan. θ being greater than 1, θ will exceed 45°. Also (19.) tan. ½ (A — B)

$$= \frac{a-b}{a+b}\cot.\tfrac{1}{2}\mathrm{C} = \frac{\frac{a}{b}-1}{\frac{a}{b}+1}\cot.\tfrac{1}{2}\mathrm{C} = \frac{\tan.\,\theta-1}{\tan.\,\theta+1}\cot.\tfrac{1}{2}\mathrm{C}$$

$= \tan.\,(\theta - 45°)\cot.\tfrac{1}{2}\mathrm{C}$ (p. 33).

Hence, introducing the radius, A — B is determined by these two formulas, viz. $\log.\tan.\theta = 10 + \log.\,a - \log.\,b$

$$\log.\tan.\tfrac{1}{2}(\mathrm{A}-\mathrm{B}) = \log.\tan.(\theta - 45°) + \log.\cot.\tfrac{1}{2}\mathrm{C} - 10.$$

Thus, taking the example in the last problem, we have

$$\begin{array}{lr l}
10 + \log.\ 562 \quad . \quad . & 12{\cdot}7497363 & \\
\log.\ 320 \quad . \quad . & 2{\cdot}5051500 & \\
\hline
\log.\tan.\theta \quad . & 10{\cdot}2445863 & \therefore \theta = 60°\ 20'\ 35''
\end{array}$$

$$\therefore \theta - 45° = 15°\ 20'\ 35''.$$

$$\begin{array}{lr l}
\textbf{Again, } \log.\tan.\ 15°\,20'\,35'' & 9{\cdot}4383476 & \\
\log.\cot.\ 64\ \ 2 & 9{\cdot}6875402 & \\
\hline
\log.\tan.\tfrac{1}{2}(\mathrm{A}-\mathrm{B}) & 9{\cdot}1258878 & \therefore \begin{cases}\tfrac{1}{2}(\mathrm{A}-\mathrm{B}) = \ \ 7°\ 36'\ 40' \\ \tfrac{1}{2}(\mathrm{A}+\mathrm{B}) = 25\ \ 58\end{cases}
\end{array}$$

$$\mathrm{A} = 33\ 34\ 40$$
$$\mathrm{B} = 18\ 21\ 20.$$

This method of determining the angles A and B will always be the shortest, when instead of their sides their logarithms are given. Thus the solution of problem x., p. 31, becomes much facilitated by the application of this process.

PROBLEM III.

To determine the area of a plane triangle when any three parts except the three angles are given.

1. Let two sides b, c, and the included angle A, be given. (See fig. p. 17.)

The area of the triangle is expressed by ½ AB · CD; but CD = AC sin. A; hence the expression for the area, in terms of the given quantities, is Area $= \tfrac{1}{2}bc\sin.\,\mathrm{A}$.

2. Let two angles, A, B, and the interjacent side c, be given.

Then, since sin. C : sin. B :: c : b,

we have $b = \frac{\sin.\,\mathrm{B}}{\sin.\,\mathrm{C}}\,c \ \therefore bc\sin.\,\mathrm{A} = \frac{\sin.\,\mathrm{A}\sin.\,\mathrm{B}}{\sin.\,\mathrm{C}}\,c^2$;

hence the expression for the area is Area $= \frac{\sin.\,\mathrm{A}\ \sin.\,\mathrm{B}}{2\sin.\,\mathrm{C}}\,c^2$.

3. Let the three sides be given.

By art. (20), $\sin.\tfrac{1}{2}\mathrm{A} = \sqrt{\frac{(\frac{1}{2}\mathrm{S}-b)\ (\frac{1}{2}\mathrm{S}-c)}{bc}}$, $\cos.\tfrac{1}{2}\mathrm{A} = \sqrt{\frac{\frac{1}{2}\mathrm{S}\,(\frac{1}{2}\mathrm{S}-a)}{bc}}$

$\therefore 2\sin.\tfrac{1}{2}\mathrm{A}\cos.\tfrac{1}{2}\mathrm{A}$, or (art. 31) $\sin.\,\mathrm{A} = \frac{2}{bc}\sqrt{\tfrac{1}{2}\mathrm{S}\,(\tfrac{1}{2}\mathrm{S}-a)\,(\tfrac{1}{2}\mathrm{S}-b)\,(\tfrac{1}{2}\mathrm{S}-c)}$.

Consequently, by substituting this value of sin. A in the first expression, we have, Area $= \sqrt{\tfrac{1}{2}\mathrm{S}\,(\tfrac{1}{2}\mathrm{S}-a)\,(\tfrac{1}{2}\mathrm{S}-b)\,(\tfrac{1}{2}\mathrm{S}-c)}$; which formula furnishes the well known rule, given in all books on mensuration, for the area of a triangle when the three sides are given. (See Geom. p. 202.) These expressions for the area of a plane triangle are all adapted to logarithmic computation.

PROBLEM IV.

To find the logarithmic sine of a very small arc.

By article (30) the expression for the sine of any arc x is,

$$\sin. x = x - \frac{x^3}{1\cdot2\cdot3} + \frac{x^5}{1\cdot2\cdot3\cdot4\cdot5} -, \text{ \&c.}$$ Now as the length of an arc of one degree is ·01745329, (see p. 36–7,) it is plain that, even when x is so great as this, the third term of the above series can have no significant figure in the first ten places of decimals.

Retaining therefore only the first two terms, we have, when x is small,

$$\sin. x = x - \frac{x^3}{1\cdot2\cdot3} = x\,(1 - \frac{x^2}{2\cdot3}) = x\,\Big\{1 - \frac{x^2}{2} + \frac{x^4}{2\cdot3\cdot4}\Big\}^{\frac{1}{3}} \text{ nearly;}$$

that is, (p. 36,) $\sin. x = x \cos.^{\frac{1}{3}} x$; hence, by introducing the radius,

$$\log. \sin. x = \log. x - \tfrac{1}{3}\,(10 - \log. \cos. x) \;\ldots.\; (1).$$ (1

Let the arc x contain n seconds, then $x = \dfrac{n \cdot \pi}{180 \times 60 \times 60}$;

hence, by introducing the radius,

$$\log. x = \log. n + \log. 3{\cdot}14159, \text{ \&c.} + 10 - \log. \overline{180 \times 60^2}$$
$$= \log. n + 4{\cdot}6855749; \text{ therefore, from (1),}$$

$$\log. \sin. x = \log. n + 4{\cdot}6855749 - \tfrac{1}{3} \text{ arith. comp. log. cos. } x \;\ldots\; (2);$$ (2

hence this rule. To the logarithm of the arc reduced into seconds, with the decimal annexed, add the constant quantity 4·6855749, and from the sum subtract one third of the arithmetical complement of the log. cosine; the remainder will be the logarithmic sine of the given arc.

This rule will determine the log. sine of a very small arc with great accuracy; it was first given, without demonstration, by Dr. Maskelyne, in his Introduction to Taylor's Logarithms. The above proof is from Woodhouse's Trigonometry.

PROBLEM V.

To find the logarithmic tangent of a very small arc.

Let x be the arc; then, as we have found in last problem,

$\sin. = x \cos.^{\frac{1}{3}} x \mathrel{\therefore} \dfrac{\sin. x}{\cos. x} = \tan. x = \dfrac{x}{\cos.^{\frac{2}{3}} x}$. Hence, introducing the radius, $\log. \tan. x = \log. x + \tfrac{2}{3}\,(10 - \log. \cos. x)$.

The second member of this equation is equal to the second member of (1) in last problem, *plus* the arithmetical complement of log. cos. x; hence, since the second member of (2) is equivalent to the second member of (1), we have

$$\log. \tan. x = \log. n + 4{\cdot}6855749 + \tfrac{2}{3} \text{ arith. comp. log. cos. } x \;\ldots\; (3);$$

which furnishes this rule. To the logarithm of the arc reduced to seconds add the constant quantity 4·6855749, and two thirds of the arithmetical complement of the log. cosine, the sum is the log. tangent of the given arc.

PROBLEM VI.

To find a small arc from its log. sine or its log. tangent.

10

x = length of arc to radius 1.

1. Let the log. sine be given; then n being the number of seconds in the arc, the expression (2), in problem IV., gives

log. n = log. sin. x — 4·6855749 + ⅓ arith. comp. log. cos. x = log. sin. x + 5·3144251 — 10 + ⅓ arith. comp. log. cos. x; therefore, to find the arc from the log. sine the rule is this. To the log. sine of the small arc add 5·3144251, and ⅓ of the arithmetical complement of the log. cosine; subtract 10 from the index of the sum, and the remainder will be the logarithm of the number of seconds in the arc.

2. Let the log. tangent be given; then from the expression (3), last problem, we have

log. n = log. tan. x — 4·6855749 — ⅔ arith. comp. log. cos. x
= log. tan. x + 5·3144251 — 10 — ⅔ arith. comp. log. cos. x;

that is, to the log. tangent of the small arc add 5·3144251, and from the sum subtract ⅔ of the arithmetical complement of the log. cosine, take 10 from the index of the remainder, and we shall have the logarithm of the number of seconds in the arc.

Let us now apply each of the foregoing rules to an example.

1. Required the log. sine of 1′ 4·8754″.

By the rule in problem IV. the process is as follows:

log. 64·8754 - - -	1·8120801
Constant No. - -	4·6855749
	6·4976550
⅓ arith. comp. log. cos. - -	0
log. sin. 1′ 4·8754″ - - -	6·4976550.

By the tables the log. sine is found as follows:

log. sin. 1′ 5″ - - -	6·4984882
log. sin. 1′ 4″ -	6·4917548
Difference -	·0067334

∴ log. sin. 1′ 4·8754″ = 6·4917548 + ·8754 × ·0067334 = 6·4976489.

2. Required the log. tangent of 7′ 2·38″.

By the Rule in Problem V.		*By the tables.*	
log. 422·38 -	2·6257033	log. tan. 7′ 3″	7·3119158
Constant No.	4·6855749	log. tan. 7 2	7·3108879
⅔ arith. comp. log. cos.	0		·0010279
log. tan. 7′ 2·38″	7·3112782.		

∴ log. tan. 7′ 2·38″ = 7·3108879 + ·38 × ·0010279 = 7·3112785.

3. Required the arc whose log. sine is 6·4976550.

By the Rule, Problem V. log. sine .	6·4976550
Constant No. .	5·3144251
⅓ arith. comp. .	0
log. 64·8754 . .	1·8120801

∴ the arc is 1′ 4·8754″.

By the Tables.

The proposed log. sine lies between log. sine 1′ 4″ and log. sine 1′ 5″, and the difference between these logs is ·0067334; also the difference between the proposed log. and log. sine 1′ 4″ is 59002; hence

$$\text{required arc} = 1' \, 4'' + \frac{59002}{67334} = 1' \, 4{\cdot}876''.$$

4. Required the arc whose log. tangent is 7·1644398.

By the Rule.	log. tan. . .	7·1644398
	Constant No. . . .	5·3144251
	⅔ arith. comp. . .	— 3
	log. 301·2067 . .	2·4788646

∴ the arc is 5′ 1·2067″.

By the Tables.

The proposed log. is between log. tan. 5′ 1″ and log. tan. 5′ 2″; the difference of these logs. is ·0014404, and the difference of the proposed and log. tan. 5′ 1″ is ·0002981.

$$\therefore \text{ the arc is } 5' \, 1'' + \frac{2981}{14404} = 5' \, 1{\cdot}2069''.$$

CHAPTER II.

INVESTIGATIONS OF EXPRESSIONS FOR THE SURFACE OF A SPHERICAL TRIANGLE AND FOR THE SPHERICAL EXCESS.

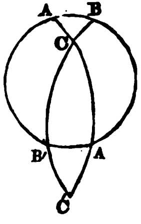
(84.) It has been already shown (36) that two great circles always intersect in two points at the distance of a semicircle from each other. The space thus included by two great circles is called a *lune*, (see the fig. at p. 42.)

The surface of a lune is to the surface of the whole sphere as the arc QQ′, or as the angle P of the lune, is to the whole circumference IQHI. This is pretty obvious, but it may be rigorously proved in the same way as it is proved in plane Geometry, that in the same circle any sector is to the whole circle as its arc is to the circumference, (Geom. prob. 23. Book 6). Hence, if we call the surface of the sphere S, and the angle of the lune ω degrees, the expression from its area will be $S \frac{\omega}{360}$; or if, instead of degrees, ω represents the absolute length of those degrees to radius 1, then the expression may be written $S \frac{\omega}{2\pi}$, where π stands for the number 3·14159, &c.

It can be proved, although not by the elementary principles of Trigonometry, that the surface of a sphere is equal to four times the area of one of its great circles;* that is, r being the radius of the sphere $S = 4\pi r^2$, so that the expression for the area of the lune is $2r^2\omega$. If we suppose r to be unity, the surface will be expressed by 2ω, that of the whole sphere being 4π.

PROBLEM I.

To express the area of a spherical triangle in terms of its three angles.



Let ABC be any spherical triangle, and produce the sides AC, BC, till they meet again in C′, forming the lune CC′. The triangle CAB will be a portion of an opposite lune equal to the lune CC′; and this portion will obviously be equal to the portion C′A′B′, provided the arcs CA, CB, are equal to the arcs C′A′, C′B′. Now AA′ is equal to CC′, each being a semicircle; hence, taking from each the common part CA′, we have CA = C′A′. In like manner CB = C′B′, and, therefore, the triangles ABC, A′B′C′, are equal. Hence the surface of the hemisphere, whose base is AB′A′B′, is equal to the sum of the three lunes AA′, BB′, CC′, minus twice the triangle ABC;

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* See "The Elements of the Integral Calculus," page 144.

2π −

that is, calling the surface of this triangle Σ.

$$\tfrac{1}{2}S = 2r^2(A+B+C) - 2\Sigma$$ $= 2r^2(A+B+C) - 2\Sigma$

$$\therefore \Sigma = r^2(A+B+C) - \tfrac{1}{4}S = r^2\{(A+B+C) - \pi\}.$$

where it must be observed that A, B, C, denote the lengths of the arcs which measure the angles of the proposed triangle to radius unity.

But, if we take A, B, C, and π in degrees, then since

$$180^\circ : \pi :: A+B+C-180^\circ : \{A+B+C-180^\circ\}\frac{\pi}{180^\circ}$$

#

the expression for Σ will be $\Sigma = r^2\{A+B+C-180^\circ\}\dfrac{\pi}{180^\circ}$ . . (1).

If the radius of the sphere, on which the triangle is, be taken for unity, then calling the area in this case ε, we have

$$\varepsilon = A+B+C-180^\circ \quad . \;\; . \;\; . \;\; . \quad (2);$$

which indicates that *the area of a triangle, on the surface of a sphere, whose radius is unity, is equal to the excess of its three angles above two right-angles.* This quantity is technically called the *spherical excess*, and the theorem (2) is known by the name of *Girard's* theorem.

It follows from this proposition that two spherical triangles are equal in surface, if the angles of the one are severally equal to those of the other, or, indeed, if the sum of the angles of the one triangle is equal to the sum of the angles of the other.

PROBLEM II.

To express the area of a spherical triangle, or the spherical excess in terms of two sides, and the included angle.

angles

Calling as before the surface of the triangle to radius unity ε, and, the sum of its three ~~sides~~ s, we have, by last problem,

!

$$\varepsilon = s - 180^\circ \therefore \cot.\frac{\varepsilon}{2} = -\tan.\tfrac{1}{2}s.$$

But (27), $\tan.\tfrac{1}{2}s = \tan.\tfrac{1}{2}(A+B+C) = \dfrac{\tan.\tfrac{1}{2}(A+B)+\tan.\tfrac{1}{2}C}{1-\tan.\tfrac{1}{2}(A+B)\tan\tfrac{1}{2}C}$;

and, by Napier's analogy, $\tan.\tfrac{1}{2}(A+B) = \dfrac{\cos.\tfrac{1}{2}(a-b)}{\cos.\tfrac{1}{2}(a+b)}\cot.\tfrac{1}{2}C$;

hence, by substitution,

$$\cot.\frac{\varepsilon}{2} = \frac{\cos.\tfrac{1}{2}(a-b)\cot.\tfrac{1}{2}C + \cos.\tfrac{1}{2}(a+b)\tan.\tfrac{1}{2}C}{\cos.\tfrac{1}{2}(a-b) - \cos.\tfrac{1}{2}(a+b)};$$

or multiplying the numerator by 2 sin. ½C cos. ½C, and the denominator by its equal, sin. C, (equa. 18, p. 37,)

$$\cot.\frac{\varepsilon}{2} = \frac{2\cos.\tfrac{1}{2}(a-b)\cos.^2\tfrac{1}{2}C + 2\cos.\tfrac{1}{2}(a+b)\sin.^2\tfrac{1}{2}C}{\cos.\tfrac{1}{2}(a-b)\sin.C - \cos.\tfrac{1}{2}(a+b)\sin C};$$

that is, by the formulas (1) and (2), page 32,

$$\cot.\frac{\varepsilon}{2} = \frac{\cos.\tfrac{1}{2}a\cos.\tfrac{1}{2}b + \sin.\tfrac{1}{2}a\sin.\tfrac{1}{2}b(\cos.^2\tfrac{1}{2}C - \sin.^2\tfrac{1}{2}C)}{\sin.\tfrac{1}{2}a\sin.\tfrac{1}{2}b\sin.C};$$

but, (from 19, p. 37,) $\cos.^2\tfrac{1}{2}C - \sin.^2\tfrac{1}{2}C = \cos.C$; hence

$$\cot.\frac{\varepsilon}{2} = \frac{\cot.\tfrac{1}{2}a\cot.\tfrac{1}{2}b + \cos.C}{\sin.C} = \left\{\frac{\cot.\tfrac{1}{2}a\cot.\tfrac{1}{2}b}{\cos.C} + 1\right\}\cot.C.$$

To adapt this expression to logarithmic computation suppose first that cos. C is positive, and that we assume $\dfrac{\cot.\tfrac{1}{2}a\cot.\tfrac{1}{2}b}{\cos.C} = \tan.^2\theta$,

then $\cot.\tfrac{1}{2}\varepsilon = \sec.^2\theta\cot.C$; suppose, secondly, that cos. C is negative, then if $\dfrac{\cot.\tfrac{1}{2}a\cot.\tfrac{1}{2}b}{\cos.C}$ is numerically less than radius, assume it equal

it [illegible] π = 180°, when radius = 1, as it evidently is

it when r > 1, semicircumference = r°π

to $\sin.^2\theta$, and we shall have $\cot.\frac{\varepsilon}{2} = \cos.^2\theta \cot. C$; but if the same expression be numerically greater than radius, then assume it equal to $\sec.^2\theta$, when we shall have $\cot.\frac{\varepsilon}{2} = \tan.^2\theta \cot. C$.

It may be remarked that, with the proposed data, the excess may be otherwise easily determined, by first finding, by the common formula, the third angle of the triangle, and then applying Girard's theorem.

PROBLEM III.

To determine the spherical excess when the three sides are given.

By formula 25, p. 38,

$$\cot.\tfrac{1}{2}a \cot.\tfrac{1}{2}b = \frac{1+\cos. a}{\sin. a}\cdot\frac{1+\cos. b}{\sin. b} = \frac{1+\cos. a+\cos. b+\cos. a\cos. b}{\sin. a\sin. b}.$$

By formula (A) p. 47, $\cos. C = \dfrac{\cos. c - \cos. a \cos. b}{\sin. a \sin b}$.

By formulas (1), (2), p. 49

$$2\sin.\tfrac{1}{2}C\cos.\tfrac{1}{2}C = \sin. C = \frac{2}{\sin. a\sin. b}\sqrt{\sin.\tfrac{1}{2}S\sin.(\tfrac{1}{2}S-a)\sin.(\tfrac{1}{2}S-b)\sin.(\tfrac{1}{2}S-c)}.$$

Substituting these values in the expression for $\cot.\frac{\varepsilon}{2}$, last problem, we have

$$\cot.\frac{\varepsilon}{2} = \frac{1+\cos. a+\cos. b+\cos. c}{2\sqrt{\sin.\tfrac{1}{2}S\sin.(\tfrac{1}{2}S-a)\sin.(\tfrac{1}{2}S-b)\sin.(\tfrac{1}{2}S-c)}}. \quad (2).$$

We may investigate another expression for the excess, as follows:

By the formulas (1), (2), page 49,

$$\sin.\tfrac{1}{2}A\cos.\tfrac{1}{2}B = \frac{\sin.(\tfrac{1}{2}S-b)}{\sin. c}\sqrt{\frac{\sin.\tfrac{1}{2}S\sin.(\tfrac{1}{2}S-c)}{\sin. b\sin. a}}$$

$$\sin.\tfrac{1}{2}B\cos.\tfrac{1}{2}A = \frac{\sin.(\tfrac{1}{2}S-a)}{\sin. c}\sqrt{\frac{\sin.\tfrac{1}{2}S\sin.(\tfrac{1}{2}S-c)}{\sin. b\sin. a}}$$

By adding, $\sin.\tfrac{1}{2}(A+B) = \dfrac{\sin.(\tfrac{1}{2}S-b)+\sin.(\tfrac{1}{2}S-a)}{2\sin.\tfrac{1}{2}c\cos.\tfrac{1}{2}c}\cos.\tfrac{1}{2}C.$

By subtracting, $\sin.\tfrac{1}{2}(A-B) = \dfrac{\sin.(\tfrac{1}{2}S-b)-\sin.(\tfrac{1}{2}S-a)}{2\sin.\tfrac{1}{2}c\cos.\tfrac{1}{2}c}\cos.\tfrac{1}{2}C.$

But by formula (27), page 39,

$$\sin.(\tfrac{1}{2}S-b)+\sin.(\tfrac{1}{2}S-a) = 2\sin.\tfrac{1}{2}c\cos.\tfrac{1}{2}(a-b).$$
$$\sin.(\tfrac{1}{2}S-b)-\sin.(\tfrac{1}{2}S-a) = 2\cos.\tfrac{1}{2}c\sin.\tfrac{1}{2}(a-b).$$

Hence by substitution, $\sin.\tfrac{1}{2}(A+B = \dfrac{\cos.\tfrac{1}{2}(a-b)}{\cos.\tfrac{1}{2}c}\cos.\tfrac{1}{2}C$

$$\sin.\tfrac{1}{2}(A-B = \frac{\sin.\tfrac{1}{2}(a+b)}{\sin.\tfrac{1}{2}c}\cos.\tfrac{1}{2}C.$$

Proceeding in the same way with the expressions for $\cos.\tfrac{1}{2}A\cos.\tfrac{1}{2}B$, and $\sin.\tfrac{1}{2}A\sin.\tfrac{1}{2}B$,

there results $\cos.\tfrac{1}{2}(A-B) = \dfrac{\sin.\tfrac{1}{2}(a+b)}{\sin.\tfrac{1}{2}c}\sin.\tfrac{1}{2}C,$

$\cos.\tfrac{1}{2}(A+B) = \dfrac{\cos.\tfrac{1}{2}(a+b)}{\cos.\tfrac{1}{2}c}\sin.\tfrac{1}{2}C.$ Now, since

$\sin.\frac{\varepsilon}{2} = -\cos.\frac{1}{2}(A+B+C) = \sin.\frac{1}{2}(A+B)\sin.\frac{1}{2}C - \cos.\frac{1}{2}(A+B)\cos.\frac{1}{2}C$, we have, by substituting in the second member the first and last of the foregoing expressions, $\sin.\frac{\varepsilon}{2} = \frac{\sin.\frac{1}{2}a\sin.\frac{1}{2}b}{\cos.\frac{1}{2}c}\sin.C$; or substituting for sin. C its value, as exhibited in last problem, and recollecting that (31), $\sin.a\sin.b = 4\sin.\frac{1}{2}a\cos.\frac{1}{2}a\sin.\frac{1}{2}b\cos.\frac{1}{2}b$,

we have $\sin.\frac{\varepsilon}{2} = \sqrt{\frac{\sin.\frac{1}{2}S\sin.(\frac{1}{2}S-a)\sin.(\frac{1}{2}S-b)\sin.(\frac{1}{2}S-c)}{2\cos.\frac{1}{2}a\cos.\frac{1}{2}b\cos.\frac{1}{2}c}}$ (3);

an expression adapted to logarithms.

By combining the formulas (2) and (3) various others may be deduced. Thus, by multiplying them together, we have

$$\cos.\frac{\varepsilon}{2} = \frac{1+\cos.a+\cos.b+\cos.c}{4\cos.\frac{1}{2}a\cos.\frac{1}{2}b\cos.\frac{1}{2}c} \quad . \; . \; . \quad (4).$$

But, formula (20), page 37,

$\cos.\frac{1}{2}a = \sqrt{(\frac{1}{2}+\frac{1}{2}\cos.a)}$, $\cos.\frac{1}{2}b = \sqrt{(\frac{1}{2}+\frac{1}{2}\cos.b)}$, $\cos.\frac{1}{2}c = \sqrt{(\frac{1}{2}+\frac{1}{2}\cos.c)}$;

hence, $\cos.\frac{\varepsilon}{2} = \frac{1+\cos.a+\cos.b+\cos.c}{\sqrt{2(1+\cos.a)(1+\cos.b)(1+\cos.c)}}$ . . . . (5).

Squaring this, $2\cos.^2\frac{\varepsilon}{2} = \frac{(1+\cos.a+\cos.b+\cos.c)^2}{(1+\cos.a)(1+\cos.b)(1+\cos.c)}$;

which, since $\cos.\varepsilon = 2\cos.^2\frac{\varepsilon}{2} - 1$, gives

$$\cos.\varepsilon = \frac{(1+\cos.a+\cos.b+\cos.c)^2 - (1+\cos.a)(1+\cos.b)(1+\cos.c)}{(1+\cos.a)(1+\cos.b)(1+\cos.c)} \quad . \; . \quad (6);$$

also because $1 - \cos.\varepsilon = \text{vers.}\,\varepsilon$, we may change this into

$$\text{vers.}\,\varepsilon = \frac{1-\cos.^2a-\cos.^2b-\cos.^2c+2\cos.a\cos.b\cos.c}{(1+\cos.a)(1+\cos.b)(1+\cos.c)} \quad . \; . \; . \quad (7).$$

Lastly, by squaring the expression (3) and multiplying by 2, we have

$$2\sin.^2\frac{\varepsilon}{2} = 1-\cos.\varepsilon = \text{vers.}\,\varepsilon = \frac{\sin.\frac{1}{2}S\sin.(\frac{1}{2}S-a)\sin.(\frac{1}{2}S-b)\sin.(\frac{1}{2}S-c)}{2\cos.^2\frac{1}{2}a\cos.^2\frac{1}{2}b\cos.^2\frac{1}{2}c} \quad . \; . \quad (8).$$

The expression, marked (2), is due to *De Gua*, as are those marked (4), (5), (6), and (7). The expression (3) is from *Cagnoli*, (Trigon. page 329.) Since, $\tan.\frac{\varepsilon}{4} = \frac{1-\cos.\frac{1}{2}\varepsilon}{\sin.\frac{1}{2}\varepsilon}$, we have, by combining the expressions (3) and (5),

$$\tan.\frac{\varepsilon}{4} = \frac{1-\cos.^2\frac{1}{2}a-\cos.^2\frac{1}{2}b-\cos.^2\frac{1}{2}c+2\cos.\frac{1}{2}a\cos.\frac{1}{2}b\cos.\frac{1}{2}c}{\sqrt{\sin.\frac{1}{2}S\sin.(\frac{1}{2}S-a)\sin.(\frac{1}{2}S-b)\sin.(\frac{1}{2}S-c)}}$$

Now, $1-\cos.^2\frac{1}{2}a-\cos.^2\frac{1}{2}b = \sin.^2\frac{1}{2}a-\cos.^2\frac{1}{2}b$,

by equa. 5, p. 32, $= \sin.^2\frac{1}{2}a\sin.^2\frac{1}{2}b-\cos.^2\frac{1}{2}a\cos.^2\frac{1}{2}b$;

hence, the numerator of the above expression is equal to $\sin.^2\frac{1}{2}a\sin.^2\frac{1}{2}b-(\cos.\frac{1}{2}a\cos.\frac{1}{2}b-\cos.\frac{1}{2}c)^2$, which is the same as

$$\{\sin.\frac{1}{2}a\sin.\frac{1}{2}b+\cos.\frac{1}{2}a\cos.\frac{1}{2}b-\cos.\frac{1}{2}c\}\times$$
$$\{\sin.\frac{1}{2}a\sin.\frac{1}{2}b-\cos.\frac{1}{2}a\cos.\frac{1}{2}b+\cos.\frac{1}{2}c\}\text{; or as}$$

$\{\cos.\frac{1}{2}(a-b)-\cos.\frac{1}{2}c\}\times\{\cos.\frac{1}{2}c-\cos.\frac{1}{2}(a+b)\} =$ (by equa. 27, p. 39);

$$2\sin.\frac{1}{2}(\frac{1}{2}S-b)\sin.\frac{1}{2}(\frac{1}{2}S-a)\times 2\sin.\frac{1}{2}S\sin.\frac{1}{2}(\frac{1}{2}S-c).$$

Consequently, since (page 38), $\sqrt{\frac{1}{2}\tan.\frac{1}{2}A} = \frac{\sin.\frac{1}{2}A}{\sqrt{\sin.A}}$,

the foregoing expressions for $\tan.\frac{\varepsilon}{4}$ takes this very remarkable form, viz.

$$\tan.\ \frac{\epsilon}{4} = \sqrt{\tan.\ \tfrac{1}{2}\mathrm{S}\tan.\ \tfrac{1}{2}(\tfrac{1}{2}\mathrm{S}-a)\tan.\ \tfrac{1}{2}(\tfrac{1}{2}\mathrm{S}-b)\tan.\ \tfrac{1}{2}(\tfrac{1}{2}\mathrm{S}-c)};$$

which is *Lhullier's* expression.

It follows from this problem that two spherical triangles are always equal in surface when the sides of the one are severally equal to those of the other, whether the triangles admit of coincidence or not.

PROBLEM IV.

Given the area of a spherical triangle on the surface of the earth in square feet, to determine the spherical excess.

Let the area of the triangle in feet be Σ, then, by problem I.,

$$\epsilon = \frac{\Sigma}{r^2} \cdot \frac{180^\circ}{3{\cdot}14159}.$$

Now the length of a degree, supposing the earth to be a perfect sphere, is 365154·6 feet; hence the earth's radius is $\frac{180}{3{\cdot}14159} \times 365154{\cdot}6$ feet; consequently, $\epsilon = \frac{3{\cdot}14159\,\Sigma}{180 \times (365154{\cdot}6)^2}$ degrees; or if the excess be expressed in seconds, then $\epsilon = \frac{3{\cdot}14159\,\Sigma \times 60^2}{180\ (365154{\cdot}6)^2}$ seconds.

$$\begin{aligned} \therefore \log.\ \epsilon &= \log.\ \Sigma + \log.\ 62{\cdot}83185\ \&c. - 2\log.\ 365154{\cdot}6 \\ &= \log.\ \Sigma + 1{\cdot}7981799 - 11{\cdot}1249536 \\ &= \log.\ \mathrm{E} - 9{\cdot}3267737. \end{aligned}$$

Hence, *from the logarithm of the area of the triangle in feet, subtract the constant logarithm* 9·3267737, *and the remainder will be the logarithm of the excess in seconds.*

This rule, which usually goes by the name of *General Roy's* rule, is in fact due to the late professor *Dalby*, by whom it was communicated to the General, when engaged with him in conducting the Trigonometrical Survey. (See the "Life of Mr. Dalby," in Leybourn's Repository, vol. v.)

By means of the rule just given we may very readily compute the spherical excess, provided that we previously know the area of the triangle in feet. In trigonometrical surveying, the triangle on the surface of the earth, composed between any three stations, is necessarily so limited a portion of the whole sphere that its area, computed as a plane triangle from the measured data, cannot be affected with any error of consequence. On this hypothesis, therefore, the area of the triangle may be determined by one or other of the methods in prob. III., last chapter, and thence the excess ascertained by the above rule. Should the excess, thus deduced, exactly equal the excess of the three observed angles above two right-angles, we may be assured of the accuracy of the observations; but if they differ, the difference must be regarded as the amount of the errors with which the three observed angles are affected. If all of them were observed with equal care, so that there appear no reason why one should be more erroneous than another, the correction thus found must be distributed equally among them; but if it be suspected that one of the angles is less to be depended on than the others, then to this angle must be applied the greater part of the whole correction. The data being thus corrected, the required side or sides of the spherical triangle may be computed by the rules of spherical trigonometry; or the same object may be effected by plane trigonometry, with all requisite accuracy, provided we employ in the computation, not the corrected spherical angles, but these angles diminished each by one third of the spherical excess found as above, a truth which has been established by Legendre, (See the Appendix to Brewster's translation of Legendre's Geometry.) Trigonometrical surveying is a very important application of the theory of trigonometry, but is too ample a subject to admit of being discussed in the present volume. The student will find a condensed account of these geodetical operations in the tenth section of Dr. Lardner's Trigonometry, and every requisite information in the *Geodesie* of *M. Puissant* and *Col. Mudge's* account of the *Trigonometrical Survey of England and Wales.*[*]

Miscellaneous Expressions involving the Sides and Angles of a Spherical Triangle.

(85). We shall terminate the present chapter by the insertion of a few general expressions, involving the three sides and the three angles of a spherical triangle. Those formulas which have already been given in the second part of the work, are amply sufficient for the solution of every case in spherical trigonometry, but the sides and angles of a spherical triangle possess many other remarkable relations which are often called in aid, in higher investigations concerning a sphere. A few of these, therefore, it may be proper to give. Let s represent half the sum of the sides a, b, c, and S, half the sum of the angles A, B, C, of a spherical triangle; then by multiplying together the expressions for $\sin. \frac{1}{2}A$, $\cos. \frac{1}{2}A$, in art. (47), and those for $\sin. \frac{1}{2}a$, $\cos. \frac{1}{2}a$, in art. (49), and squaring the results, we have these equations;

$\sin.^2 b \sin.^2 c \sin.^2 A = 4 \sin. s \sin. (s-a) \sin. (s-b) \sin. (s-c) = 4n^2$. (1)

$\sin.^2 B \sin.^2 C \sin.^2 a = -4 \cos. S \cos. (S-A) \cos. (S-B) \cos. (S-C) = 4N^2$ (2).

By multiplication, $\therefore \sin. a \sin. b \sin. c \sin. A \sin. B \sin. C = 4Nn$ (3).

By division, $\frac{\sin. b}{\sin. B} \cdot \frac{\sin. c}{\sin. C} : \frac{\sin. A}{\sin. a} = \frac{n}{N}$.

But the first two factors of this expression are each of them the reciprocal of the last. $\therefore \frac{\sin. b}{\sin. B} = \frac{\sin. c}{\sin. C} = \frac{\sin. a}{\sin. A} = \frac{n}{N} \ldots$ (4).

But from (1), $\frac{\sin. A}{\sin. a} = \frac{2n}{\sin. a \sin. b \sin. c} \therefore \frac{N}{n} = \frac{2n}{\sin. a \sin. b \sin. c}$. (5).

and from (2),

$\frac{\sin. a}{\sin. A} = \frac{2N}{\sin. A \sin. B \sin. C} \therefore \frac{n}{N} = \frac{2N}{\sin. A \sin. B \sin. C} \ldots$ (6).

Substituting in (6) the value of N deduced from (5), and in (5) the value of n deduced from (6), we have, from the resulting equations, these expressions for n and N viz.

$$n = \tfrac{1}{2} \left\{ \sin.^2 a \sin.^2 b \sin.^2 c \sin. A \sin. B \sin. C \right\}^{\frac{1}{3}} \quad . : . \quad (7)$$

$$N = \tfrac{1}{2} \left\{ \sin.^2 A \sin.^2 B \sin.^2 C \sin. a \sin. b \sin. c \right\}^{\frac{1}{3}} \quad . . : \quad (8);$$

and, for their ratio $\frac{n}{N}$, we have from these, as also from (4),

$$\frac{n}{N} = \left\{ \frac{\sin. a \sin. b \sin. c}{\sin. A \sin. B \sin. C} \right\}^{\frac{1}{3}} \quad \ldots \quad (9);$$

expressions which are remarkable for their symmetry.

Again, referring to the expressions for $\sin. \frac{1}{2}A$, and $\cos. \frac{1}{2}A$, at (47), we see that $\sin. \frac{1}{2} A \sin. \frac{1}{2} B \sin. \frac{1}{2} C = \frac{n^2}{\sin. s \sin. a \sin. b \sin. c}$ (10)

$$\cos \tfrac{1}{2} A \cos. \tfrac{1}{2} B \cos. \tfrac{1}{2} C = \frac{n \sin. s}{\sin. a \sin. b \sin. c} \quad \ldots \quad (11)$$

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[*] Some additional particulars respecting the spherical excess will be found in the supplement.

$$\therefore \text{tan.}\,\tfrac{1}{2}\,\text{A}\,\text{tan.}\,\tfrac{1}{2}\,\text{B}\,\text{tan.}\,\tfrac{1}{2}\,\text{C} = \frac{n}{\text{sin.}^2\,s} \quad .\;.\;.\;.\;. \quad (12).$$

And by referring to the expressions for sin. $\frac{1}{2}a$, cos. $\frac{1}{2}a$, at (49), we see the truth of the following analogous equations; viz.,

$$\text{sin.}\,\tfrac{1}{2}\,a\,\text{sin.}\,\tfrac{1}{2}\,b\,\text{sin.}\,\tfrac{1}{2}\,c = \frac{-\text{N}\,\text{cos.}\,\text{S}}{\text{sin.}\,\text{A}\,\text{sin.}\,\text{B}\,\text{sin.}\,\text{C}}\,* \quad .\;. \quad (13)$$

$$\text{cos.}\,\tfrac{1}{2}\,a\,\text{cos.}\,\tfrac{1}{2}\,b\,\text{cos.}\,\tfrac{1}{2}\,c = \frac{\text{N}^2}{-\text{cos.}\,\text{S}\,\text{sin.}\,\text{A}\,\text{sin.}\,\text{B}\,\text{sin.}\,\text{C}} \quad . \quad (14)$$

$$\therefore \text{tan.}\,\tfrac{1}{2}\,a\,\text{tan.}\,\tfrac{1}{2}\,b\,\text{tan.}\,\tfrac{1}{2}\,c = \frac{\text{cos.}^2\,\text{S}}{\text{N}} \quad .\;. \quad (15).$$

From (10), $\text{sin.}\,s = \dfrac{n^2}{\text{sin.}\,a\,\text{sin.}\,b\,\text{sin.}\,c\,\text{sin.}\,\frac{1}{2}\,\text{A}\,\text{sin.}\,\frac{1}{2}\,\text{B}\,\text{sin.}\,\frac{1}{2}\,\text{C}} \quad . \quad (15)$

$$= \text{, by (5),} \frac{\text{N}}{2\,\text{sin.}\,\tfrac{1}{2}\,\text{A}\,\text{sin.}\,\tfrac{1}{2}\,\text{B}\,\text{sin.}\,\tfrac{1}{2}\,\text{C}} \quad .\;. \quad (16).$$

From (11), $\text{sin.}\,s = \dfrac{\text{sin.}\,a\,\text{sin.}\,b\,\text{sin.}\,c\,\text{cos.}\,\frac{1}{2}\,\text{A}\,\text{cos.}\,\frac{1}{2}\,\text{B}\,\text{cos.}\,\frac{1}{2}\,\text{C}}{n}$

$$= \text{, by (5), } 2\,\frac{n}{\text{N}}\,\text{cos.}\,\tfrac{1}{2}\,\text{A}\,\text{cos.}\,\tfrac{1}{2}\,\text{B}\,\text{cos.}\,\tfrac{1}{2}\,\text{C} \quad .\;. \quad (17);$$

and, from (12), $\text{sin.}^2\,s = \dfrac{n}{\text{tan.}\,\frac{1}{2}\,\text{A}\,\text{tan.}\,\frac{1}{2}\,\text{B}\,\text{tan.}\,\frac{1}{2}\,\text{C}} \quad .\;. \quad (18).$

In like manner, from (13), (14), (15), we have

$$\text{cos.}\,\text{S} = -\frac{\text{sin.}\,\text{A}\,\text{sin.}\,\text{B}\,\text{sin.}\,\text{C}\,\text{sin.}\,\tfrac{1}{2}\,a\,\text{sin.}\,\tfrac{1}{2}\,b\,\text{sin.}\,\tfrac{1}{2}\,c}{\text{N}}$$

$$= \text{, by (6),} -2\,\frac{\text{N}}{n}\,\text{sin.}\,\tfrac{1}{2}\,a\,\text{sin.}\,\tfrac{1}{2}\,b\,\text{sin.}\,\tfrac{1}{2}\,c \quad .\;. \quad (19)$$

$$\text{cos.}\,\text{S} = -\frac{\text{N}^2}{\text{sin.}\,\text{A}\,\text{sin.}\,\text{B}\,\text{sin.}\,\text{C}\,\text{cos.}\,\tfrac{1}{2}\,a\,\text{cos.}\,\tfrac{1}{2}\,b\,\text{cos.}\,\tfrac{1}{2}\,c}$$

$$= \text{, by (6),} -\frac{n}{2\,\text{cos.}\,\tfrac{1}{2}\,a\,\text{cos.}\,\tfrac{1}{2}\,b\,\text{cos.}\,\tfrac{1}{2}\,c} \quad .\;. \quad (20)$$

$$\text{cos.}^2\,\text{S} = \text{N}\,\text{tan.}\,\tfrac{1}{2}\,a\,\text{tan.}\,\tfrac{1}{2}\,b\,\text{tan.}\,\tfrac{1}{2}\,c = \frac{\text{N}}{\text{cot.}\,\tfrac{1}{2}\,a\,\text{cot.}\,\tfrac{1}{2}\,b\,\text{cot.}\,\tfrac{1}{2}\,c} \quad . \quad (21).$$

(86). In addition to these we shall here put down a few other useful expressions immediately deducible from the four equations which we had occasion to investigate at p. 114; and which are as follows:

$$\text{sin.}\,\tfrac{1}{2}\,(\text{A}+\text{B}) = \frac{\text{cos.}\,\tfrac{1}{2}\,\text{C}}{\text{cos.}\,\tfrac{1}{2}\,c}\,\text{cos.}\,\tfrac{1}{2}\,(a-b) \quad .\;.\;. \quad (22)$$

$$\text{sin.}\,\tfrac{1}{2}\,(\text{A}-\text{B}) = \frac{\text{cos.}\,\tfrac{1}{2}\,\text{C}}{\text{sin.}\,\tfrac{1}{2}\,c}\,\text{sin.}\,\tfrac{1}{2}\,(a-b) \quad .\;:\;. \quad (23)$$

$$\text{cos.}\,\tfrac{1}{2}\,(\text{A}+\text{B}) = \frac{\text{sin.}\,\tfrac{1}{2}\,\text{C}}{\text{cos.}\,\tfrac{1}{2}\,c}\,\text{cos.}\,\tfrac{1}{2}\,(a+b) \quad .\;.\;. \quad (24$$

$$\text{cos.}\,\tfrac{1}{2}\,(\text{A}-\text{B}) = \frac{\text{sin.}\,\tfrac{1}{2}\,\text{C}}{\text{sin.}\,\tfrac{1}{2}\,c}\,\text{sin.}\,\tfrac{1}{2}\,(a+b) \quad .\;.\;. \quad (25).$$

From these equations we immediately deduce the following analogous ones, viz. $\sin.\tfrac{1}{2}(a+b)=\dfrac{\sin.\frac{1}{2}c}{\sin.\frac{1}{2}C}\cos.\tfrac{1}{2}(A-B)$. (26)

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* This expression, as well as those marked 19, is usually given with an improper sign, viz. + instead of —, a mistake which seems to have arisen from confounding $\sqrt{(\text{cos. S} \cdot \text{cos. S})}$ with $\sqrt{(-\text{cos. S} \times -\text{cos. S})}$, which are, in fact, distinct expressions; the one being + cos. S, and the other — cos. S. See the chapter on *Imaginary Quantities*, in Young's Algebra, just published by Carey, Lea, & Co. Philadelphia.

$$\sin.\tfrac{1}{2}(a-b)=\frac{\sin.\frac{1}{2}c}{\cos.\frac{1}{2}C}\sin.\tfrac{1}{2}(A-B)\ \ .\ .\ .\ \ (27)$$

$$\cos.\tfrac{1}{2}(a+b)=\frac{\cos.\frac{1}{2}c}{\sin.\frac{1}{2}C}\cos.\tfrac{1}{2}(A+B)\ \ .\ .\ .\ \ (28)$$

$$\cos.\tfrac{1}{2}(a-b)=\frac{\cos.\frac{1}{2}c}{\cos.\frac{1}{2}C}\sin.\tfrac{1}{2}(A+B)\ \ .\ .\ .\ \ (29).$$

From (22) and (23) we have,

$$\sin.^2\tfrac{1}{2}(A+B)\cos.^2\tfrac{1}{2}c=\cos.^2\tfrac{1}{2}C\cos.^2\tfrac{1}{2}(a-b)$$
$$\sin.^2\tfrac{1}{2}(A-B)\sin.^2\tfrac{1}{2}c=\cos.^2\tfrac{1}{2}C\sin.^2\tfrac{1}{2}(a-b).$$

Hence, by addition,

$$\sin.^2\tfrac{1}{2}(A-B)\sin.^2\tfrac{1}{2}c+\sin.^2\tfrac{1}{2}(A+B)\cos.^2\tfrac{1}{2}c=\cos.^2\tfrac{1}{2}C\ \ .\ .\ .\ .\ \ (30).$$

In like manner, from (24) and (25),

$$\cos.^2\tfrac{1}{2}(A-B)\sin.^2\tfrac{1}{2}c+\cos.^2\tfrac{1}{2}(A+B)\cos.^2\tfrac{1}{2}c=\sin.^2\tfrac{1}{2}C\ \ .\ .\ \ (31).$$

Again, from (26) and (27), we have

$$\sin.^2\tfrac{1}{2}(a+b)\sin.^2\tfrac{1}{2}C=\sin.^2\tfrac{1}{2}c\cos.^2\tfrac{1}{2}(A-B)$$
$$\sin.^2\tfrac{1}{2}(a-b)\cos.^2\tfrac{1}{2}C=\sin.^2\tfrac{1}{2}c\sin.^2\tfrac{1}{2}(A-B);$$

and, by addition,

$$\sin.^2\tfrac{1}{2}(a-b)\cos.^2\tfrac{1}{2}C+\sin.^2\tfrac{1}{2}(a+b)\sin.^2\tfrac{1}{2}C=\sin.^2\tfrac{1}{2}c\ \ .\ .\ .\ \ (32);$$

and, in like manner, from (28) and (29) we get

$$\cos.^2\tfrac{1}{2}(a-b)\cos.^2\tfrac{1}{2}C+\cos.^2\tfrac{1}{2}(a+b)\sin.^2\tfrac{1}{2}C=\cos.^2\tfrac{1}{2}c\ .\ .\ .\ \ (33).$$

CHAPTER III.

ON THE RELATIONS BETWEEN THE CORRESPONDING VARIATIONS OF THE PARTS OF A TRIANGLE.

In the present chapter we propose briefly to examine into the effect produced on the sides and angles of a triangle, by a small change taking place in the magnitude of one of them; that is to estimate the amount of error affecting any part which may have been determined from data, not strictly accurate, and thence to ascertain under what circumstances a small inaccuracy in a proposed datum will least affect the accuracy of the result. This becomes a very essential matter of inquiry in all the more delicate practical operations of trigonometry, because, since the data furnished by observation necessarily fall short of strict accuracy, on account of the imperfections of instruments, and other unavoidable defects, we ought to know under what circumstances our observation should be made, so that the small error with which it is affected may have the least possible influence on the quantity to be determined from it. The following problems will sufficiently show the method of arriving at this knowledge.

PROBLEM I.

In a right-angled plane triangle, whose base is b, and altitude a, it is required to determine the error committed in calculating a by means of the given base b, and the observed angle opposite to a.

Let us consider α to represent the true angle opposite a, from which that given by observation varies by a small quantity, which we shall represent by $\delta\alpha$, and call *the variation of* α, then the sought side which would be given by the equation $a=b\tan.\alpha$, is affected by an error δa, so that instead of a it is $a + \delta a$, and this we determine from the equation $a + \delta a = b \tan. (a + \delta a)$; in which, by subtracting the preceding equation, we find the value of δa to be,

$$\delta a = b \rbrace \tan. (a + \delta a) - \tan. a \lbrace = \frac{b \sin. \delta a}{\cos. a \cos. (a + \delta a)} \text{ (art. 27):}$$

Now, by hypothesis, δa is very small, so that we may substitute it for its sine, and cos. a instead of cos. $(a + \delta a)$, $\therefore \delta a = \frac{b \delta a}{\cos.^2 a}$; in which expression δa is the length of the arc to radius 1, which measures the angular error.

To determine what length b must be to render the variation ,θa the least possible under the same amount of error δa in a, we have

$$b = a \cot. a \therefore \delta a = \frac{a \cot. a \delta a}{\cos.^2 a} = \frac{a \delta a}{\sin. a \cos. a} = 2 \frac{a \delta a}{\sin. 2a};$$

hence δa will be the least possible when sin. $2a$ is the greatest possible, that is when $a = 45°$: so that in order to determine the height of a tower or steeple, &c. with the utmost accuracy, by means of an observation of its angular altitude, we should make the observation at a distance from the object as nearly as possible equal to its height.

PROBLEM II.

In a right-angled spherical triangle is given one of the oblique angles to determine the variation of the opposite side, arising from a small variation of the hypotenuse.

Let A be the constant angle, a its opposite side, and c the hypotenuse; then $\sin. a = \sin. A \sin. c$, $\sin. (a + \delta a) = \sin. A \sin. (c + \delta c)$.
$\therefore$ by subtraction, $\sin. (a + \delta a) - \sin. a = \sin. A \rbrace \sin. (c + \delta c) - \sin. c \lbrace$; that is, (page 39, equa. 27,)

$$2 \cos. (a + \tfrac{1}{2} \delta a) \sin. \tfrac{1}{2} \delta a = 2 \sin. A \cos. (c + \tfrac{1}{2} \delta c) \sin. \tfrac{1}{2} \delta c$$

$\therefore \sin. \tfrac{1}{2} \delta a = \frac{\sin. A \cos. (c + \frac{1}{2} \delta c) \sin. \frac{1}{2} \delta c}{\cos. (a + \frac{1}{2} \delta a)}$; and if δa, δc, be very small, $\delta a = \frac{\sin. A \cos. c}{\cos. a} \delta c$; or, substituting for sin. A its value from the first equation, $\delta a = \frac{\sin. a}{\cos. a} \cdot \frac{\cos. c}{\sin. c} \delta c = \tan. a \cot. c \, \delta c$; which variation will be the least possible when cot. c is least, or when $c = 90°$. It would seem from the expression for δa, that in this case δa is absolutely 0, which we know cannot be. Indeed, no result deduced like that above, from a process in which certain small quantities are rejected, can be considered as perfectly accurate, although they may approximate so nearly to the truth as to be practically admissible as such. If we restore the $\tfrac{1}{2} \delta c$ which has been neglected, and write the above result thus, $\delta a = \tan. a \cot. (c + \tfrac{1}{2} \delta c) \delta c$; then, in the case of $c = 90°$, the expression becomes $\delta a = - \tan. a \tan. \tfrac{1}{2} \delta c \cdot \delta c$; or, considering the very small arc $\tfrac{1}{2} \delta c$ to be equal to its tangent, we have in the case supposed $\delta a = - \tfrac{1}{2} \tan. a (\delta c)^2$, the same expressions otherwise determined by *Professor Airy* in his Treatise on Trigonometry, in the Encyclopædia Metropolitana.

PROBLEM III.

In an oblique-angled spherical triangle are given two sides to determine the variation produced in the third side by a small variation of the opposite angle.

Let a, b, be the two given sides, C the included angle, and c the side

opposite to it. Then $\cos. c = \cos. a \cos. b + \sin. a \sin. b \cos. C$,
$\cos. (c + \delta c) = \cos. a \cos. b + \sin. a \sin. b \cos. (C + \delta c)$;
$\therefore$ by subtraction, $\cos. (c + \delta c) - \cos. c = \sin. a \sin. b \{(C + \delta C) - \cos. C\}$;
that is, $2 \sin. (c + \frac{1}{2} \delta c) \sin. \frac{1}{2} \delta c = 2 \sin. a \sin. b \sin. (C + \frac{1}{2} \delta C) \sin. \frac{1}{2} \delta C$

Hence, if δC be very small, $\sin. c\, \delta c = \sin. a \sin. b \sin. C \delta C$

$$\therefore \delta c = \frac{\sin. a \sin. b \sin. C}{\sin. c} \delta C = \sin. a \sin. B \delta C;$$

and δc is therefore the least possible when $\sin. C$ is the least possible, that is, when $C = 0$. To find the expression for δc, in this case, restore what has been rejected, and we shall have

$\delta c = \dfrac{\sin. a \sin. b \sin. (C + \frac{1}{2} \delta C)}{\sin. c} \delta C$; which, when $C = 0$, and $\frac{1}{2} \delta C$ very small, becomes $\delta c = \dfrac{\sin. a \sin. b}{2 \sin. c} (\delta C)^2$.

PROBLEM IV.

In an oblique-angled spherical triangle are given, as before, two sides and the included angle, to find the variation produced in one of the opposite angles by a small variation in the included angle.

Let a, b, be the given sides, C the included angle, then we have to find what influence a small variation in the value of the angle C will have on the angle A opposite a. For this purpose we shall deduce a suitable formula, as follows: substitute the expression for $\cos. c$, on the opposite page, in the corresponding expression for $\cos. a$, and we shall have the equation $\cos. A \sin c = \cos. a \sin. b - \sin. a \cos. b \cos. C$;
$\therefore \cos. A \dfrac{\sin. c}{\sin. a} = \cot. a \sin. b - \cos. b \cos. C$. But $\dfrac{\sin. c}{\sin. a} = \dfrac{\sin. C}{\sin. A}$;
hence by substitution, $\cot. A \sin. C = \cot. a \sin. b - \cos. b \cos. C$,
$\cot. (A + \delta A) \sin. (C + \delta C) = \cot. a \sin. b - \cos. b \cos. (C + \delta C)$;
and by subtraction,
$\cot. (A + \delta A) \sin. (C + \delta C) - \cot. A \sin. C = \cos. b \{\cos. C - \cos. (C + \delta C)\}$.

The first side of this equation is the same as
$\cot. (A + \delta A) \{\sin. (C + \delta C) - \sin. C\} + \sin. C \{\cot. (A + \delta A) - \cot. A\}$;
and the quantities within the brackets are respectively the same as

$$2 \cos. (C + \tfrac{1}{2} \delta C) \sin. \tfrac{1}{2} \delta C \text{ and } \frac{-\sin. \delta A}{\sin. A \sin. (A + \delta A)}$$

Also the second side of the same equation is the same as
$\cos. b \cdot 2 \sin. (C + \frac{1}{2} \delta C) \sin. \frac{1}{2} \delta C$; consequently,

$$2 \cot. (A + \delta A) \cos. (C + \tfrac{1}{2} \delta C) \sin. \tfrac{1}{2} \delta C - \frac{\sin. C \sin. \delta A}{\sin. A \sin. (A + \delta A)} =$$

$2 \cos. b \sin. (C + \frac{1}{2} \delta C) \sin. \frac{1}{2} \delta C$; and, therefore, when δC and δA are very small, $\cot. A \cos. C \delta C - \dfrac{\sin. C}{\sin.^2 A} \delta A = \cos. b \sin. C \delta C$

$$\therefore \delta A = \frac{\sin.^2 A}{\sin. C} (\cot. A \cos. C - \cos. b \sin. C) \delta C.$$

The foregoing examples are those selected by ***Professor Airy*** in his Treatise on Trigonometry, before referred to, and we have here adopted his processes. But the instruments of investigation generally the best adapted to inquiries of this kind is the ***Differential Calculus***.

END OF TRIGONOMETRY.

SUPPLEMENT

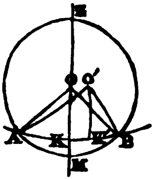
ON SPHERICAL GEOMETRY, POLAR TRIANGLES, &c,

BY T. S. DAVIES F. R. S. E., F. R. A. S. &c.

CHAPTER I.

ON SPHERICAL GEOMETRY.

In the commencement of the Spherical Trigonometry, a small collection of propositions, such as were necessary in the character of fundamental principles upon which to build the subsequent analytical investigations was given. At the request of the author, we here propose to add a few others, and shall endeavour to select such as may serve the double purpose of facilitating our future inquiries, and of interesting the mind of the student in some of the most beautiful classes of Geometrical research that are yet known to exist; we shall commence with a few properties analogous to the more elementary propositions in Euclid, and which are very often *assumed* by writers in spherical trigonometry, both unnecessarily and improperly.



1. Let O be the spherical centre of a circle, and AB any great circle chord: the perpendicular* OK demitted from the centre upon AB will bisect it. Draw AO, BO. Then from the right-angled triangles AKO, BKO, we have,

$$\frac{\text{cos. OA}}{\text{cos. OB}} = \frac{\text{cos. AK cos. OK}}{\text{cos. BK cos. OK}} = \frac{\text{cos. AK}}{\text{cos. BK}}.$$

But OA = OB, and ∴ AK = BK.

2. Conversely, if OK bisect AB, it will cut it at right angles.

† For $\text{cos. AKO} = \dfrac{\text{cos. AO} - \text{cos. AK cos. KO}}{\text{sin. AK sin. KO}}$

$$\text{cos. BKO} = \frac{\text{cos. BO} - \text{cos. BK cos. KO}}{\text{sin. BK sin. KO}}.$$

But the right-hand sides of these equations are equal, term for term, and therefore cos. AKO = cos. BKO, or AKO = BKO; and as AK, KB are one great circle, the angles at K are right angles: the tenth definition of the first book of Euclid applying to spherical as well as to plane angles.

3. If the great circle chord AB, be bisected at right angles at K, by the great circle ZM, this perpendicular shall pass through the centre of the circle. For, assume for a moment that the centre is at O′, not in the circle ZM; and draw the perpendicular O′K′. Now, we have seen that O′K′ bisects AB in K′ when O′ is the centre, or that AK + KK′ = BK − KK′; But, by hypothesis, AK = BK, and therefore, subtracting the latter equation, KK′ = − KK′, which is only true when K, K′ coincide, that is, when O′K′ coincides with OK, or when O′ is in ZM. The centre is therefore in ZM.

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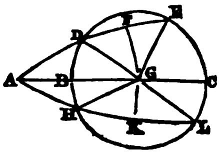
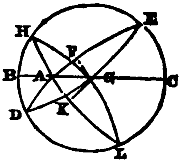
* Always meaning a great circular perpendicular, except expressly stated otherwise.

Equation (A/page 4)
for AK = BK, AK + KK′ = BK − KK′ when KK′ =



4. If two great circles which cut one another at A, be intersected by a circle of the sphere in D, E, and H, L respectively, the rectangles of the tangents of the semi-segments into which they are divided shall be equal.

That is,
tan. $\frac{1}{2}$ AE tan. $\frac{1}{2}$ AD = tan. $\frac{1}{2}$ AH tan. $\frac{1}{2}$ AL.



For find the centre G, draw AG meeting the circle in B and C, draw the perpendiculars GF, GK, and join GD, GE, GH, GL.

Then, cos. AK cos. KG = cos. AG = cos. AF cos. FG, cos. LK cos. KG = cos. GL = cos. FE cos. FG.

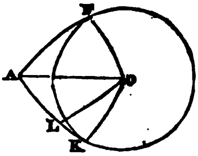
From these we have, by subtraction, addition, and subsequent division.

$$\frac{(\cos. AK - \cos. KL)}{(\cos. AK + \cos. KL)} = \frac{(\cos AF - \cos. FE)}{(\cos. AF + \cos. FE)};$$

and hence, by dividing (28) by (29), page 39.
tan. $\frac{1}{2}$(AK−KL) tan. $\frac{1}{2}$(AK + KL) = tan. $\frac{1}{2}$(AF − FE) tan. $\frac{1}{2}$(AF + FE),
that is, tan. $\frac{1}{2}$ AH tan. $\frac{1}{2}$ AL = tan. $\frac{1}{2}$ AD tan. $\frac{1}{2}$ AE.

The analogue to Euclid III. 35, may be seen in another form in the Math. Repository, No. 23, part II., p. 131, 2.

5. Let the secants in the case where A is without the circle take the position of tangents. Then D, E, F, coalesce, and so do H, K, L. Then the equation just obtained becomes
$\tan.^2 \frac{1}{2}$ AF = $\tan.^2 \frac{1}{2}$ AK, or AF = AK.



The tangents from any points to the circle are therefore equal. The case when the point is within the circle is demonstrated by *Cagnoli*, in his Trigonometry, but the other case he has not noticed.*

6. We may easily prove, also, that the great circle drawn through K at right angles to the radius, OK, touches the circle. For draw any other arc from O, as OL. Then, because K is a right-angle, we have cos. LK cos. KO = cos. LO. But cos. LK < 1, and therefore cos. LO < cos. KO, or LO > KO; and L will therefore fall *without* the circle; or, no part of KA falls within the circle whence KA is a tangent.

It is unnecessary to dwell at greater length upon these simple subjects; the nature of the inquiry, and the method of pursuing it, as well as its close analogy to the corresponding properties in the Elements of Euclid, must be at once apparent.

We may add that in the *Repository*, as above referred to, some other remarkable analogies to plain properties are derived by similar methods, to which we refer the inquiring reader.

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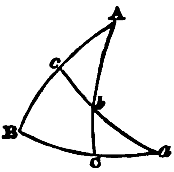
* Since this paper was written, *Professor Lowry* has sent me the enunciation of the proposition in the text, accompanied by the foregoing remark, and with the following corollaries subjoined, viz.

1. If an arc be drawn perpendicular to the diameter of a small circle of the sphere, the square of the tangent of half this arc will equal the product of the tangents of half the segments into which it divides the diameter.

2. If, from the extremities of the diameter of the small circle, arcs be drawn cutting the circle in the same point as the perpendicular, then the sum of the squares of the sines of half these arcs will equal the square of the sine of half the diameter of the small circle.



7. Let any spherical triangle be cut by a transversal *cba*. Then the products of the sines of the alternate segments will be equal. That is,

$$\sin. Ac \sin. Ba \sin. Cb = \sin. cB \sin. aC \sin bA.$$

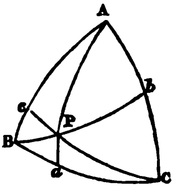
For $\sin. Ac : \sin. Ab :: \sin. \angle b : \sin. \angle c$

$\sin. aB : \sin. Bc :: \sin. \angle c : \sin. \angle a$

$\sin. Cb : \sin. Ca :: \sin. \angle a : \sin. \angle b.$

Hence, multiplying, and effacing the common terms in the antecedent and consequent of the result, we have the properties stated above.

8. The student can show, *ex absurdo*, that if the equality above stated takes place, the three points *a*, *b*, *c*, are in the same great circle.



9. If through any point P on the surface of the sphere three great circles be described, which also pass through the angles of the triangle ABC, and cut the opposite sides in *a*, *b*, *c*, respectively, then

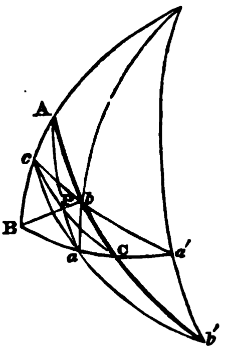
$$\sin. Ac \sin. Ba \sin. bC = \sin. aC \,.\, \sin. Bc \sin. bA.$$

For the two spherical triangles B*a*A and C*a*A cut by the two transversals C*c*, B*b*, give respectively

$$\sin. AP \cdot \sin aB \sin. bC = \sin. aP \sin. Bc \sin. bA,$$

$$\sin. aP \cdot \sin. cB \sin. cA = \sin. AP \sin. aC \sin. cB;$$

which multiplied, and the common terms effaced, give the enunciated property.



10. If through any point P in a given great circle A*a*, which passes through an angle, of a spherical triangle, great circles be drawn to the remaining angles cutting the opposite sides in *b*, *c*, respectively, then the great circle *bc* will always pass through the same point *a'* in the great circle BC, and so divide it that

$$\sin. Ba : \sin. aC :: \sin. Ba' : \sin. a'C.$$

For by (9 and 7) we have, respectively,

$$\sin. Ba : \sin. aC :: \sin. Bc \sin. Ab : \sin. cA \sin. bC$$

$$\sin. Ba' : \sin. a'C :: \sin. Bc \sin. Ab : \sin. cA \sin bC;$$

when, by equality of ratios, we have

$$\sin. Ba : \sin. aC :: \sin. Ba' : \sin. a'C.^*$$

11. If three great circles be drawn through the angles of a spherical triangle and through the same point on the surface of the sphere, cutting the sides in three points; three other great circles, each passing through two of these points, will intersect the sides of the triangle (produced or not as the case may require,) in the circumference of one and the same great circle.

By (10) we have $\sin. Ba : \sin. aC :: \sin. Ba' : \sin. a'C$

$\sin. Cb : \sin. bA :: \sin. Cb' : \sin. b'A$

$\sin. cA : \sin. cB :: \sin. c'A : \sin. c'B;$

or by multiplying vertically, and bearing (9) in mind,

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* This division of an arc is analogous to that which in *plano* is called the *harmonical* division of a line. Some of the most interesting properties of elementary geometry flow from considerations respecting the mode of division; and the spherical properties have perfect analogies to those. A few of these may be seen in the paper above mentioned in the Repository; and others will appear in a future number. Some curious investigations on this subject, by Professor Lowry, may also be seen in vol. II., new series of the same work, quest. 223. His processes however, are totally different from those just adverted to.

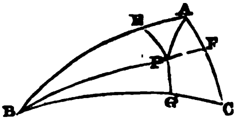
△ ABC ; point P, cutting sides in a, b, c ; cb meets BC in a', ca meets AC in b', ab meets BA in c' & a', b', c' are in one great circle.

$$\sin. Ba' \cdot \sin. Cb' \sin. c'A = \sin. a'C \cdot \sin. b'A \cdot \sin. c'B;$$
and hence by (8) the pr3position is established.*

12. If great circles be drawn from the angular points of any spherical polygon to a point on the surface of the sphere, the product of the sines of the alternate angles will be equal. In the triangle, (fig. to 9),

$$\sin. BP : \sin. PA :: \sin. BAP : \sin. ABP$$
$$\sin. PA : \sin. PC :: \sin. PCA : \sin. PAC$$
$$\sin. PC : \sin. PB :: \sin. PBC : \sin. PCB;$$
and, by multiplication, $\sin. BAP \sin. PCA \cdot \sin. CBP = \sin. ABP \sin. PAC \sin. PCB$.

The student is required to prove it for four, five, &c. sided figures, and is recommended to complete the argument from the suggestions furnished by the particular cases of the general truth. This theorem is due to *Professor Lowry*, Math. Rep. old series, vol. I., page 90.

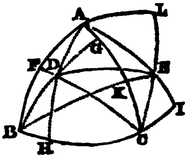


Let ABC be a spherical triangle, and P a point on the surface of the sphere, from which perpendiculars PE, PF, PG, are drawn to the sides of the triangle: then the product of the cosines of the alternate segments will be equal to one another. For

$$\cos. AE \cos. EP = \cos. AP = \cos. AF \cos. FP$$
$$\cos. BG \cos. GP = \cos. BP = \cos. BE \cos. EP$$
$$\cos. CF \cos. FP = \cos. PC = \cos. CG \cos. GP.$$
and multiplying the first and last columns vertically, we find
$$\cos. AE \cos. BG \cos. CF = \cos. AF \cos. BE \cos. CG.$$

Cor. If the triangle in *triquadrantal*, we shall have
$$\tan. AE \tan. BG \tan. CF = 1 = \cot. AF \cot. BE \cot. CG.$$

14. We shall here give (although we forget from whom we take it, and what kind of demonstration was given of it,) another such property of the triquadrantal triangle; and the student who is versed in Analytical Geometry, will recognise in it the trigonometrical demonstration of a remarkable property of a point referred to rectangular coordinates.



Let D, E, be two points on the sphere, and ABC a triquadrantal triangle. Then we have this property, viz.
$$\cos. DE = \cos. DA \cos. AE + \cos. DB + \cos. BE + \cos. DC \cos. CE.$$

For $\cos. DE = \cos. CD \cos. CE + \sin. CD \sin. CE \cos. DCE \quad .\ .\ .\ .\ (a);$
and, by right angled triangles,
$$\sin. CD = \frac{\cos. AD}{\cos. AF} \text{ and } \sin. CE = \frac{\cos. AE}{\cos. AL} \ .\ .\ (b).$$
the angle DCE is measured by $FL = AF \pm AL$; and hence (a) becomes
$$\cos. DE = \cos. CD \cos. CE + \frac{\cos. AD}{\cos. AF} \cdot \frac{\cos. AE}{\cos. AL} \left\{ \cos. AF \cos. AL \mp \sin. AF \sin. AL \right\}$$
$$= \cos. CD \cos. CE + \cos. AD \cos. AE \mp \frac{\cos. AD \cos. AE \sin. AF \sin. AL}{\cos. AF \cos. AL} \ .\ .\ .\ .\ (c).$$

But, by right angled triangles,

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* This remarkable proposition appears to have been discovered by *Carnot*, and was first published by him in 1803, and afterwards in 1806, in the *Geometry of Position*, and the *Essay upon Transversals*. It was subsequently and independently discovered by an eminent mathematician in this country, *Mr. Whitley*, who inferred it from the corresponding plane one, in the *Ladies' Diary*, 1817. The demonstration above given is taken entirely from Carnot, and it is a beautiful model for the method of conducting such inquiries. More ample information on these subjects may be had in the Repository, *ub supra*.

$$\cos. AD = \cos. AF \cos. FD, \text{ and } \cos. AE = \cos. AL \cos. LE;$$
also, $\sin. AF = \cos. BF$, and $\mp \sin. AL = \cos. BL$.
hence the last term of (*c*) reduces to

$$\cos. FD \cos. BF \cos. LE \cos. BL,$$

and by right-angled triangles, the first pair of these factors is equal to cos. BD, and the second pair to cos. BE, and thus is the proposed theorem established.*

Cor. 1. When D and E coincide, cos. DE = 1, and we have

$$\cos.^2 AD + \cos.^2 BD + \cos.^2 CD = 1,$$
$$\sin.^2 DH + \sin.^2 DG + \sin.^2 DF = 1.$$

Cor. 2. By (9) we have

$$\sin. AF \sin. BH \sin. CG = \sin. BF \sin. AG \sin. CH$$
$$= \cos. AF \cos. BH \cos. CG \text{ or, by division,}$$
$$\tan. AF \tan. BH \tan. CG = 1 = \tan. AL \tan. B1 \tan. CK.$$ ///

Cor. 3. When $DE = \frac{\pi}{2}$ we have, cos. DA cos. AE + cos. DB cos. BE + cos. DC cos. CE = 0; (vide Young's Anal. Geom., p. 228, art. 182—just published by Carey, Lea, & Co. Philadelphia.)

15. The following propositions, dependent upon what has been done here, or else upon similar methods, are left as exercises for the student.

(*a.*) Let a transversal great circle cut any spherical polygon; dividing each side into two segments; the product of the sines of the one set of alternate segments will be equal to the product of those of the other set. ||| or each produc

(*b.*) If a great circle bisect the angle of a triangle, (either interior or exterior,) the sines of the segments of the base have the same ratio as the sines of the sides.

(*c.*) The three bisectors pass through the same point on the sphere.

(*d.*) Perpendiculars to the middles of the sides pass through the same point, (centre of circumscribing circle.)

(*e.*) Perpendiculars from the angles of the triangle to the opposite sides pass through the same point.

(*f.*) Great circles joining the middles of the sides to the opposite angles intersect in the same point.

(*g.*) Great circles joining the points of contact of the inscribed circles with the sides, and the opposite angles pass through the same point.

(*h.*) Great circles passing through the points of contact of the circle which touches a triangle exteriorly, and the opposite angles, pass through the same point. ///

(*i.*) Great circles bisecting the interior angles of a spherical triangle meet the opposite sides in three points, which are situated in one great circle of the sphere. ///

(*k.*) Show under what conditions the propositions (12) and (13) admit of conversion.

(*l.*) Perpendiculars from the angles of a triangle upon the opposite sides intersect in three points, and the triangle formed by joining these points has its angles bisected by the said perpendiculars.

It would have been easy to extend and to vary these subjects almost without limit. As the method of Transversals is the most powerful one yet known for the investigation of spherical determinate theorems, (seeming to make up for the deficiency of parallels and similar triangles, the great organon in plane researches,) we thought it better to dwell

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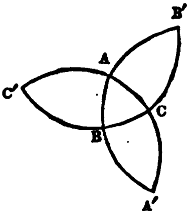
[*] It may be proper to mention here, that since the above demonstration was written I have remarked the same property in Dr. Luby's Trigonometry, p. 61-2; but I must have first met with it elsewhere, as I well recollect that it was unaccompanied with any proof. Dr. Luby's demonstration is a good deal similar to mine.

upon this sufficiently to give the student a real insight into the character of its processes and to furnish him with a few suitable exercises for his own improvement in such investigations.*

CHAPTER II.

ASSOCIATED TRIANGLES.



1. Let ABC be a spherical triangle, having its sides produced to meet again in A′ B′ C′, respectively opposite to the angles A, B, C. Four triangles are thus formed which have a necessary relation to one another. These we propose to call *the associated system of triangles*, or simply *the associated triangles.*

That which was first drawn (ABC), and which serves as a basis of the rest, we call the *fundamental triangle of the associated system*, or simply *the fundamental triangle.*

The others, two sides of each being supplements of two sides of the fundamental, and two angles of two angles, we call the *supplemental triangles of the associated system*, or simply the *supplemental triangles.*†

Moreover, when we wish to specify any one of the supplemental triangles, we shall do it by reference to the side in it which is common to a side, or an angle which is equal to an angle of the fundamental triangle. Thus, to designate the triangles BA′C, CB′A, AC′B, we say the supplemental triangle taken with respect to A′, (or $a_{\prime}$ as the case may be,) with respect to B′, or with respect to C′.

As a uniformity of notation is essential in inquiries like these, related to classes of similar objects, we shall attempt to conform to the established notation as a basis. Thus, *abc* are the sides of ABC,

$a \quad b_{\prime} \quad c_{\prime}$ - - - BA′C
$a_{\prime\prime} \quad b \quad c_{\prime\prime}$ - - - - CB′A
$a_{\prime\prime\prime} \quad b_{\prime\prime\prime} \quad c$ - - - AC′B.

In which the number of subscribed accents points out the particular triangle designated, considering them to be ranged round the fundamental one in the order of the letters A′, B′, C′.

Again, for the angles, we have the angles

$A \quad B \quad C$ of the fundamental triangle ABC
$A \quad B_{\prime} \quad C_{\prime}$ those of BA′C
$A_{\prime\prime} \quad B \quad C_{\prime\prime}$ those of CB′A
$A_{\prime\prime\prime} \quad B_{\prime\prime\prime} \quad C$ those of AC′B. The values of these are

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* A number of important properties of spherical triangles, demonstrated geometrically, by Professor Lowry, may be seen in the first vol. of the old series of the Mathematical Repository, and some others in Howard's Spherical Geometry, 1798. The subject, however, is still open to indefinite research, and offers ample reward to those whose taste may lead them to cultivate it. See also note A.

† The term "*supplemental*" has been used by English Mathematicians to designate that triangle which is now universally denominated the "*polar triangle.*" The word has ceased to be used in that sense for some years, and as it is so peculiarly adapted to express the triangles which are formed by producing the sides of the fundamental, we have not hesitated to adopt it. We give this notice, however, of the change in its appropriation, lest some confusion should arise in the mind of the young mathematician when he sees in Trigonometrical works, of the last age, a use different from our own of the word supplemental.

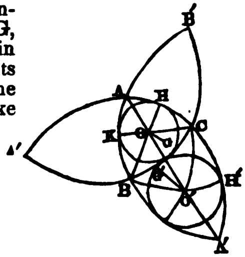
It may be remarked that the choice of the word for that purpose was not happy; for though it was so far a defining property as to give the *species* of the triangle, it did not give its *position*; an element quite as important, in many investigations, as the species itself, and, indeed, that upon which several of its valuable properties depend.

$$\begin{array}{ll|ll} b_{,} = \pi - b; & c_{,} = \pi - c & B_{,} = \pi - B; & C_{,} = \pi - C \\ a_{,,} = \pi - a; & c_{,,} = \pi - c & A_{,,} = \pi - A; & C_{,,} = \pi - C \\ a_{,,,} = \pi - a; & b_{,,,} = \pi - b & A_{,,,} = \pi - A; & B_{,,,} = \pi - B \end{array}$$

Again, for the radii of the circles inscribed in the four triangles, taken in the aforesaid order, we write r, $r_{,}$, $r_{,,}$, $r_{,,,}$: whilst, for the radii of the circumscribed circles, we put R, $R_{,}$, $R_{,,}$, $R_{,,,}$, respectively. The unaccented letters referring to the circles of the fundamental triangles.

These triangles possess many beautiful properties when considered in their mutual association, which render them worthy of greater attention than has yet been bestowed upon them. Indeed, till very recently their existence has scarcely been alluded to by writers on spherical subjects, and even to the present day, not more than three of their properties have, we believe, been published.

2. Let O be the centre of the circle inscribed in the fundamental triangle, and G, H, K, its points of contact with the sides, join AO, BO, CO, and draw the radii to the points of contact. Then the tangents from A to the circle are equal; that is, AK = AH; in like manner BK = BG, and CG = CH.



$$\text{Put} \left\{ \begin{array}{l} AK = AH = \alpha \\ BK = BG = \beta \\ CG = CH = \gamma \end{array} \right\}$$

$$\text{From which} \left\{ \begin{array}{l} a = \beta + \gamma \\ b = \alpha + \gamma \\ c = \alpha + \beta \end{array} \right.$$

Whence

$$\left. \begin{array}{rl} \alpha + \beta + \gamma = & \dfrac{a+b+c}{2} = s \\ \alpha = & \dfrac{-a+b+c}{2} = \overline{s-a} \\ \beta = & \dfrac{a-b+c}{2} = \overline{s-b} \\ \gamma = & \dfrac{a+b-c}{2} = \overline{s-c} \end{array} \right\} \quad . \; . \; . \; (1).$$

Again, in the right-angled triangle BOG, we have tan. OG = sin. BG tan. OBG, that is, tan. r = sin. β tan. $\frac{1}{2}$ B; or by (1) just given and (3), upon page 49, applied to B, we have

$$\tan. r = \sin. \overline{s-b} \left\{ \frac{\sin. \overline{s-a} \sin. \overline{s-c}}{\sin. s \sin. \overline{s-b}} \right\}^{\frac{1}{2}}$$

$$= \frac{\sqrt{\sin. s \sin. \overline{s-a} \sin. \overline{s-b} \sin. \overline{s-c}}}{\sin. s} \quad . \; . \; . \; . \; (2).$$

Again, in the supplemental triangle BA′C, denoting the quantities BG′, CH′, and AK′, by $\beta_{,}$, $\gamma_{,}$, $\alpha_{,}$, we shall have

$$a_{,} = \beta_{,} + \gamma_{,}$$
$$b_{,} = \alpha_{,} + \gamma_{,}$$
$$c_{,} = \alpha_{,} + \beta_{,}$$

and hence, as before, $s_{,} = \alpha_{,} + \beta_{,} + \gamma_{,} =$

$$\frac{a_{,} + b_{,} + c_{,}}{2} = \frac{a + \pi - b + \pi - c}{2} = \pi - \frac{-a+b+c}{2} = \pi - \overline{s-a},$$

$$s_{,} - a_{,} = \alpha_{,}$$

$$=\frac{-a_{,}+b_{,}+c_{,}}{2}=\pi-\frac{-a+\overline{\pi-b}+\overline{\pi-c}}{2}=\frac{a+b+c}{2}=s,$$

$$\overline{s_{,}-b_{,}}=\beta_{,}$$

$$=\frac{a_{,}-b_{,}+c_{,}}{2}=\frac{a-\overline{\pi-b}+\overline{\pi-c}}{2}=\frac{a+b-c}{2}=\overline{s-c},$$

$$\overline{s_{,}-c_{,}}=\gamma_{,}$$

$$=\frac{a_{,}+b_{,}-c_{,}}{2}=\frac{a+\overline{\pi-b}-\overline{\pi-c}}{2}=\frac{a-b+c}{2}=\overline{s-b}.$$

Also $B_{,}=\pi-B$, and $\tan.\tfrac{1}{2}B_{,}=\cot.\tfrac{1}{2}B$. Hence, in the right angled triangle BO′G′, we have tan. O′G′ = sin. BG tan. O′BG′, that is, $\tan. r_{,}=\sin.\beta_{,}\tan.\tfrac{1}{2}B_{,}=\sin.\overline{s-c}\left\{\frac{\sin. s\sin.\overline{s-b}}{\sin.\overline{s-a}\sin.\overline{s-c}}\right\}^{\frac{1}{2}}$

$$=\sqrt{\frac{\sin. s\sin.\overline{s-a}\sin.\overline{s-b}\sin.\overline{s-c}}{\sin.(s-a)}}.$$

In exactly the same way we find the other associated inscribed radii, and the whole tabulated gives

$$\left.\begin{array}{l}
\tan. r=\sqrt{\dfrac{\{\sin. s\sin.(s-a)\sin.(s-b)\sin.(s-c)\}}{\sin. s}}\\
\tan. r_{,}=\sqrt{\dfrac{\{\sin. s\sin.(s-a)\sin.(s-b)\sin.(s-c)\}}{\sin.(s-a)}}\\
\tan. r_{,,}=\sqrt{\dfrac{\{\sin. s\sin.(s-a)\sin.(s-b)\sin.(s-c)\}}{\sin.(s-b)}}\\
\tan. r_{,,,}=\sqrt{\dfrac{\{\sin. s\sin.(s-a)\sin.(s-b)\sin.(s-c)\}}{\sin.(s-c)}}
\end{array}\right\}\ldots(3).$$

These formulæ were first given by *Professor Lowry* (1829), Leybourn's Repository, vol. v. p. 3. Multiply these together, then we obtain

$$\tan. r\tan. r_{,}\tan. r_{,,}\tan. r_{,,,}=\sin. s\sin.(s-a)\sin.(s-b)\sin.(s-c).\ (4).$$

Divide (4) by the squares of each of the equations in art. (3), the first side by the first side, and the second by the second: then

$$\left.\begin{array}{r}
\sin.^2 s=\cot. r\tan. r_{,}\tan. r_{,,}\tan. r_{,,,}\\
\sin.^2(s-a)=\tan. r\cot. r_{,}\tan. r_{,,}\tan. r_{,,,}\\
\sin.^2(s-b)=\tan. r\tan. r_{,}\cot. r_{,,}\tan. r_{,,,}\\
\sin.^2(s-c)=\tan. r\tan. r_{,}\tan. r_{,,}\cot. r_{,,,}
\end{array}\right\}\ldots(5);$$

which remarkable formulæ are due to Mr. Lowry (1819), vide Repository, *ub. sup.*

Again, by multiplication of the terms in (3), we have

$$\tan. r\tan. r_{,}+\tan. r_{,,}\tan. r_{,,,}=\sin.(s-b)\sin.(s-c)+\sin. s\sin.(s-a)$$

$$=\sin.\frac{a+(b-c)}{2}\sin.\frac{a-(b-c)}{2}+\sin.\frac{b+c+a}{2}\sin.\frac{(b+c)-a}{2}$$

$$=\sin.^2\frac{a}{2}-\sin.^2\frac{b-c}{2}+\sin.^2\frac{b+c}{2}-\sin.^2\frac{a}{2}=\sin. b\sin. c.$$

Taking also each of the other corresponding combinations, we obtain in all the three following equations,

$$\left.\begin{array}{l}\tan. r \tan. r_{,} + \tan. r_{,,} \tan. r_{,,,} = \sin. b \sin. c\\ \tan. r \tan. r_{,,} + \tan. r_{,} \tan. r_{,,,} = \sin. a \sin. c\\ \tan. r \tan. r_{,,,} + \tan. r_{,} \tan. r_{,,} = \sin. a \sin. b\end{array}\right\} . \quad (6).$$

Or, by addition, we have at once the following theorem.

$$\tan. r \tan. r_{,} + \tan. r \tan. r_{,,} + \tan. r \tan. r_{,,,} + \tan. r_{,} \tan. r_{,,} + \tan. r_{,} \tan. r_{,,,} + \tan. r_{,,} \tan. r_{,,,}$$
$$= \sin. a \sin. b + \sin. a \sin. c + \sin. b \sin. c \quad . \; . \; . \quad (7).$$

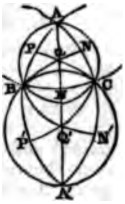
That is, in words, *the sum of the binary products of the tangents of the four inscribed radii are equal to the sum of the binary products of the sines of the sides.*

We may notice one beautiful theorem more, which is due to Mr. Lowry, *ubi supra*. It is $\tan. r_{,} \tan. r_{,,} + \tan. r_{,} \tan. r_{,,,} + \tan. r_{,,} \tan. r_{,,,} =$

$$\sin. s \,\{\sin. (s - a) + \sin. (s - b) + \sin. (s - c)\} \quad . \quad (8).$$

For the several consequences of these theorems, and a continuation of the inquiry, we must refer to the number xxiv. of Leybourn's Repository, now in the press, where expressions for the various trigonometrical functions of the sides and angles of the triangle, will be given in terms of the inscribed radii.

3. We now proceed to consider the *circumscribed* radii of the associated triangles. We shall immediately find these in terms of the angles, as we did those of the inscribed in terms of the sides.



Let Q be the centre of the circumscribing circle of the fundamental triangle, and draw the perpendiculars QM, QN, QP. Then M, N, P, bisect the sides a, b, c, respectively, and the several triangles BQC, CQA, AQB, are isosceles. Let the angles made by the radii QB, QC, with the side a be denoted by α; those with b, by β, and those with c by γ.* Then $A = \beta + \gamma$, $B = \alpha + \gamma$, $C = \alpha + \beta$.

From which we have

$$\left.\begin{array}{ll}\alpha + \beta + \gamma & = \dfrac{A + B + C}{2} = S,\\ \alpha & = \dfrac{-A + B + C}{2} = (S - A)\\ \beta & = \dfrac{A - B + C}{2} = (S - B)\\ \gamma & = \dfrac{A + B - C}{2} = (S - C)\end{array}\right\} \quad . \; . \; . \quad (9).$$

But, by right-angled triangles BMQ, we have $\cot. QB = \cos. QBM \cot. BM$, or $\cot. R = \cos. \alpha \cot. \frac{1}{2} a$; or, by (9) and (p. 50-1), we have at once

$$\cot. R = \sqrt{\frac{- \cos. S \cos. \overline{S - A} \cos. \overline{S - B} \cos. \overline{S - C}}{- \cos. S}}.$$

Proceeding with respect to the triangle BA′C, in a manner analogous to that employed in obtaining the three last equations of (3), using the values of α, β, γ, just given in (9), we shall have the following tablet of values,

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* These quantities being merely introduced as subsidiary ones, to be replaced in all general formulæ by their values in terms of A, B, C, we have not transgressed our general rule in employing them to designate two different sets of quantities in this place as in art (2) which belong finally to the inquiry.

$$\left.\begin{aligned}
\text{cot. R} &= \sqrt{\frac{-\text{cos. S cos. }\overline{\text{S}-\text{A}}\text{ cos. }\overline{\text{S}-\text{B}}\text{ cos. }\overline{\text{S}-\text{C}}}{-\text{cos. S}}}\\
\text{cot. R}_{,} &= \sqrt{\frac{-\text{cos. S cos. }\overline{\text{S}-\text{A}}\text{ cos. }\overline{\text{S}-\text{B}}\text{ cos. }\overline{\text{S}-\text{C}}}{\text{cot. }\overline{\text{S}-\text{A}}}}\\
\text{cot. R}_{,,} &= \sqrt{\frac{-\text{cos. S cos. }\overline{\text{S}-\text{A}}\text{ cos. }\overline{\text{S}-\text{B}}\text{ cos. }\overline{\text{S}-\text{C}}}{\text{cos. }\overline{\text{S}-\text{B}}}}\\
\text{cot. R}_{,,,} &= \sqrt{\frac{-\text{cos. S cos. }\overline{\text{S}-\text{A}}\text{ cos. }\overline{\text{S}-\text{B}}\text{ cos. }\overline{\text{S}-\text{C}}}{\text{cos. }\overline{\text{S}-\text{C}}}}
\end{aligned}\right\} \quad ; \ (10);$$

which, with the following beautiful theorem, analogous to Lowry's, at p. 128, (obtained by multiplying these together) is due to Dr. Lardner, (1826). Trig. p. 153. $\text{cot. R cot. R}_{,}\text{ cot. R}_{,,}\text{ cot. R}_{,,,} =$

$$-\text{cos. S cos. }\overline{\text{S}-\text{A}}\text{ cos. }\overline{\text{S}-\text{B}}\text{ cos. }\overline{\text{S}-\text{C}} \ldots\ldots (11).$$

Divide (11) by the squares of each of the equations in (10), and we have

$$\left.\begin{aligned}
\text{cos.}^2\,\text{S} &= \text{tan. R cot. R}_{,}\text{ cot. R}_{,,}\text{ cot. R}_{,,,}\\
\text{cos.}^2\,\overline{\text{S}-\text{A}} &= \text{cot. R tan. R}_{,}\text{ cot. R}_{,,}\text{ cot. R}_{,,,}\\
\text{cos.}^2\,\overline{\text{S}-\text{B}} &= \text{cot. R cot. R}_{,}\text{ tan. R}_{,,}\text{ cot. R}_{,,,}\\
\text{cos.}^2\,\overline{\text{S}-\text{C}} &= \text{cot. R cot. R}_{,}\text{ cot. R}_{,,}\text{ tan. R}_{,,,}
\end{aligned}\right\} \quad \ldots (12).$$

These elegant theorems, which are here published for the first time, were discovered by my learned friend, the *Rev. H. F. C. Logan*, Professor of Mathematics in the Catholic College of Prior Park. The first # of them is a remarkable expression for the *spherical excess in terms of the four circumscribed radii.* The spherical excess in terms of the *inscribed radii* may be seen in the Repository before alluded to; and some theorems connected with the same function of the triangle will be given in a future page of this supplement. By combining (10) in the same way as (5) was combined to obtain (6), we shall have

$$\left.\begin{aligned}
\text{cot. R cot. R}_{,} + \text{cot. R}_{,,}\text{ cot. R}_{,,,} &= \text{sin. B sin. C}\\
\text{cot. R cot. R}_{,,} + \text{cot. R}_{,}\text{ cot. R}_{,,,} &= \text{sin. A sin. C}\\
\text{cot. R cot. R}_{,,,} + \text{cot. R}_{,}\text{ cot. R}_{,,} &= \text{sin. A sin. B}
\end{aligned}\right\} \ldots . (13).$$

Hence, by addition, we get $\text{cot. R cot. R}_{,} + \text{cot. R cot. R}_{,,} + \text{cot. R cot. R}_{,,,} + \text{cot. R}_{,}\text{ cot. R}_{,,} + \text{cot. R}_{,}\text{ cot. R}_{,,,} + \text{cot. R}_{,,}\text{ cot R}_{,,,}$
$= \text{sin. A sin. B} + \text{sin. B sin. C} + \text{sin. C sin. A} \quad \ldots . (14).$

In the Repository (xxiv) will also be found expressions for the trigonometrical functions of the elements of the triangle, in terms of R, $\text{R}_{,}$, $\text{R}_{,,}$, $\text{R}_{,,,}$; and we may here remark that by means of a theorem to be given at page 133-4 of this treatise, the expressions (10, 14, incl. and all of the same class) may be derived, by inspection, from those given in terms of sides and inscribed radii. It is by means of a property of the polar triangle. We shall, however, before proceeding to the theory of polar triangles, point the student's attention to two interesting propositions, the analytical expressions for which we have passed by without particular notice. We allude to the values of tan. r, and of cot. R, at pages 127, 129; and which are

$$\left.\begin{aligned}
\text{tan. } r &= \text{sin. } \beta \text{ tan. } \tfrac{1}{2}\text{ B} = \text{sin. } \tfrac{1}{2}\,(a + c - b)\text{ tan. } \tfrac{1}{2}\text{ B}\\
\text{cot. R} &= \text{cos. } \alpha \text{ cot. } \tfrac{1}{2}\, a = \text{cos. } \tfrac{1}{2}\,(\text{B} + \text{C} - \text{A})\text{ cot. } \tfrac{1}{2}\, a
\end{aligned}\right\}$$

1. From the first of these we infer that if B and $a + c - b$ are constant, r will he constant; that is to say, *in any spherical triangle if the vertical angle* (B) *be constant, as also the difference between the base and sum of the other two sides, the radius and centre of the inscribed circle will continue fixed.*

$\sin^2 \frac{E}{2} = \cos^2 S.$ — 14

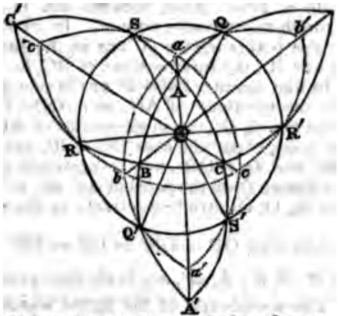
(a) be constant, as also the difference between the vertical angle, and the sum of the other two, the radius and centre of the circumscribed circle will be fixed; that is, the locus of the vertex will be a circle.

3. This last property suggests a remarkable simple method of demonstrating the beautiful theorem of *Lexell* which is this, viz. that *if the base and area of the spherical triangle be constant, the locus of the vertex will be a circle.* For, referring to the figure at page 129, let BC be the constant base, and ABC any one of the triangles. Produce the sides to meet in A′, and call the angles at B and C *below* the base B′ and C′. For the area of the triangle ABC we have the expressions $A + B + C - \pi =$ constant. But $A = A'$, $B = \pi - B'$, $C = \pi - C'$; hence, by substitution, $A' - (B' + C') + \pi =$ const. $\therefore B' + C' - A' =$ const. and, therefore, as the base BC or a' is also constant, it follows from the theorem just demonstrated that the locus of the vertex A′ is the circle A′BC, and, consequently, the locus of A, which is the antipodes of A′, must be an equal circle. We ought to remark here that this demonstration is the same in substance as that given by M. Lowry in Leybourn's Repository, vol. I.*

Polar Triangles.

We have already seen, (p. 45,) that if from the three angular points of a spherical triangle ABC we describe three great circles, they will form an associated system of triangles, *one* of which also has a remarkable relation to the triangle ABC: but it has not, so far as we know, been noticed, that if we complete the associated system, whose fundamental is ABC, then the two sets of associated triangles thus produced, will be separable into four pairs, (one of each system forming a pair) which will be related to one another precisely in the same way as the above named pair are related. Thus, if in the annexed figure, ABC be the fundamental triangle, *abc* its polar; then four pairs of polars will simultaneously be produced, viz.

(1)	ABC *abc*	(2)	AB′C *ab′c*	(3)	A′BC *a′bc*	(4)	ABC′. *abc′*.



Now the first pair ABC, *abc* are by hypothesis polars.† Hence B is the pole of *ca*, that is of *ac′*; A is the pole of *bc*, that is of *bc′*; and since C is the pole of *ab*, therefore C′ is the other pole of *ab*. The points A, B, C′ are the poles of the sides, therefore, of the triangle *abc′*. Whence, also, it follows by the reciprocity of the polar system, that *a*, *b*, *c′* are the poles of ABC′.

In the same way it is shown that ACB′ and *acb′* form a polar system, and that BA′C, *ba′c*, form another. Hence it follows that if one triangle of an associated system be polar to one of another associated system, then each of the triangles of one system is polar to one triangle of the other associated system.

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[*] In the Edinburgh Transactions the author of this supplement has investigated Lexell's theorem by a novel analysis,—*the geometry of spherical coordinates*, which determines in a direct manner the equation of the locus, and then shows its identity with the previously found equation of a circle.

[†] When we speak of parallel lines without specifying which is taken as the line of

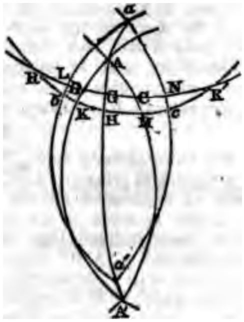
This is not the only curious property of the figure before us: and we shall put down a small selection from those we are in possession of, not doubting that on many accounts they will be interesting to those geometers who indulge in trigonometrical speculations. An addition to these will appear in the 25th number of the Mathematical Repository.

Draw the great circle aA (next figure), and produce it to meet BC, bc in G and H. Then, because a is the pole of BC, and that A is the pole of bc, the arcs aG, AH, are quadrants. Hence, $aG + AH = aH + AG = \pi$, and the angles at G and H are right angles.

Let R, R′, be the intersections of BC, bc; then, because the angles at G and H are right, R, R′, are the poles of aA.

Let AB meet bc in K, BC meet ab in L, AC meet bc in M, and ac meet BC in N.

$$\left\{\begin{array}{l} \text{Then } bH \backsim Hc = GAC \backsim GAB \\ bH + GAB = \dfrac{\pi}{2} = Hc + GAC. \end{array}\right\}$$



For $bM = bH + HM = bH + HAM = bH + GAC$. In like manner, $cK = cH + HK = cH + HAK = cH + GAB$

Also $bM = \dfrac{\pi}{2} = cK$, since b and c are the poles AM and CK.

Hence the propositions as stated are true.

These are due to *Professor Lowry*, and were given by him, in 1800, in Leybourn's Repos. old series, No. 1, p. 44, 5. They have not, that we are aware, been noticed in any other place.

Returning now to our original figure, let the three arcs, Aa, Bb, Cc, be drawn; these will pass through the same point O, because as has been just shown, they are perpendicular to the three sides BC, AC, AB, (vide p. 125). Also, because aA is perpendicular to bc, it will pass through the opposite pole a'. In like manner it will pass through A′. Or the points $aAOa'A'$ are in the same great circle, whose poles are R, and R′, the intersections of BC, bc.

In like manner $bBOb'B'$ are in one great circle, whose poles are S, S′, the intersections of AC, ac; $cCOc'C'$ are in one great circle, whose poles are Q, Q′, the intersections of AB, ab. Again, because R, R′, are the poles of aa' the arcs RO, OR′, are quadrants. In like manner SO, OS′, and QO, OQ′, are respectively quadrants; and as the quadrants are drawn from the poles of aa', bb', cc', respectively, R, O, R′; S, O, S′, and Q, O, Q′, are respectively in the same great circles.

Also since $OR = OR' = OS = OS' = OQ = OQ' = \dfrac{\pi}{2}$, *the points* R, R′, S, S′, A, Q′, *are in the same great circle, and* O *is its pole.*

The complexity of the figure which would represent from a further detail of these interesting researches, compels us to leave them for the

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reference, and knowing that the second is related to the first in the same way that the first is to the second, we simply denominate them parallels. The same practice also holds in speaking of two mutually supplemental angles. But when we previously fix upon one line or one angle, we say the parallel, or the supplemental in the singular number to express the other line or angle. Just so in respect to the two triangles which constitute the polar system, when we speak of both without assigning the referee, we call them polars, as a common epithet; but when we have fixed upon one already, as that to which the other is referred, we call it the *primary* and the other the *polar triangle.*

present to the industry and ingenuity of the student. They are exceedingly easy and furnish an excellent exercise in spherical investigations: and we therefore hope he will give it a proper degree of consideration. We proceed to view the subject algebraically.

Let *abc*, ABC, be the sides and angles respectively of a spherical triangle; R*r*, the radii of the circumscribing and the inscribed circles; *s*, S, the semi sums of the sides and angles respectively; also let $a'\,b'\,c'$; $A'\,B'\,C'$; R', r'; S', s', be the same things in the triangle which is polar to *abc*. Put, as at page 116,

$$n^2 = \sin.\, s \sin.\, \overline{s-a} \sin.\, \overline{s-b} \sin.\, \overline{s-c}$$

$$n'^2 = \sin.\, s \sin.\, \overline{s'-a'} \sin.\, \overline{s'-b'} \sin.\, \overline{s'-c'}$$

$$N^2 = -\cos.\, S \cos.\, \overline{S-A} \cos.\, \overline{S-B} \cos.\, \overline{S-C}$$

$$N'^2 = -\cos.\, S' \cos.\, \overline{S'-A'} \cos.\, \overline{S'-B'} \cos.\, \overline{S'-C'}.$$

Then we propose to prove that $n^2 = N'^2$, and $n'^2 = N^2$.

For
$$\begin{aligned} s &= \tfrac{1}{2}(a+b+c) \\ s-a &= \tfrac{1}{2}(-a+b+c) \\ s-b &= \tfrac{1}{2}(a-b+c) \\ s-c &= \tfrac{1}{2}(a+b-c)\text{; and, in the polar tri-} \end{aligned}$$

angle, we have immediately $S' = \frac{3\pi}{2} - \frac{1}{2}(a+b+c) = \frac{3\pi}{2} - s$,

$$S'-A' = \frac{\pi}{2} - \tfrac{1}{2}(-a+b+c) = \frac{\pi}{2} - \overline{s-a},$$

$$S'-B' = \frac{\pi}{2} - \tfrac{1}{2}(a-b+c) = \frac{\pi}{2} - \overline{s-b},$$

$$S'-C' = \frac{\pi}{2} - \tfrac{1}{2}(a+b-c) = \frac{\pi}{2} - \overline{s-c}.$$

$\#$ Hence, $-\cos.\, S' = -\cos.\, (\frac{3\pi}{2} - s) = \sin.\, s$,

$$\cos.\, \overline{S'-A'} = \cos.\, (\frac{\pi}{2} - \overline{s-a}) = \sin.\, \overline{s-a},$$

$$\cos.\, \overline{S'-B'} = \cos.\, (\frac{\pi}{2} - \overline{s-b} = \sin.\, \overline{s-b},$$

$$\cos.\, \overline{S'-C'} = \cos.\, (\frac{\pi}{2} - \overline{s-c}) = \sin.\, \overline{s-c}.$$

Whence, by multiplication of the extreme vertical columns we have the quantities designated by N'^2, and n^2; or, $N'^2 = n^2$. In exactly the same way, we find that $N^2 = n'^2$. That is, $N' = n$, and $N = n'$.*

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* These properties might have been more simply obtained, thus: by (18) and (21), page 117.

$$\tan.\, \tfrac{1}{2} A' \tan.\, \tfrac{1}{2} B' \tan.\, \tfrac{1}{2} C' = \frac{n'}{\sin.^2 S'}\text{; } \cot.\, \tfrac{1}{2} a \cot.\, \tfrac{1}{2} b \cot.\, \tfrac{1}{2} c = \frac{N}{\cos.^2 S}.$$

But $\frac{1}{2}(A'+a) = \frac{\pi}{2}$ $\therefore \cot.\, \frac{1}{2} a = \tan. \frac{1}{2} A'$, and hence the left sides are equal. Also, $\cos.\, ^2S = \sin.\, ^2s$, therefore $N = n'$.

Had the mere proof of the property been our object, this would have been the preferable method: but the stages through which in the text we have passed are necessary in other inquiries, and we prefer therefore to give them in that form which, it may be remarked too, is the original one by which the property was obtained. See note B.

$-\cos(3\pi - 2S) = -\cos(\pi - 2S) = -\cos(\frac{\pi}{2} - S$

$\cos(90 - S = \sin S$

Again, by Lowry's theorem, page 128,

$$\tan. r = \frac{n}{\sin. s},\ \tan. r' = \frac{n'}{\sin. s'};$$ and, therefore, by (17, p. 117),

$$\tan. r' = \frac{N'}{2 \cos. \frac{1}{2} A' \cos. \frac{1}{2} B' \cos. \frac{1}{2} C'} = \frac{N'}{2 \sin. \frac{1}{2} a \sin. \frac{1}{2} b \sin. \frac{1}{2} c}.$$

Also (10) p. 130, and (19) p. 117,

$$\tan. R = -\frac{\cos. S}{N} = \frac{2 \sin. \frac{1}{2} a \sin. \frac{1}{2} b \sin. \frac{1}{2} c}{n};\ \tan. R' = -\frac{\cos. S'}{N} = \frac{\sin. s}{N'}.$$

Whence, recollecting that $N = n$, $\frac{\tan. r}{\tan. r'} = \frac{2 \sin. \frac{1}{2} a \sin. \frac{1}{2} b \sin. \frac{1}{2} c}{\sin. \frac{1}{2}(a+b+c)}$,

and $\frac{\tan. R}{\tan. R'} = \frac{2 \sin. \frac{1}{2} a \sin. \frac{1}{2} b \sin. \frac{1}{2} c}{\sin. \frac{1}{2}(a+b+c)} \therefore \frac{\tan. r}{\tan. r'} = \frac{\tan. R}{\tan. R'}$.

But we have a more useful result, as follows:

$$\tan. R \tan. r' = \frac{2 \sin. \frac{1}{2} a \sin. \frac{1}{2} b \sin. \frac{1}{2} c}{n} \cdot \frac{N'}{2 \sin. \frac{1}{2} a \sin. \frac{1}{2} b \sin. \frac{1}{2} c} \cdot = 1,$$

or $\tan. R = \cot. r'$, which is fulfilled by the relations

$R + r' = \frac{\pi}{2}$, and $R + r' = \frac{3\pi}{2}$. The former of these relations holds when the circles are referred to their nearest poles, and the latter when to their farther poles. In like manner, we also have, in the corresponding cases, $R' + r = \frac{\pi}{2}$ and $R' + r = \frac{3\pi}{2}$

From these we get

$$\begin{aligned} R + r' &= R' + r \\ R - R' &= r - r' \\ R - r &= R' - r'. \end{aligned}$$

Thus the remarkable relation between the polar triangles is continued even amongst the radii of the inscribed and circumscribing circles, viz. *that the inscribed radius of one is the complement of the circumscribing radius of the other; or, taking diameters, the supplement of the other.*

We have seen that in the two associated systems, having one pair of mutually polar fundamental triangles, the two systems, are triangle for triangle mutually polar; and hence collecting the whole result into one table, we have

$$\begin{array}{ll|ll}
R' + r & = \frac{\pi}{2} & R' + r & = \frac{3\pi}{2} \\
R + r' & = \frac{\pi}{2} & R + r' & = \frac{3\pi}{2} \\
R'_{,} + r_{,} & = \frac{\pi}{2} & R'_{,} + r_{,} & = \frac{3\pi}{2} \\
R_{,} + r'_{,} & = \frac{\pi}{2} & R_{,} + r'_{,} & = \frac{3\pi}{2} \\
R'_{,,} + r_{,,} & = \frac{\pi}{2} & R'_{,,} + r_{,,} & = \frac{3\pi}{2} \\
R_{,,} + r'_{,,} & = \frac{\pi}{2} & R_{,,} + r'_{,,} & = \frac{3\pi}{2} \\
R'_{,,,} + r_{,,,} & = \frac{\pi}{2} & R'_{,,,} + r_{,,,} & = \frac{3\pi}{2} \\
R_{,,,} + r'_{,,,} & = \frac{\pi}{2} & R_{,,,} + r'_{,,,} & = \frac{3\pi}{2}
\end{array}$$

Adding all these together, we have

$$r + r_{,} + r_{,,} + r_{,,,} + r' + r'_{,} + r'_{,,} + r'_{,,,} + \mathrm{R} + \mathrm{R}_{,} + \mathrm{R}_{,,} + \mathrm{R}_{,,,} + \mathrm{R}' + \mathrm{R}'_{,} + \mathrm{R}'_{,,} + \mathrm{R}'_{,,,} = 4\pi \text{ (or } 12\,\pi).$$

That is, the sum of all the sixteen radii of the primary and polar associated system is equal to the surface of the hemisphere if the first system of values is taken, or to three times that sum if the second system be taken.

The consideration of the latter system of values I owe to the Rev. Professor Logan; the former with its results is my own independent discovery, and was the origin of my researches on the subject of polar triangles, and of the associated triangles too.

Another property also comes immediately from this: viz. *that the product of the tangents of the sixteen radii is equal to unity.* Which is seen at once by $\tan.\, r . \tan.\, \mathrm{R}' = 1$, &c.: or it may again be put

$$\tan.\, r \tan.\, r_{,} \tan.\, r_{,,} \tan.\, r_{,,,} \tan.\, r' \tan.\, r'_{,} \tan.\, r'_{,,} \tan.\, r_{,,,} =$$
$$\cot.\, \mathrm{R} \cot.\, \mathrm{R}_{,} \cot.\, \mathrm{R}_{,,} \cot.\, \mathrm{R}_{,,,} . \cot.\, \mathrm{R}' \cot.\, \mathrm{R}'_{,} \cot.\, \mathrm{R}'_{,,} \cot.\, \mathrm{R}'_{,,,};$$

or still differently,

$$\frac{\tan.\, r \tan.\, r_{,} \tan.\, r_{,,} \tan.\, r_{,,,}}{\cot.\, \mathrm{R} \cot.\, \mathrm{R}_{,} \cot.\, \mathrm{R}_{,,} \cot.\, \mathrm{R}_{,,,}} = \frac{\cot.\, r' \cot.\, r'_{,} \cot.\, r'_{,,} \cot.\, r'_{,,,}}{\tan.\, \mathrm{R}' \tan.\, \mathrm{R}'_{,} \tan.\, \mathrm{R}'_{,,} \tan.\, \mathrm{R}'_{,,,}}.$$

Another neat relation may be put down here; we have already seen that $\frac{\tan.\, \mathrm{R}}{\tan. \mathrm{R}'} = \frac{2 \sin.\, \frac{1}{2} a \sin.\, \frac{1}{2} b \sin.\, \frac{1}{2} c}{\sin.\, \frac{1}{2} (a + b + c)}$; and by interchanging the primary and secondary polar triangles, still retaining the accent upon the same letters to distinguish them as the sides of the same triangle as before, we have $\frac{\tan.\, \mathrm{R}'}{\tan.\, \mathrm{R}} = \frac{2 \sin.\, \frac{1}{2} a' \sin.\, \frac{1}{2} b' \sin.\, \frac{1}{2} c'}{\sin.\, \frac{1}{2} (a' + b' + c')}$.

Dividing these, we have

$$\frac{\tan.\, ^2\mathrm{R}}{\tan.\, ^2\mathrm{R}'} = \frac{\sin.\, \frac{1}{2} a \sin.\, \frac{1}{2} b \sin.\, \frac{1}{2} c}{\sin.\, \frac{1}{2} a' \sin.\, \frac{1}{2} b' \sin.\, \frac{1}{2} c'} . \frac{\sin.\, \frac{1}{2} (a' + b' + c')}{\sin.\, \frac{1}{2} (a + b + c)}$$

But $\frac{1}{2} a = \frac{\pi}{2} - \frac{1}{2} \mathrm{A}$, &c. because the triangles are polar;

$$\therefore \frac{\tan.\, ^2\mathrm{R}}{\tan.\, ^2\mathrm{R}'} = \frac{\sin.\, \frac{1}{2} a \sin.\, \frac{1}{2} b \sin.\, \frac{1}{2} c}{\cos.\, \frac{1}{2} \mathrm{A} \cos.\, \frac{1}{2} \mathrm{B} \cos.\, \frac{1}{2} \mathrm{C}} . \frac{-\cos.\, \mathrm{S}}{\sin.\, \mathrm{S}} .$$

$= \frac{\sin.\, \frac{1}{2} a \sin.\, \frac{1}{2} b \sin.\, \frac{1}{2} c}{\sin.\, \mathrm{S}} . \frac{-\cos.\, \mathrm{S}}{\cos.\, \frac{1}{2} \mathrm{A} \cos.\, \frac{1}{2} \mathrm{B} \cos.\, \frac{1}{2} \mathrm{C}}$. Multiplying the same equations we get $4 \cos.\, \frac{1}{2}\mathrm{A} \cos.\, \frac{1}{2} \mathrm{B} \cos.\, \frac{1}{2}\mathrm{C} \sin.\, \frac{1}{2}a \sin.\, \frac{1}{2}b \sin.\, \frac{1}{2} c = -\cos.\, \mathrm{S} \sin.\, s$; or in every spherical triangle we have

$$-\frac{2 \cos.\, \frac{1}{2} \mathrm{A} \cos.\, \frac{1}{2} \mathrm{B} \cos.\, \frac{1}{2} \mathrm{C}}{\cos.\, \mathrm{S}} = \frac{\sin.\, s}{2 \sin.\, \frac{1}{2} a \sin.\, \frac{1}{2} b \sin.\, \frac{1}{2} c},$$

which, by the bye, is also an immediate consequence of the relations (17), (19), at page 117. Innumerable other interesting results may be obtained with equal facility, by means of the property of polar radii given above; but the limits of a work, like the present, prevent our enlarging upon them here. We may, however, refer for some of them to the Mathematical Repository, No. xxiv.

CHAPTER III.

SOME ADDITIONAL INQUIRIES RESPECTING THE SPHERICAL EXCESS.

We shall now devote a short chapter to some miscellaneous inquiries respecting the Spherical Excess, in continuation of what has been already done in Chapter II. Part IV. All the usual formulæ for the spherical excess have there been amply discussed; but there are still certain other combinations of data which have not yet been considered: these are, 1st, Two angles and the interjacent side; 2d, Two angles and a side opposite to one of them: and lastly, Two sides and an angle opposite to one of them. Expressions for the spherical excess in each of these cases may be readily deduced. In the last two, however, the formulæ which I have obtained are neither sufficiently symmetrical nor sufficiently simple to render them deserving of much notice, either for analytical beauty or for practical convenience; they involve, however, but one radical. The formula for the first of the above cases I investigate as follows.

To determine the Spherical Excess when two angles and the interjacent side are given.

Here we have $\frac{E}{2}=\frac{A+B+C-180^\circ}{2}$,

and therefore $\cos.\frac{E}{2}=\cos.\frac{A+B}{2}\sin.\frac{C}{2}+\sin.\frac{A+B}{2}\cos.\frac{C}{2}$,

$\sin.\frac{E}{2}=\cos.\frac{A+B}{2}\cos.\frac{C}{2}-\sin.\frac{A+B}{2}\sin.\frac{C}{2}$.

But $\sin.^2\frac{C}{2}$ and $\cos.^2\frac{C}{2}$ may take either of the following forms:

$$\sin.^2\frac{C}{2}=\frac{1-\cos.C}{2}=\frac{1+\cos.A\cos.B-\sin.A\sin.B\cos.c}{2}$$

$$=\cos.^2\frac{A}{2}\cos.^2\frac{B}{2}-2\cos.\frac{A}{2}\sin.\frac{A}{2}\cos.\frac{B}{2}\sin.\frac{B}{2}\cos.c+$$

$$\sin.^2\frac{A}{2}\sin.^2\frac{B}{2}=\cos.^2\frac{A-B}{2}\sin.^2\frac{c}{2}+\cos.^2\frac{A+B}{2}\cos.^2\frac{c}{2}.$$

$$\cos.^2\frac{C}{2}=\frac{1+\cos.C}{2}=\frac{1-\cos.A\cos.B+\sin.A\sin.B\cos.c}{2}$$

$$=\cos.^2\frac{A}{2}\sin.^2\frac{B}{2}-2\cos.\frac{A}{2}\sin.\frac{A}{2}\cos.\frac{B}{2}\sin.\frac{B}{2}\cos.c+\sin.^2\frac{A}{2}\cos.^2\frac{B}{2}$$

$$=\sin.^2\frac{A-B}{2}\sin.^2\frac{c}{2}+\sin.^2\frac{A+B}{2}\cos.^2\frac{c}{2}.$$

Thus we shall have a choice of three forms, to suit the specific purpose we have in view. The last is the preferable on the ground of *algebraical symmetry*. Substituting these, we have

$$\cos.\frac{E}{2}=\cos.\frac{A+B}{2}\left\{\cos.^2\frac{A-B}{2}\sin.^2\frac{c}{2}+\cos.^2\frac{A+B}{2}\cos.^2\frac{c}{2}\right\}^{\frac{1}{2}}$$

$$+\sin.\frac{A+B}{2}\left\{\sin.^2\frac{A-B}{2}\sin.^2\frac{c}{2}+\sin.^2\frac{A+B}{2}\cos.^2\frac{c}{2}\right\}^{\frac{1}{2}}$$

$$\sin.\frac{E}{2}=\cos.\frac{A+B}{2}\left\{\sin.^2\frac{A-B}{2}\sin.^2\frac{c}{2}+\sin.^2\frac{A+B}{2}\cos.^2\frac{c}{2}\right\}^{\frac{1}{2}}$$

Substitute the value of cos. c in terms of [illegible] & multiply both terms by 1 or $\sin^2\frac{c}{2}+\cos^2\frac{a}{2}$

$$+\sin.\frac{A+B}{2}\left\{\cos.^2\frac{A-B}{2}\sin.^2\frac{c}{2}+\cos.^2\frac{A+B}{2}\cos.^2\frac{c}{2}\right\}^{\frac{1}{2}}$$

To determine the excess, when the three sides are given.

This case has been already discussed, but the following investigation may not be unacceptable. By (46, 4,)

$$\tan.\frac{E}{4}=\frac{2\sin.\frac{A+B+C-\pi}{4}\cos.\frac{A+B-C+\pi}{4}}{2\cos.\frac{A+B+C-\pi}{4}\cos.\frac{A+B-C+\pi}{4}}$$

$$=\frac{\sin.\frac{A+B}{2}-\sin.\left(\frac{\pi}{2}-\frac{C}{2}\right)}{\cos.\frac{A+B}{2}+\cos.\left(\frac{\pi}{2}-\frac{C}{2}\right)}=\frac{\sin.\frac{A+B}{2}-\cos.\frac{C}{2}}{\cos.\frac{A+B}{2}+\sin.\frac{C}{2}}\text{ or by (art. 86)}$$

$$=\frac{\cos.\frac{a-b}{2}-\cos.\frac{c}{2}}{\cos.\frac{a+b}{2}+\cos.\frac{c}{2}}\cot.\frac{C}{2}=\frac{\sin.\frac{\overline{s-a}}{2}\sin.\frac{\overline{s-b}}{2}}{\cos.\frac{s}{2}\cos.\frac{s-c}{2}}.\cot.\frac{C}{2}\,..(a).$$

$$\text{But,}\qquad\cot.\frac{C}{2}=\sqrt{\frac{\sin.s\sin.\overline{s-c}}{\sin.\overline{s-a}\sin.\overline{s-b}}}$$

$$=\sqrt{\frac{\sin.\frac{s}{2}\cos.\frac{s}{2}\sin.\frac{s-c}{2}\cos.\frac{s-c}{2}}{\sin.\frac{s-a}{2}\cos.\frac{s-a}{2}\sin.\frac{s-b}{2}\sin.\frac{s-c}{2}}}\;...\;(b).$$

Inserting (*b*) in (*a*), we have, after slight reductions,

$$\tan.\frac{E}{4}=\sqrt{\tan.\frac{s}{2}\tan.\frac{s-a}{2}\tan.\frac{s-b}{2}\tan.\frac{s-c}{2}}$$ which is the remarkable formula of *Lhuillier*.* Applying this to the polar triangles, some interesting results may be obtained as follows:

Denoting by $S_{,}$, $S_{,,}$, $S_{,,,}$ the semi-sums of the sides of the supplementary triangles; by a, $b_{,}$, and $c_{,}$ the sides of BA′C; $a_{,,}$, b, $b_{,,}$ the sides of AB′C; and by $a_{,,,}$, $b_{,,,}$, c, the sides of AC′B. Then (see p. 127)

$$s_{,}=\frac{a+\overline{\pi-b}+\overline{\pi-c}}{2}=\frac{a-b-c}{2}+\pi=$$

$$\pi-\frac{-a+b+c}{2}=\pi-\overline{s-a}$$

$$s_{,}-a=\frac{a-b-c}{2}+\pi-a=\pi-\frac{a+b+c}{2}=\pi-s,$$

$$s_{,}-b_{,}=\frac{a-b-c}{2}+\pi-\overline{\pi-b}=\frac{a+b-c}{2}=s-c,$$

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[*] The excess has also been obtained, by means of the modern analysis, by Euler in the Memoirs of the Royal Academy at Berlin, vol. ix. p. 256, and by Tedenat, in Gergonne's Annals of Mathematics, vol. vi. p. 48.

$\frac{-b}{2}$ & $\frac{c}{2}$ is sum & difference of

$\frac{+c-b}{2}$ & $\frac{b+c-a}{2}$ or $\frac{s-b}{2}$ & $\frac{s-a}{2}$

$$s_{,} - c_{,} = \frac{a-b-c}{2} + \pi - \overline{\pi - c} = \frac{a-b+c}{2} = s-b.$$

Hence if $E_{,}$, $E_{,,}$, $E_{,,,}$ denote the excesses of BA′C, AB′C, BC′A, we have by Lhuillier's theorem,

$$\tan.\frac{E_{,}}{4} = \sqrt{\tan.\frac{s_{,}}{2}\tan.\frac{s_{,}-a_{,}}{2}\tan.\frac{s_{,}-b_{,}}{2}\tan.\frac{s_{,}-c_{,}}{2}},$$

that is;

$$\left.\begin{aligned}
\tan.\frac{E_{,}}{4} &= \sqrt{\cot.\frac{s}{2}\cot.\frac{s-a}{2}\tan.\frac{s-b}{2}\tan.\frac{s-c}{2}} \\
\tan.\frac{E_{,,}}{4} &= \sqrt{\cot.\frac{s}{2}\tan.\frac{s-a}{2}\cot.\frac{s-b}{2}\tan.\frac{s-c}{2}} \\
\tan.\frac{E_{,,,}}{4} &= \sqrt{\cot.\frac{s}{2}\tan.\frac{s-a}{2}\tan.\frac{s-b}{2}\cot.\frac{s-c}{2}} \\
\tan.\frac{E}{4} &= \sqrt{\tan.\frac{s}{2}\tan.\frac{s-a}{2}\tan.\frac{s-b}{2}\tan.\frac{s-c}{2}}
\end{aligned}\right\} \dots (1).$$

in which Lhuillier's theorem is applied to each of the triangles in succession. If we multiply these together, we find

$$\tan.\frac{E}{4}\tan.\frac{E_{,}}{4}\tan.\frac{E_{,,}}{4}\tan\frac{E_{,,,}}{4} = \cot.\frac{s}{2}\tan.\frac{s-a}{2}\tan.\frac{s-b}{2}\tan.\frac{s-c}{2}. (2).$$

Again, the angles of the triangle BA,C are

$$A_{,} = A,\ B_{,} = \pi - B, \text{ and } C_{,} = \pi - C.$$

$$\text{Hence similarly} \left\{\begin{aligned}
\frac{E_{,}}{4} &= \frac{\pi + A - B - C}{4}. \\
\frac{E_{,,}}{4} &= \frac{\pi - A + B - C}{4} \\
\frac{E_{,,,}}{4} &= \frac{\pi - A - B + C}{4} \\
\frac{E}{4} &= \frac{-\pi + A + B + C}{4}
\end{aligned}\right\} \dots (3).$$

Also $\dfrac{\pi + A - B - C}{4} + \dfrac{\pi - A + B + C}{4} = \dfrac{\pi}{2}$,

whence $\tan.\dfrac{\pi - A + B + C}{4} = \cot.\dfrac{E_{,}}{4} =$

$\sqrt{\tan.\frac{s}{2}\tan.\frac{s-a}{2}\cot.\frac{s-b}{2}\cot.\frac{s-c}{2}}$; and by similar processes with the other triangles, we get the following table;

$$\left.\begin{aligned}
\tan.\frac{-A+B+C+\pi}{4} &= \sqrt{\tan.\frac{s}{2}\tan.\frac{s-a}{2}\cot.\frac{s-b}{2}\cot.\frac{s-c}{2}} \\
\tan.\frac{A-B+C+\pi}{4} &= \sqrt{\tan.\frac{s}{2}\cot.\frac{s-a}{2}\tan.\frac{s-b}{2}\cot.\frac{s-c}{2}} \\
\tan.\frac{A+B-C+\pi}{4} &= \sqrt{\tan.\frac{s}{2}\cot.\frac{s-a}{2}\cot.\frac{s-b}{2}\tan.\frac{s-c}{2}} \\
\tan.\frac{A+B+C-\pi}{4} &= \sqrt{\tan.\frac{s}{2}\tan.\frac{s-a}{2}\tan.\frac{s-b}{2}\tan.\frac{s-c}{2}}
\end{aligned}\right\} (4).$$

The last of which is the common form of the area of a triangle given by *Lhuillier*, applied to the fundamental triangle.

Multiply all these together, and we shall have,

$$\tan.^2 \frac{a+b+c}{4} = \tan. \frac{A+B+C-\pi}{4} \tan. \frac{A+B-C+\pi}{4}$$
$$\tan. \frac{A-B+C+\pi}{4} \tan. \frac{-A+B+C+\pi}{4} \quad . \; . \; . \quad (5).$$

Also, giving to the terms of Lhuillier's theorem, their unabbreviated values, we shall see a striking analogy in their general form between that and the one just obtained. For in $\tan.^2 \frac{A+B+C-\pi}{4} =$

$$\tan. \frac{a+b+c}{4} \tan. \frac{a+b-c}{4} \tan. \frac{a-b+c}{4} \tan. \frac{-a+b+c}{4} \quad (6).$$

we see the only difference, as to general form, is, that π enters into all the angular functions, and not into those of the sides. Again, since the three last factors in the right hand member of equation (5) are $\cot. \frac{E_{,,,}}{4}$, $\cot. \frac{E_{,,}}{4}$, and $\cot. \frac{E_{,}}{4}$; and the remaining factor is $\tan. \frac{E}{4}$, we have (5) converted into

$$\tan.^2 \frac{a+b+c}{4} = \tan. \frac{E}{4} \cot. \frac{E_{,}}{4} \cot. \frac{E_{,,}}{4} \cot. \frac{E_{,,,}}{4} \quad . \; . \; . \quad (7).$$

By the principle of the symmetry of the triangles, and of their expressions, we at once infer from (7) that

$$\cot. \frac{E}{4} \tan. \frac{E_{,}}{4} \cot. \frac{E_{,,}}{4} \cot. \frac{E_{,,,}}{4} = \tan.^2 \frac{s_{,}}{2}.$$

But $$s_{,} = \frac{a + \overline{\pi - b} + \overline{\pi - c}}{2} = \pi - \frac{-a+b+c}{2}$$

$$\therefore \tan. \frac{s_{,}}{2} = \cot. \frac{-a+b+c}{4} = \cot. \frac{s-a}{2}.$$

Applying the same principle of reduction to the other supplemental triangles, and collecting the results, we have

$$\left.\begin{array}{l} \tan.^2 \frac{s}{2} = \tan. \frac{E}{4} \cot. \frac{E_{,}}{4} \cot. \frac{E_{,,}}{4} \cot. \frac{E_{,,,}}{4} \\ \cot.^2 \frac{s-a}{2} = \cot. \frac{E}{4} \tan. \frac{E_{,}}{4} \cot. \frac{E_{,,}}{4} \cot. \frac{E_{,,,}}{4} \\ \cot.^2 \frac{s-b}{2} = \cot. \frac{E}{4} \cot. \frac{E_{,}}{4} \tan. \frac{E_{,,}}{4} \cot. \frac{E_{,,,}}{4} \\ \cot.^2 \frac{s-c}{2} = \cot. \frac{E}{4} \cot. \frac{E_{,}}{4} \cot. \frac{E_{,,}}{4} \tan. \frac{E_{,,,}}{4} \end{array}\right\} \quad . \; . \; . \; . \quad (8).$$

Let us resume equations (3), and multiply by (4) then we have

$$\left.\begin{array}{r} A+B+C-\pi = E \\ A-B-C+\pi = E_{,} \\ -A+B-C+\pi = E_{,,} \\ -A-B+C+\pi = E_{,,,} \end{array}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9).$$

add them, then $E + E_{,} + E_{,,} + E_{,,,} = 2\pi$ . . . . . (10).

Add the first of these to each of the others successively, then

$S = \frac{a+b+c}{2}$

$$\{A = \frac{E+E_{,}}{2},\ B = \frac{E+E_{,,}}{2}, \text{and } C = \frac{E+E_{,,,}}{2};\} \ldots\ldots (11).$$

$S = \frac{A+B+C}{2} = \frac{3E+E_{,}+E_{,,}+E_{,,,}}{4}$, or, inserting the value of $E_{,}+E_{,,}+E_{,,,}$ from (10), it becomes $S = \frac{1\pi}{2} + \frac{E}{2}$; and, in like manner, we have

$$\left\{\begin{array}{l} S - A = \frac{E - E_{,} + E_{,,} + E_{,,,}}{4} = \frac{\pi}{2} - \frac{E_{,}}{2} \\ S - B = \frac{E + E_{,} - E_{,,} + E_{,,,}}{4} = \frac{\pi}{2} - \frac{E_{,,}}{2} \\ S - C = \frac{E + E_{,} + E_{,,} - E_{,,,}}{4} = \frac{\pi}{2} - \frac{E_{,,,}}{2} \end{array}\right\} (12).$$

$$\left.\begin{array}{l} \therefore -\cos. S. = \sin. \frac{E}{2} \\ \cos. \overline{S - A} = \sin. \frac{E_{,}}{2} \\ \cos. \overline{S - B} = \sin. \frac{E_{,,}}{2} \\ \cos. \overline{S - C} = \sin. \frac{E_{,,,}}{2} \end{array}\right\} \ldots (13).$$

Inserting these values in the usual formula for finding a side, we get

$$\left.\begin{array}{l} \cot.^2 \tfrac{1}{2} a = \dfrac{\sin. \frac{E_{,,}}{2} \sin. \frac{E_{,,,}}{2}}{\sin. \frac{E}{2} \sin. \frac{E_{,}}{2}}, \quad \cot.^2 \tfrac{1}{2} b = \dfrac{\sin. \frac{E_{,}}{2} \sin. \frac{E_{,,,}}{2}}{\sin. \frac{E}{2} \sin. \frac{E_{,,}}{2}} \\ \text{and } \cot.^2 \tfrac{1}{2} c = \dfrac{\sin. \frac{E_{,}}{2} \sin. \frac{E_{,,}}{2}}{\sin. \frac{E}{2} \sin. \frac{E_{,,,}}{2}}, \end{array}\right\} \cdot (14).$$

$$\cot.^2 \tfrac{1}{2} a \cot.^2 \tfrac{1}{2} b \cot.^2 \tfrac{1}{2} c =$$

$$\left.= \frac{\sin. \frac{E_{,}}{2}}{\sin. \frac{E}{2}} \cdot \frac{\sin. \frac{E_{,,}}{2}}{\sin. \frac{E}{2}} \cdot \frac{\sin. \frac{E_{,,,}}{2}}{\sin. \frac{E}{2}} = \frac{\sin. \frac{E_{,}}{2} . \sin. \frac{E_{,,}}{2} \sin. \frac{E_{,,,}}{2}}{\sin.^3 \frac{E_{,} + E_{,,} + E_{,,,}}{2}}\right\} \ldots (15).$$

The sides and angles of the triangle are thus found, (the angles in 11,) in terms of the areas of the four triangles: and the equation of condition also which subsists among these four triangles is assigned in (10).

By (12) the values of the factors in N are found, and by (8) there is another trigonometrical function of the factors of n assigned. From this, those factors themselves may be assigned, but the process is troublesome and the result inelegant. We have obtained a simpler form, but even then neither the form nor the method is well suited to this place. The values of the inscribed and circumscribed radii in terms of the excesses will be discussed in the Repository, and we shall conclude this section with assigning the connexion between the polar systems of associated triangles, in respect to the areas.

The sides of the primary fundamental triangle being a, b, c, we have

$-b+c$), and hence $\frac{E'}{4}$

$\frac{+b+c)}{4} = \frac{\pi}{2} - \frac{a+b+c}{4}$

$= \cot.\frac{s}{2}$. Again, in the supplemental

th respect to A for instance,) we have

$-b)+(\pi-\pi-c = \pi - a + b + c$

$+c) = \frac{-a+b+c}{4} = \frac{(s-a)}{2}$

In performing the same changes upon $2S'_{\prime\prime\prime}$

ues of $\frac{E'_{\prime\prime}}{4}$ and $\frac{E'_{\prime\prime\prime}}{4}$; the whole form of

her below.

$$\left.\begin{array}{l}
\tan.\frac{E'}{4} = \cot.\frac{s}{2} \\
\tan.\frac{E'_{\prime}}{4} = \tan.\frac{s-a}{2} \\
\tan.\frac{E'_{\prime\prime}}{4} = \tan.\frac{s-b}{2} \\
\tan.\frac{E'_{\prime\prime\prime}}{4} = \tan.\frac{s-c}{2}
\end{array}\right\} \ldots\ldots (16).$$

gether; then $\tan.\frac{E'}{4}\tan.\frac{E'_{\prime}}{4}\tan.\frac{E'_{\prime\prime}}{4}\tan.\frac{E'_{\prime\prime\prime}}{4}$

$\cot.\frac{s}{2}\tan.\frac{s-a}{2}\tan.\frac{s-b}{2}\tan.\frac{s-c}{2} \ldots (16).$

By comparison of (2) and (16,) we find

$\tan.\frac{E'_{\prime\prime}}{4}\tan.\frac{E'_{\prime\prime\prime}}{4} = \tan.\frac{E}{4}\tan.\frac{E_{\prime}}{4}\tan.\frac{E_{\prime\prime}}{4}\tan.\frac{E_{\prime\prime\prime}}{4} (17).$

ison of the component equations of table (16) with those we shall get the value of the area of any triangle in terms system which is polar to it. Thus,

$$\left.\begin{array}{l}
\cot.^2\frac{E'}{4} = \tan.\frac{E}{4}\cot.\frac{E_{\prime}}{4}\cot.\frac{E_{\prime\prime}}{4}\cot.\frac{E_{\prime\prime\prime}}{4} \\
\cot.^2\frac{E'_{\prime}}{4} = \cot.\frac{E}{4}\tan.\frac{E_{\prime}}{4}\cot.\frac{E_{\prime\prime}}{4}\cot.\frac{E_{\prime\prime\prime}}{4} \\
\cot.^2\frac{E'_{\prime\prime}}{4} = \cot.\frac{E}{4}\cot.\frac{E_{\prime}}{4}\tan.\frac{E_{\prime\prime}}{4}\cot.\frac{E_{\prime\prime\prime}}{4} \\
\cot.^2\frac{E'_{\prime\prime\prime}}{4} = \cot.\frac{E}{4}\cot.\frac{E_{\prime}}{4}\cot.\frac{E_{\prime\prime}}{4}\tan.\frac{E_{\prime\prime\prime}}{4}
\end{array}\right\} \ldots\ldots (18)$$

versely, by interchanging the system of reference in the polar

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accented letters S', E', &c. denote quantities in the polar triangles which are the primary by S, E, &c.

triangles we have

$$\left.\begin{aligned}\text{cot.}^2\frac{E}{4}&=\text{tan.}\frac{E'}{4}\text{cot.}\frac{E'_{\prime}}{4}\text{cot.}\frac{E'_{\prime\prime}}{4}\text{cot.}\frac{E'_{\prime\prime\prime}}{4}\\ \text{cot.}^2\frac{E_{\prime}}{4}&=\text{cot.}\frac{E'}{4}\text{tan.}\frac{E'_{\prime}}{4}\text{cot.}\frac{E'_{\prime\prime}}{4}\text{cot.}\frac{E'_{\prime\prime\prime}}{4}\\ \text{cot.}^2\frac{E_{\prime\prime}}{4}&=\text{cot.}\frac{E'}{4}\text{cot.}\frac{E'}{4}\text{tan.}\frac{E'_{\prime\prime}}{4}\text{cot.}\frac{E'_{\prime\prime\prime}}{4}\\ \text{cot.}^2\frac{E_{\prime\prime\prime}}{4}&=\text{cot.}\frac{E'}{4}\text{cot.}\frac{E'_{\prime}}{4}\text{cot.}\frac{E'_{\prime\prime}}{4}\text{tan.}\frac{E'_{\prime\prime\prime}}{4}\end{aligned}\right\}(19)$$

By multiplying either of these sets, we should also obtain the reciprocal of equa. (17). By means of (10) applied to both systems of associated triangles, we have $E+E_{\prime}+E_{\prime\prime}+E_{\prime\prime\prime}+E'+E'_{\prime}+E'_{\prime\prime}+E'_{\prime\prime\prime}$ #

$$=4\pi=\text{surface of the sphere}\;\ldots\;(20).$$

Taking the values of $\text{tan.}\frac{E}{4}$ $\text{tan.}\frac{E'}{4}$ &c. from (1) and (16), we get

$$\left.\begin{aligned}\text{tan.}^2\frac{E}{4}\text{tan.}^2\frac{E'}{4}&=\text{cot.}\frac{s}{2}\text{tan.}\frac{s-a}{2}\text{tan.}\frac{s-b}{2}\text{tan.}\frac{s-c}{2}\\ \text{tan.}^2\frac{E_{\prime}}{4}\text{tan.}^2\frac{E'_{\prime}}{4}&=\text{cot.}\frac{s}{2}\text{tan.}\frac{s-a}{2}\text{tan.}\frac{s-b}{2}\text{tan.}\frac{s-c}{2}\\ \text{tan.}^2\frac{E_{\prime\prime}}{4}\text{tan.}^2\frac{E'_{\prime\prime}}{4}&=\text{cot.}\frac{s}{2}\text{tan.}\frac{s-a}{2}\text{tan.}\frac{s-b}{2}\text{tan.}\frac{s-c}{2}\\ \text{tan.}^2\frac{E_{\prime\prime\prime}}{4}\text{tan.}^2\frac{E'_{\prime\prime\prime}}{4}&=\text{cot.}\frac{s}{2}\text{tan.}\frac{s-a}{2}\text{tan.}\frac{s-b}{2}\text{tan.}\frac{s-c}{2}\end{aligned}\right\}\;\ldots\;(21)$$

From the equality of the right sides of the last equations we find

$$\text{tan.}\frac{E}{4}\text{tan.}\frac{E'}{4}=\text{tan.}\frac{E_{\prime}}{4}\text{tan.}\frac{E'_{\prime}}{4}=\text{tan.}\frac{E_{\prime\prime}}{4}\text{tan.}\frac{E'_{\prime\prime}}{4}$$

$$=\text{tan.}\frac{E_{\prime\prime\prime}}{4}\text{tan.}\frac{E'_{\prime\prime\prime}}{4}\;\ldots\;(22).$$

When the geographical positions of the three angles of a spherical triangle are given to determine the area, we have the following expression, first given by the author of this Supplement, in the 12th volume of the Edinburgh Transactions, viz.

$$\text{cos.}\frac{E}{2}=\left\{\frac{\begin{aligned}1&+\text{cos.}\,\alpha_{\prime}\,\text{cos.}\,\alpha_{\prime\prime}+\text{sin.}\,\alpha_{\prime}\,\text{sin.}\,\alpha_{\prime\prime}\,\text{cos.}\,(\beta_{\prime}-\beta_{\prime\prime})\\ &+\text{cos.}\,\alpha_{\prime}\,\text{cos.}\,\alpha_{\prime\prime\prime}+\text{sin.}\,\alpha_{\prime}\,\text{sin.}\,\alpha_{\prime\prime\prime}\,\text{cos.}\,(\beta_{\prime}-\beta_{\prime\prime\prime})\\ &+\text{cos.}\,\alpha_{\prime\prime}\,\text{cos.}\,\alpha_{\prime\prime\prime}+\text{sin.}\,\alpha_{\prime\prime}\,\text{sin.}\,\alpha_{\prime\prime\prime}\,\text{cos.}\,(\beta_{\prime\prime\prime}-\beta_{\prime\prime})\end{aligned}}{\begin{aligned}2\,(1&+\text{cos.}\,\alpha_{\prime}\,\text{cos.}\,\alpha_{\prime\prime\prime}+\text{sin.}\,\alpha_{\prime}\,\text{sin.}\,\alpha_{\prime\prime\prime}\,\text{cos.}\,\overline{\beta_{\prime}-\beta_{\prime\prime}})\times\\ (1&+\text{cos.}\,\alpha_{\prime}\,\text{cos.}\,\alpha_{\prime\prime\prime}+\text{sin.}\,\alpha_{\prime}\,\text{sin.}\,\alpha_{\prime\prime\prime}\,\text{cos.}\,\overline{\beta_{\prime}-\beta_{\prime\prime\prime}})\times\\ (1&+\text{cos.}\,\alpha_{\prime\prime}\,\text{cos.}\,\alpha_{\prime\prime\prime}+\text{sin.}\,\alpha_{\prime\prime}\,\text{sin.}\,\alpha_{\prime\prime\prime}\,\text{cos.}\,\overline{\beta_{\prime\prime\prime}-\beta_{\prime\prime}})\end{aligned}}\right\}^{\frac{1}{2}}\;23$$

where $\alpha_{\prime}, \beta_{\prime}$; $\alpha_{\prime\prime}, \beta_{\prime\prime}$; and $\alpha_{\prime\prime\prime}, \beta_{\prime\prime\prime}$, are the colatitudes and the longitudes of the vertices.

The spherical excess may also be very readily exhibited under another form by a direct investigation, but which, in my paper, in the Mathematical Repository now publishing, is obtained by inference from another property. Thus,

$$\overset{\text{tan}}{\cancel{\text{cot.}}}\,\frac{E}{2}=\frac{\text{sin.}\,\frac{A+B+C-\pi}{2}}{\text{cos.}\,\frac{A+B+C-\pi}{2}}=\frac{\overset{\text{cos}}{\cancel{\text{sin.}}}\,\frac{A+B+C}{2}}{\overset{\text{sin}}{\cancel{\text{cos.}}}\,\frac{A+B+C}{2}}=$$

* by last preceding equation (16)

$$= \frac{-\cos.\frac{A}{2}\cos.\frac{B}{2}\cos.\frac{C}{2}+\cos.\frac{A}{2}\sin.\frac{B}{2}\sin.\frac{C}{2}+\sin.\frac{A}{2}\cos.\frac{B}{2}\sin.\frac{C}{2}+\sin.\frac{A}{2}\sin.\frac{B}{2}\cos.\frac{C}{2}}{-\sin.\frac{A}{2}\sin.\frac{B}{2}\sin.\frac{C}{2}+\sin.\frac{A}{2}\cos.\frac{B}{2}\cos.\frac{C}{2}+\cos.\frac{A}{2}\sin.\frac{B}{2}\cos.\frac{C}{2}+\cos.\frac{A}{2}\cos.\frac{B}{2}\sin.\frac{C}{2}} \quad . \text{(24)}.$$

Now, for these several functions of the angles, insert their values from (1, 2, page 49,) keeping in mind that for the quantity there marked $\frac{1}{2}S$ (half the sum), we have, in this supplement, written s; and, performing the same upon each of the excesses, we shall obtain

$$\left.\begin{aligned}
\cot.\frac{E}{2} &= \frac{-\text{cosec.}\, s+\text{cosec.}\,(s-a)+\text{cosec.}\,(s-b)+\text{cosec.}\,(s-c)}{-\sin.\, s+\sin.\,(s-a)+\sin.\,(s-b)+\sin.\,(s-c)}\,\{\sin.\, s\sin.\,(s-a)\sin.\,(s-b)\sin.\,(s-c)\}^{\frac{1}{2}}\\
\cot.\frac{E_{,}}{2} &= \frac{\text{cosec.}\, s+\text{cosec.}\,(s-a)+\text{cosec.}\,(s-b)+\text{cosec.}\,(s-c)}{\sin.\, s+\sin.\,(s-a)+\sin.\,(s-b)+\sin.\,(s-c)}\,\{\sin.\, s\sin.\,(s-a)\sin.\,(s-b)\sin.\,(s-c)\}^{\frac{1}{2}}\\
\cot.\frac{E_{,,}}{2} &= \frac{\text{cosec.}\, s+\text{cosec.}\,(s-a)-\text{cosec.}\,(s-b)+\text{cosec.}\,(s-c)}{\sin.\, s+\sin.\,(s-a)-\sin.\,(s-b)+\sin.\,(s-c)}\,\{\sin.\, s\sin.\,(s-a)\sin.\,(s-b)\sin.\,(s-c)\}^{\frac{1}{2}}\\
\cot.\frac{E_{,,,}}{2} &= \frac{\text{cosec.}\, s+\text{cosec.}\,(s-a)+\text{cosec.}\,(s-b)+\text{cosec.}\,(s-c)}{\sin.\, s+\sin.\,(s-a)+\sin.\,(s-b)+\sin.\,(s-c)}\,\{\sin.\, s\sin.\,(s-a)\sin.\,(s-b)\sin.\,(s-c)\}^{\frac{1}{2}}
\end{aligned}\right\}\text{(25)}.$$

Again, if for s, $s-a$, $s-b$, $s-c$, we insert their values in terms of r, $r_{,}$, $r_{,,}$, $r_{,,,}$ from (5, p. 128,) we shall have the several areas of the associated triangles as functions of their inscribed radii.

$$\left.\begin{aligned}
\cot.\frac{E}{2} &= \frac{-\cot.\, r+\cot.\, r_{,}+\cot.\, r_{,,}+\cot.\, r_{,,,}}{-\tan.\, r+\tan.\, r_{,}+\tan.\, r_{,,}+\tan.\, r_{,,,}}\sqrt{\tan.\, r\tan.\, r_{,}\tan.\, r_{,,}\tan.\, r_{,,,}}\\
\cot.\frac{E_{,}}{2} &= \frac{\cot.\, r-\cot.\, r_{,}+\cot.\, r_{,,}+\cot.\, r_{,,,}}{\tan.\, r-\tan.\, r_{,}+\tan.\, r_{,,}+\tan.\, r_{,,,}}\sqrt{\tan.\, r\tan.\, r_{,}\tan.\, r_{,,}\tan.\, r_{,,,}}\\
\cot.\frac{E_{,,}}{2} &= \frac{\cot.\, r+\cot.\, r_{,}-\cot.\, r_{,,}+\cot.\, r_{,,,}}{\tan.\, r+\tan.\, r_{,}-\tan.\, r_{,,}+\tan.\, r_{,,,}}\sqrt{\tan.\, r\tan.\, r_{,}\tan.\, r_{,,}\tan.\, r_{,,,}}\\
\cot.\frac{E_{,,,}}{2} &= \frac{\cot.\, r+\cot.\, r_{,}+\cot.\, r_{,,}-\cot.\, r_{,,,}}{\tan.\, r+\tan.\, r_{,}+\cot.\, r_{,,}-\cot.\, r_{,,,}}\sqrt{\tan.\, r\tan.\, r_{,}\tan.\, r_{,,}\tan.\, r_{,,,}}
\end{aligned}\right\}\quad \ldots\ldots \text{(26)}.$$

We are also enabled, by means of the last set of equations, combined with (10), to ascertain the relation that subsists among the four radii r, $r_{,}$, $r_{,,}$, $r_{,,,}$, a relation which I believe has never before been assigned.

For we have $\frac{E+E_{,}+E_{,,}+E_{,,,}}{2}=\pi$; and hence,

$$\tan.\frac{E+E_{,}+E_{,,}+E_{,,,}}{2}=0=\frac{\Sigma_4(t)-\Sigma_4(t\,t_{,}\,t_{,,})^*}{1-\Sigma_4(t\,t_{,})+\Sigma_4(t\,t_{,}\,t_{,,}\,t\,t_{,,,})}.$$

Now this may be fulfilled in two different ways, either making the denominator infinite, whilst the numerator is finite; or making the numerator zero, whilst the denominator is a real quantity, finite or infinite. It would exceed the confined limits of a work like the present to discuss the circumstances of this function, and I shall, therefore, *assume* (though I shall elsewhere *prove* it) that the only condition that obtains is the latter. We hence have $\Sigma_4(t)-\Sigma_4(t\,t_{,}\,t_{,,})=0$; which may be written as follows:

(having previously divided it by $\tan.\frac{E}{2}\tan.\frac{E_{,}}{2}\tan.\frac{E_{,,}}{2}\tan.\frac{E_{,,,}}{2}$):

$$\left.\begin{array}{l}
-\cot.\frac{E}{2}+\cot.\frac{E_{,}}{2}\cot.\frac{E_{,,}}{2}\cot.\frac{E_{,,,}}{2}\\
-\cot.\frac{E_{,}}{2}+\cot.\frac{E}{2}\cot.\frac{E_{,,}}{2}\cot.\frac{E_{,,,}}{2}\\
-\cot.\frac{E''}{2}+\cot.\frac{E}{2}\cot.\frac{E_{,}}{2}\cot.\frac{E_{,,,}}{2}\\
-\cot.\frac{E_{,,,}}{2}+\cot.\frac{E}{2}\cot.\frac{E_{,}}{2}\cot.\frac{E_{,,}}{2}
\end{array}\right\}=0.$$

Insert for these cotangents their values, and reduce the expression to its simplest form. The work is somewhat laborious, but the result is comparatively simple; and hence I shall leave it as an exercise, for the student to perform alone. In another place I have given a different investigation of this formula, and several collateral topics are also combined with it, which will render it needless to enter into further detail upon this class of subjects, in the present necessarily very incomplete sketch. I trust, however, that enough is done to excite the interest of the mathematical student, whilst the extent of the subject itself will afford sufficient exercise for his ingenuity, and reward to his perseverance.

I am obliged to terminate these researches abruptly, on account of the space which they would occupy, if developed with any approach to completeness. I take the opportunity afforded me by reading the proofs, to state that my friend and neighbour, the Rev. Professor Logan, has also engaged in these and several collateral researches, and that the results to which both he and I may ultimately be found to have arrived, upon comparison of our MSS., will be published in a single dissertation to be considered as our joint production. These researches will extend to every other function of parts of the spherical triangle, as well as those which have been in this supplement discussed; and to a considerable extension of each of these. It will then be seen that Spherical Geometry offers one of the most ample fields of research that has yet been discovered; and I hope I shall not be thought too sanguine in anticipating that the properties of figures, traced upon the surface of the sphere, will, in a very few years, become as familiar to English Geometers as the correlative figures *in plano* now are.*

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* It is left for the student to prove, from the expression for the tan. of the sum of two arcs, at p. 33, that the expression for the tan. of four arcs is that in the text, in which E (t) denotes the sum of the tangents of those arcs E $(t\,t_{,})$, the sum of their products taken two and two, and so on. The same may be generalized for any number of arcs.

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* Not only have Spherical Geometry and Spherical Trigonometry been greatly neglected in England, but also upon the Continent. The continental Geometers have, however, been truly assiduous in the cultivation of the Geometry of three divisions, and have imagined and discussed almost every variety of *method* for conducting their investigations in this branch of science: whilst, on the other hand, it will be difficult to point to any one British Geometer who ever added a single important theorem to our stock, much less devised a single original method of investigation. Of the *causes* of this humiliating fact, the present is not the place to speak. It may, however, be allowed me to mention what appears to be a barrier to our removing the discreditable charge. We have no work, expressly devoted to the subject, in which either the methods themselves are developed, or the spirit of them at all displayed. Mere *illustrations*, taken in a considerable degree at random from different works, in which they were originally very appropriately placed, when brought together without due regard to the principles themselves, and often without adapting the notation to any uniform standard—works like these, though they may be entitled treatises on the Geometry of Three Dimensions, can scarcely be called so without a complete perversion of the use of terms. He that renders a *method of investigation* intelligible, with whatever paucity of mere illustration, does more for the interests of science than he who collects all the illustrative examples of those methods that have ever been given into one single mass. Such collections are, indeed, too commonly calculated to confuse the young mind and to repress all the ardour it might otherwise have felt.

Long ago, impressed with the importance of the subject, the author of this supplement formed the ambitious project of supplying this desideratum, and of furnishing a work in which the *spirit of the methods* which have been employed by the continental Geometers should be the first object of his anxiety. It has been his special aim, during the preparation of his work, to explain the essential character of each general principle, and to show wherever they really differ from one another, and at the same time to illustrate each by a sufficient number of apposite examples, strictly adapted to the purpose for which they were employed. By proceeding thus with every method that has been proposed by the different continental Geometers, and by furnishing also considerable portions of original results, it is hoped that a work may be produced which will render the study of solid Geometry scarcely more difficult than the more recondite portions of plane Geometry are now, and thereby greatly extend the cultivation of that branch of science in England.

He has been, however, led to think that a subsidiary elementary work on Descriptive Geometry would not be unacceptable to British Geometers, before the other goes to press. Even on this, the simplest of all the forms under which the Geometry of these dimensions presents itself, a merely graphic form—we have no treatise in England, nor yet a single chapter in any English course of Mathematics. There was indeed published in *America*, in 1821, a thin octavo, by M. Crozet, for the use of the Military College of the United States; but it would be scarcely less difficult to devise the methods originally than to acquire them from that treatise. Such a volume will therefore be sent to press with all convenient speed, the *avant courier* of the larger work.

NOTES.

NOTE A. *p.* 126.

THE following pretty theorems I have received from Mr. Lowry, since the first chapter on Spherical Geometry was in forms.

"Let ABC be a spherical triangle,[*] D the middle of one of the sides, AC; and let AB $= d$. Then $\cos. a + \cos. c = 2 \cos. \frac{1}{2} b \cos. d$.

For
$$\frac{\cos. a - \cos. \frac{1}{2}b \cos. d}{\sin. \frac{1}{2} b \sin. d} = \cos. \mathrm{BDC}$$
$$\frac{\cos. c - \cos. \frac{1}{2} b \cos. d}{\sin. \frac{1}{2}b \sin. d} = \cos. \mathrm{BDA} = - \cos. \mathrm{BDC}$$

Hence $\cos. a \cos. \frac{1}{2}b \cos. d = - \cos. c + \cos. \frac{1}{2}b \cos. d,$

or $\cos. a + \cos. c = 2 \cos. \frac{1}{2}b \cos. d.$

Cor. 1. When the triangle is inscribed in a semi-circle, the diameter of which is b, $\cos. a + \cos. c = 2 \cos.^2 \frac{1}{2}b$, or $\cos. a + \cos. c = 1 + \cos. b$.

Cor. 2. And when $a = c$, we have $\cos. a = \cos.^2 \frac{b}{2}$.

Cor. 3. Hence, in a spherical square,[†] the cosine of the sides is equal to the square of the cosine of half the diagonal.

Cor. 4. The sine of half the area of the triangle ACB in the circle is $= \tan. \frac{a}{2} \tan. \frac{c}{2}$. *Vide form* 20, *Math. Repos.* v. *part* I. *p.* 7.

Cor. 5. Hence in a spherical rectangle,[‡] the sine of one fourth of the area is equal to the rectangle of the semi-tangents of the two sides, that is $= \tan. \frac{a}{2} \tan. \frac{c}{2}$.

Cor. 6. And in the spherical square, the sine of $\frac{1}{4}$ area $= \tan.^2 \frac{a}{2}$.

Cor. 7. In a spherical parallelogram,[¶] the sides of which are a, b, c, d and diagonals k, k', we shall have

$$\cos. a + \cos. b + \cos. c + \cos. d = 4 \cos. \frac{k}{2} \cos. \frac{k'}{2}.$$

These properties are very simple, but neat, and might serve as exercises in an elementary treatise."

NOTE B. *p.* 133.

To account for some seeming discrepances, between the notes and text of this supplement, it is necessary to state that the *text* was drawn up in its present form from my manuscript, and the demonstration remodelled, (in many cases invented), to adapt it to the disolated state of the portions here given, during brief intervals stolen from other pursuits and occupations, having but little alliance with these subjects. The *notes* were added afterwards, in a letter to Mr. Young, and distributed by him so as not to interfere (where the interference would occasion much change in the text already partly in *slips* and partly in forms,) with the part already in the compositor's hands. Where addition could be worked into the text, and appeared more adapted to incorporation, it has been done; and where it did not coalesce with the text conveniently it has been put into foot notes. Some cases have, however, occurred where the addendum could not be properly made by either method, and it has therefore been altogether omitted. Still as these omissions are rather of a historical than a mathematical nature, no inconvenience can result from them, except the possibly erroneous distribution of the names of discoverers of particular theorems. Should this be ultimately found to be the case, I trust the authors to whom they are erroneously attributed, as well as the authors to whom they are actually due, will excuse the undesigned mistake.

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[*] The figure may be easily sketched by the student.

[†] A spherical four sided figure, whose sides are all equal, and whose angles are also all equal.

[‡] A our sided spherical figure, or whose angles are equal; or, perhaps, better adapted to the term, we may call it the figure in which great circles bisecting the pairs of opposite sides intersect each other at right angles.

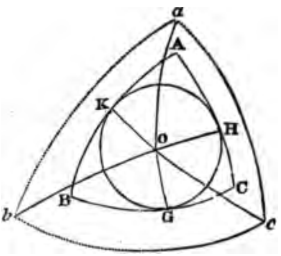
[¶] A figure whose opposite sides are equal.

These terms are adopted by analogy from Plane Geometry. Perhaps it may be found desirable ere long to modify our terminology considerably: but it does not appear to be the time.

There is, however, one particular case to which I wish more especially to refer, since I had till just now considered a theorem upon which I set some value, (and which, indeed, was the origin of my researches upon these topics), to be original, when, in fact, it had been discovered more than a quarter of a century ago, by Professor Lowry. I have just received a note from that distinguished Geometer, containing, amongst other matters, a reference to the Mathematical Repository, N.S. vol. I. p. 157.

Upon turning to this volume I find an anticipation of this beautiful property of the polar triangles: but as my copy of the Repository had been lent to a friend during the whole of the time I had the subject before my mind, my own discovery was perfectly independent of his, though so long posterior to it. I am quite sure, indeed, that I had never read that passage, or so beautiful a property must have been inevitably laid up amongst my collections. I am happy, however, to be able to render back to Professor Lowry the credit of the priority of discovery in the same volume in which I had seemed to claim not only independence but priority.

His demonstration (as was to be expected when the methods of spherical research in general at the two periods are compared) differs totally from mine; but his, as the geometrical often will have over the analytical, even when the latter is cultivated to its utmost perfection, has advantages over mine, which render it desirable to give it here. It is simple, and it proves more than mine proves, or perhaps can prove in moderate compass, viz. that the centres of the two circles, whose radii are complimentary, coalesce with one another. I will add, that *to him alone we owe every important spherical theorem that can be set down to the credit of Englishmen during at least a century past, probably even longer.*



Find the centre O of the inscribed circle in ABC, and from the points of contact G,H,K, draw the radii OG, OH, OK. Then these being perpendicular to the sides BC, CA, AB, respectively pass through the poles a, b, c, of those sides. Hence by polar triangles,

$$aOG = bOH = cOK = \frac{\pi}{2}. \quad \text{But}$$

$OG = OH = OK$, and, therefore, $Oa = Ob = Oc$, or O is also the centre of the circle about the polar triangle abc: that is, the centres of the primary inscribed, and the polar circumscribed circles are coincident. In the same manner the centres if polar inscribed and primary circumscribed are coincident. And it has been shown that these radii are complementary.

I may remark that the expression for the distance of the inscribed and circumscribed centres, *in terms of the radii themselves*, has not yet been given. *In plano* that distance was so assigned by *Mr. Landen*, and has been very elegantly investigated by *Mr. Lowry*, in the Mathematical Repository. The corresponding problem has been several times attempted, but other parts of the triangle have appeared in every result that has yet been published. The neatest form that I have seen is given anonymously in the *Annales des Matamatiques*, tom. VI. p. 223. viz.

$$\cos. D = \frac{\sin. a + \sin. b + \sin. c}{\sqrt{\sin. s \sin. \overline{s - a} \sin. \overline{s - b} \sin. \overline{s - c}}} . \sin. r \cos. R.$$

THE END.

MATHEMATICAL TABLES;

COMPREHENDING

THE LOGARITHMS OF ALL NUMBERS

FROM 1 TO 36,000;

ALSO

THE NATURAL AND LOGARITHMIC

SINES AND TANGENTS;

COMPUTED TO SEVEN PLACES OF DECIMALS, AND ARRANGED
ON AN IMPROVED PLAN;

WITH

SEVERAL OTHER TABLES,

USEFUL IN

NAVIGATION AND NAUTICAL ASTRONOMY,

AND IN

OTHER DEPARTMENTS OF PRACTICAL MATHEMATICS

BY J. R. YOUNG,

AUTHOR OF "ELEMENTS OF TRIGONOMETRY," &C.

REVISED AND CORRECTED BY

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AUTHOR OF "KEY TO HUTTON'S MATHEMATICS," &C.

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PREFACE TO THE TABLES.

THE following Tables are designed as well for the practical man as for the mathematical student. They comprehend, in a portable and cheap form, the principal information sought for in larger and more expensive collections.

The more important of these tables, viz. those immediately connected with trigonometrical and astronomical calculations, differ considerably both in form and arrangement from those in general use; and it is hoped that this departure from the usual plan, which has not been hastily made, will tend to increase the facility of reference.

In the table of the Logarithms of Numbers a new device has been adopted to mark the change of figure, and the several columns are so printed that, in seeking for the number corresponding to any proposed logarithm, the leading figures of the given logarithm may readily present them selves to the eye. Instead of omitting the several leading figures common to a number of successive logarithms, as is generally done, it has been recommended to preserve all the common figures, as at page 2 of these tables. This plan might perhaps facilitate, in a small measure, the writing out of a logarithm corresponding to a given number, but it would certainly render the detection of any given logarithm from among such a dense mass of figures much less easy.

In the table of logarithmic sines and tangents, the trigonometrical lines are inserted to every second, for the two first and two last degrees of the quadrant, and the old arrangement is followed; that is, the sines, cosines, &c. of the small arcs proceed in order from the top of the page to the bottom; and those of the large arcs, complements of the former, proceed in the reverse order, from the bottom to the top. The bulk of the table, however, is arranged differently;

the sines and tangents proceeding onwards to the end, and the cosines and cotangents in the reverse direction. This is the arrangement recommended by Professor Airy, in his Trigonometry, but it was not originally my intention to adopt it. Its advantages, however, having been more clearly pointed out to me by some scientific friends, occupied in computing the Nautical Almanack, and in the continual use of trigonometrical tables, I have been induced to depart from my first design, and to adopt the improved form. I regret that I had not come to this determination earlier, before the table for the two first degrees had been printed.

The table of natural sines and tangents is arranged upon the same plan as the former. The remaining tables of the volume require no particular observations here: a more minute detail of particulars will be found in the introductory explanation prefixed.

As accuracy in mathematical tables is of far more consequence than arrangement, it may be proper to state here that the present collection have all undergone very careful and repeated examination. The proofs of the tables of numbers, and of sines and tangents, were each compared twice with the tables of Bagay, Hutton, and Babbage, and in some cases with those of Taylor also; and the impressions from the stereotyped plates were again all compared with Hutton and Babbage. Many errors in Bagay's Tables of numbers were thus detected, and one or two in the last edition of Hutton; in Mr. Babbage's table I could find no error, and I have no doubt they amply deserve the reputation for accuracy which they have obtained.

J. R. YOUNG.

Jan. 1, 1833.

EXPLANATION OF THE TABLES.

The principal tables in this collection are the three following, 1st, a Table of the Common Logarithms of Numbers; 2d, a Table of the Logarithms of the Trigonometrical lines to radius 10^{10}; and 3d, a Table of the natural numerical values of the same lines to radius unity.

The explanation which we here propose to give of these tables must be understood to concern not the methods of *computing* them, but simply the manner of *using* them. The various methods of constructing a table of logarithms we have already discussed in a separate tract,* which will shortly be followed by a similar tract on the formation of a table of sines and tangents; our object here, therefore, will be to explain the use of tables already constructed.

TABLE I.

Of the Table of the Logarithms of Numbers.

The base of the system of Common Logarithms is 10; that is, every positive number is considered as some power, either whole or fractional, positive or negative, of the number 10, and it is the exponent of this power which is called the *logarithm* of the proposed number. If, therefore, we inquire what is the logarithm of any number, 60 for instance, we mean to ask what value the exponent x must have in order that 10^x may be equal to 60; the proper value, as far at least as seven places of decimals, is 1·7781513; that is to say $10^{1\cdot7781513} = 60$. The method of ascertaining the proper value of x, for any proposed number, is fully explained in our tract on logarithms above mentioned, but when the proposed number is any whole power of 10, whether positive or negative, it will be immediately seen to be such by mere inspection, and its logarithm will then be readily discovered. For example, the numbers.

1, 10, 100, 1000, 10000, &c.

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* An Elementary Essay on the Computation of Logarithms.

are at once seen to be positive powers of 10, which powers are

$$10^0,\ 10^1,\ 10^2,\ 10^3,\ 10^4,\ \&c.$$

and the numbers

·1, ·01, ·001, ·0001, &c.

are as readily seen to be the following negative powers of 10, viz.

$$10^{-1},\ 10^{-2},\ 10^{-3},\ 10^{-4},\ \&c.$$

Hence, of the series of numbers

. . . . , 10000, 1000, 100, 10, 1, ·1, ·01, ·001, ·0001, ...

the logs. are

. . . . , 4, 3, 2, 1, 0, — 1, — 2, — 3, — 4, ...

All this is very obvious; and it is further obvious that a number between any two terms of the first of these series will have its logarithm between the two corresponding terms of the second series. Thus the logarithm of a number between 10 and 100 will lie between 1 and 2; in other words, the *integral part* of the logarithm of any number, consisting of but two integral places of figures, however many decimals may follow, will always be 1.

In like manner, the logarithm of a number between 100 and 1000 will be between 2 and 3, of a number between 1000 and 10000 the logarithm will be between 3 and 4, and so on; that is, when the proposed number has three places of integers the integral part of its logarithm will be 2, when the number has four places of integers the integral part of its logarithm will be 4, and generally when the number has *n* places of integers, the integral part of its logarithm will be *n* — 1; and this expresses the number of places which the highest denomination, or first figure in the proposed number, is from the unit's place. Thus if 24785· 37 be the number proposed, then, seeing that its first figure 2 is four places from the unit's place, we know that its log. is 4 + a decimal. Upon the same principles the logarithm of any number between 1 and 1 is between — 1 and 0; that is, it is — 1 + a decimal, the logarithm of any number between ·01 and ·1 is — 2 + a decimal, and generally the logarithm of any number whose first significant figure is in the *n*th place of decimals is — *n*, and this expresses the number of places which the first significant figure in the proposed number is from the unit's place. Thus if ·00000736 be the number proposed, we know that as the first significant figure 7 is six places from the unit's place, its logarithm must be — 6 + a decimal. Seeing, therefore, that the integral part of a logarithm is so easily found from the proposed number, it is thought sufficient to insert in the table only the decimal part; accordingly, all the logarithms in a table of common logarithms must be understood to be decimals, although the decimal points may not appear.*

A valuable peculiarity of the common system of logarithms or that whose base is 10, is this, viz. that the logarithms of all numbers consisting of the same significant figures differ only in their characteristics. For example,

the log. of 16843 is	4·2264194
1684·3	3·2264194
168·43	2·2264194
16·843	1·2264194
1·6843	0·2264194
·16843 . . .	1·2264194
·016843 . . .	2·2264194
·0016843 . . .	3·2264194
&c.	&c.

That such must really be the case is very plain, for as

$$10^{4\cdot2264194} = 16843 \therefore 10^{3\cdot2264194} = \frac{16843}{10} = 1684\cdot3,$$

$$10^{2\cdot2264194} = \frac{1684\cdot3}{10} = 168\cdot43, \&c.$$

We may remark too, as a particular case of this property of the present system of logarithms, that the decimal part of the logarithm of a number consisting of any number of significant figures, either followed, or preceded, by ciphers, is always the same as if the ciphers were absent. Thus the decimal part of the logarithm of 358000 or of ·00358 or of 3580, &c. is the same as the decimal part of the logarithm of 358, so that, in seeking for the decimal part of the logarithm of a proposed number in the table, we are to disregard the ciphers with which it may commence or terminate.

Having stated these preliminary notions, we shall now enter more particularly into the manner of using the table of logarithms following.

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* In some few tables, however, the characteristics or integral part of the logarithms are inserted, as well as the decimal parts.

PROBLEM I.

To find the logarithm of any number from 1 to 36000.

If the proposed number either begin or end with ciphers these, as remarked above, are to be disregarded. The first significant figure to the right is to be considered as occupying the place of units, the preceding figures therefore will express so many tens. We must look for these leading figures in the column of *tens* in the table, and the horizontal row of logarithms against them will be that in which the sought logarithm occurs; it will be found under that figure, printed in the Egyptian character, which agrees with the figure in the unit's place of the proposed number. This being premised, we shall proceed at once to a few examples which will much better show the manner of using the table than any written direction.

EXAMPLE I.

Required the logarithm of 3265.

The leading figures 326 of this number I find in the column marked *tens*, at page 7; and carrying my eye along the horizontal row of logarithms, thus pointed out, I find in the vertical column headed 5 the logarithm sought, which (when the integral part 3 is supplied) is 3·5138832; for the 38832 is considered to be preceded by the 51 a little above it.

EXAMPLE II.

Required the logarithm of 3266.

The proper horizontal row of logarithms being found as before, I find that which is under the 6 to be 40162, which number is however considered to be preceded by the same figures as the number adjacent to it, or immediately before it, that is, by 51; hence supplying the index, or integral part, the required logarithm is 3·5140162.

EXAMPLE III.

Required the logarithm of 3236.

Having found the horizontal row which contains the logarithm, by means of the 323 in the *tens* column, I find the part under the **6** to be 00085 which I should proceed to complete by prefixing, as in last example, the 50 belonging to the number immediately before it, were it not that the crooked mark *ʃ* directs me to the 51 *below*, so that, supplying the index, the required logarithm is 3·5100085.

EXAMPLE IV.

Required the logarithm of 4680000.

Disregarding the terminating ciphers, I seek first for 46 in the *tens* column, and I find it in page 2 of the table; and in the same horizontal line with it, and under the 8, I find the decimal 6702459; hence, supplying the index, the required logarithm is 6·6702459.

EXAMPLE V.

Required the logarithm of ·002138.

Disregarding the ciphers, I seek first for 213 in the column of tens page 5, against which, and under the 8, I find the decimal 3300077; hence, prefixing the index, the required logarithm is 3·3300077.

PROBLEM II.

To determine the logarithm of a number beyond the limits of the table.

When the number proposed is beyond the limits of the table, that is, when it exceeds 30600. Enter the table with only the first five figures of the number, or indeed, with only the first four figures, should the five exceed the limits of the table, and find the corresponding logarithm. From the column marked *dif.* take out the number opposite to this logarithm, and multiply it by the remaining figures of the proposed number, reject from the product as many figures to the right as there are in the multiplier, and add the rest of the product to the logarithm already found: the sum will be the logarithm sought.

EXAMPLE I.

Required the logarithm of 843742.

I first seek the logarithm of 8437, the four first figures, the five first being beyond the limits of the table; this logarithm I find at page 16 to be, without the index, 9261880, and opposite to it in the column *dif.* is 515; this multiplied by 42, the remaining figures of the proposed number, produces 21630, from which product the two right-hand figures 30 being rejected, there remains 216 to be added to 9261880, which gives 9262096 for the decimal part of the required logarithm; therefore, prefixing the index 5, the complete logarithm is 5·9262096.

EXAMPLE II.

Required the logarithm of 1326927.

Log. 132690 . . .	5·1228382	*dif.* 3277
	88	27
Log. 1326927 . . .	5·1228470	2289
		654
		88,29

EXAMPLE III.

Required the logarithm of 114·1285.

Log. 114·12 . . .	2·0573618	*dif.* 381
	324	85
Log. 114·1285. . .	2·0573942	1905
		3048
		323·85.

It must be observed that as the column of differences does not commence till page 7 of the table, the preceding pages are never to be consulted for the logarithm of a number beyond the limits of the table.

PROBLEM III.

A logarithm being given, to find the corresponding number.

In this problem, too, as in the last, reference will be made to those pages only which contain the *dif.* column; among these we are to seek for the decimal part of the proposed logarithm, and we shall readily be guided to it, or else to a logarithm very near it, by means of the leading figures, which are separated in the table from the others, to attract the eye. If we find a logarithm exactly agreeing with that given, then the number, which the table shows us to belong to the logarithm found, will be the required number. If, however, as is most likely, we do not find the proposed logarithm exactly, then we are to take out the number corresponding to the next less logarithm; this number will of course fall short of that required, but the deficiency may be supplied as follows. Divide the difference between the tabular logarithm and the given one by that number in the *dif.* column which is opposite to the tabular logarithm, and add the quotient to the number already taken from the table.

EXAMPLE I.

Required the number whose logarithm is 1·2335678.

Given logarithm . . 2335678
The next less in the table is log. 17122 . 2335545

Add . . 52 tab. dif. 254)133·00 (·52
Required number . 17·12252 1270
600
508
92

EXAMPLE II.

Required the number whose logarithm is 3·1241987.

Given logarithm . . . 1241987
Next less log. 13309, . . 1241454

163 Tab. dif. 326) 533·00 (1·63
Required number 1331·063 326
2070
1956
1140

These examples will, no doubt, be found sufficient to examplify the manner of referring to the table when we are in search of a logarithm answering to a given number, or of a number answering to a given logarithm. We shall now give an example or two of the use of the table in facilitating arithmetical operations.

PROBLEM IV.

To multiply numbers together.

Add together the logarithms of the numbers, and the sum will be the logarithm of their product.

EXAMPLE I.

Required the product of 26784 and 7·865.

log. 26784	4·4278754
log. 7·865	·8956987
log. 210656·1	5·3235741.

hence the product is 210656·1.

EXAMPLE II.

Required the product of 3·586, 2·1046, ·8372, and ·0294.

log. 3·586	.5546103
2·1046	.3231696
.8372	$\bar{1}$·9228292
.0294	$\bar{2}$·4683473
Product ·1857618	$\bar{1}$·2689564.

PROBLEM V.

To divide one number by another.

Subtract the logarithm of the divisor from that of the dividend, and the remainder will be the logarithm of the quotient.

EXAMPLE I.

Divide 28·654 by 127·34.

log. 28·654	1·4571853
127·34	2·1049648
Quotient ·2250197	$\bar{1}$·3522205.

EXAMPLE II.

Divide ·06314 by ·007241.

log. ·06314	$\bar{2}$·8003046
Quotient ·007241	$\bar{3}$·8597985
8·71979.	·9405061.

PROBLEM VI.

To find the nth power of a given number.

The logarithm of the *n*th power will be equal to *n* times the logarithm of the given number.

EXAMPLE I.

Required the fourth power of ·09163.

log. ·09163 . . $\bar{2}{\cdot}9620377$
4

Power ·0000704938 . . $\bar{5}{\cdot}8481508$.

EXAMPLE II.

Required the tenth power of ·64.

log. ·64 . . $\bar{1}{\cdot}8061800$
10

Power ·011529225 . . $\bar{2}{\cdot}0618000$

PROBLEM VII.

To find the nth root of a given number.

The logarithm of the *n*th root will be equal to the *n*th part of the logarithm of the given number.

EXAMPLE I.

Required the fourth root of ·434296.

log. ·434296 . $\bar{1}{\cdot}6377858$
$\frac{1}{4}$ of it . . . $\bar{1}{\cdot}9094464$ = log. ·811795 the root.

As the negative index $\bar{1}$, of the given logarithm, is not divisible by 4, it is increased by 3 to make it so, and the 3, thus borrowed. is afterwards restored, by being prefixed to the 6, making it 3·6; that is, the proposed logarithm is viewed under the form $\bar{4} + 3{\cdot}6377858$, to which it is obviously equivalent.

EXAMPLE II.

Required the tenth root of 2.

log. 2 . . ·3010300
$\frac{1}{10}$ of it . . ·0301030 = log. 1·000121 the root.

EXAMPLE III.

Required the cube root of ·00048.

log. ·00048 . $\underline{4}\cdot 681\bar{2}412$
⅓ of it . . $\bar{2}\cdot 8937471$ = log. ·0782973 the root.

The negative index 4 not being divisible by 3, it is increased by 2 to make it so, and then the borrowed 2 restored by considering the positive part to commence with 2·6 instead of 6.

TABLE II.

Of the Table of Logarithmic Sines, Tangents, &c.

This second table consists of two parts: the first part containing the logarithmic sines, cosines, &c. of the first two and of the last two degrees of the quadrant, computed to every single second; and the other part of the table, containing the trigonometrical lines of the intermediate part of the quadrant, for every minute only.

The first part of the table, or that computed to seconds, is arranged in the usual manner; that is, the sines, cosines, tangents, and cotangents of the small arcs proceed from the top of the page to the bottom, according to the magnitude of the arcs, of which the degrees and minutes stand at the head of the columns, and the seconds occupy the *left-hand* column of every page. The sines, cosines, &c. of the large arcs, or those which are near 90°, and are the complements of the former, proceed, on the contrary, from the bottom of the page to the top, according to the magnitude of the arcs, of which the degrees and minutes stand at the bottom, and the seconds occupy the *right-hand* column of every page. In entering this part of the table, therefore, with a small arc, the eye must be directed to the *top* of the page, but on entering it with a large arc we must look to the *bottom* of the page.

The arrangement of the remaining part of the table is different from that usually adopted; for here the sines and tangents all proceed regularly, in the order of their magnitudes, from the top to the bottom of the page; while the cosines and cotangents all proceed in the contrary order, that is, from the bottom of the page to the top. This arrangement has considerable advantages over that of other trigonometrical tables, of which we may mention the following as instances. Suppose we enter this table with an arc containing seconds, as well as degrees and minutes, then if we seek its sine or tangent, that is, if we proceed *down* the table, the proportional difference, due to the seconds, will always be, *additive;* but if we want the cosine or tangent, that is, if we proceed *up* the table, then, on the contrary, the proportional difference will always be *subtractive.* Again, suppose that we enter the table with a logarithmic line, in search of the corresponding arc. We may first find the nearest tabular value less than the proposed, note the corresponding degrees and minutes, and then proportion for the seconds, which will always be additive if we proceed down the table, that is, if the given line be a sign or tangent, and always subtractive if we proceed up the table, that is, if the given line be a cosine or cotangent; of course the contrary will have place if we transcribe the nearest *greater* instead of the nearest *less* tabular value. But perhaps the principal advantage of the present arrangement is this, viz. that every opening of the table presents us with a greater number of consecutive sines, cosines, &c. than it could do under any other arrangement; and this peculiarity will always facilitate those operations which involve the sines, or the cosines, &c. of several neighbouring arcs, (as in the *lunar problem,* for instance, where the true and apparent attitudes of the bodies differ but little from each other.) The arc also, corresponding to any given logarithmic line, will be more readily found than under the old arrangement.

We must remark here, that the secants and cosecants of arcs have not been inserted, because they may be immediately supplied from the cosine and sines. For, since

$$\text{cos.} : \text{rad.} :: \text{rad.} : \text{sec.}$$

$$\therefore \text{sec.} = \frac{\text{rad.}^2}{\text{cos.}} \ \therefore \text{log. sec.} = 20 - \text{log. cos.}$$

and thus the log. secant of an arc is got by subtracting its log. consine from 20; and the log. consecant, by subtracting its log. sine from 20. Having spoken of the arrangement of this table, we shall now more particularly describe the manner of referring to it.

PROBLEM I.

To find the log. sine, &c. of a very small or of a very large arc, expressed in degrees, minutes, and seconds.

By a very small arc we mean one not exceeding two degrees; and to find its log. sine, we first search among the left-hand pages of the early part of the table, for that which presents the proposed degrees and minutes at the top; having found this, we shall have the vertical column in which the sine is; we must then pass the eye down the left-hand column, till we come to the number of seconds, then, in the same horizontal line with this number, and, in the vertical column before found, we shall find the sine required.

The tangent and cotangent are found in a similar manner among the right-hand pages.

The same pages which contain the sines, cosines, &c. of arcs below 2°, contain also those of arcs above 88°. When such a large arc is given, we must seek for that page which presents the degrees and minutes of it at *bottom*, and we shall thus find the column in which the sought trigonometrical line is; the corresponding seconds column will be seen on the right of the page; we must pass the eye *up* this till we reach the given number of seconds, opposite to which, in the vertical column already found, we shall see the sought number.

To find the log. sine, log. cosine, &c. of an arc consisting of degrees and minutes only, and between 2° and 88°

Within these limits the trigonometrical lines are given for every minute only; but columns of differences are annexed, by means of which the proper correction for seconds may be easily found.

In this part of the table the sines and tangents proceed throughout from the top to the bottom of the page; the cosines and cotangents from the bottom to the top. If we enter the table with degrees and minutes, and seek for a sine, we look for the given degrees at the top of one of the left-hand pages; if for a tangent, we look for the degrees at the top of one of the right-hand pages: the minutes are to be found in the left-hand marginal column of the page: the number sought will be under the degrees at top, and in the same horizontal row as the minutes. But if we seek for a cosine or a cotangent, we look for the degrees at the bottom of the page instead of at the top, and for the minutes in the right-hand marginal column instead of in the left.

To find the log. sine, &c. when the arc consists of degrees, minutes, and seconds.

In this case we enter the table with the degrees and minutes as before, and take out the corresponding number: between this number and that which belongs to the succeeding minute we shall find, in the adjacent column, the proper *difference*. Multiply this difference by the number of seconds, divide the product by 60, and we shall have the correction to be applied to the tabular number: this correction will be additive if we proceed down the table, or seek for a sine or tangent, but it will be subtractive if we proceed up, or look for a cosine or cotangent. We shall give an example or two of this operation.

EXAMPLE I.

Required the log. sine of 35° 27′ 24″.

Turning to page 126, we find for the log. sine of 35° 27′ the number 9·764222, and the difference between this and the sine next following is shown in the difference column to be 1774, therefore the correction for 24″ is $1774 \times \frac{24}{60} = 1774 \times \cdot 4 = 709{\cdot}6$, consequently,

$$\begin{aligned} \text{log. sin. } 35^\circ\ 27' &= 9{\cdot}7634222 \\ +\ \text{correction for } 24'' &= \phantom{9{\cdot}763}7096 \\ \hline \text{log. sin. } 35^\circ\ 27'\ 24'' &= 9{\cdot}7634932 \end{aligned}$$

EXAMPLE II.

Required the log. cosine of 48° 35′ 27″.

The log. cosine of 48° 35′ we find, at page 128, to be 9·8205496, therefore.

log. cos. 48° 35′ = 9·8205496 .... Dif. = 1433
— correction for 27″ = 645
9·8204851

1433
27
10031
2866
6,0)3869,1
645 = Cor.

EXAMPLE III.

Required the log. tangent of 15° 43′ 31″.

log. tan. 15° 43′ = 9·4493260 .... Dif. = 4842
+ correction for 31″ = 2502
9·4495762

4842
31
4842
14526
6,0)15010,2
2501·7 = Cor.

EXAMPLE IV.

Required the log. cotangent of 41° 0′ 29″.

log. cot. 41° 0′ = 10·0608369 .... Dif. = 2551
— correction for 29″ = 1233
log. cot. 41° 0′ 29″ = 10·0607136

2551
29
22959
5102
6,0)7397,9
1233 = Cor.

EXAMPLE V.

Required the log. secant of 13° 24′ 23′.

log. sec. 13°24′(=20 — log. cos.)=10·0119872. Dif.=301
+ correction for 23″ = 110
log. sec. 13° 24′ 23″ = 10·0119982

301
23
603
602
6,0)662,3
110=Cor.

EXAMPLE VI.

Required the log. cosecant of 34° 52′ 43″.

log. cosec. 34° 52′ (= 20 — log. sin.) = 10·2428556.....Dif. = 1812

43

— correction for 43″ = 1299

5436

log. cosec. 34° 52′ 43″ = 10·2427257

7248

6,0)7791,6

1298·6 = Cor.

To find the arc corresponding to a given log. sine or log. tangent, &c.

Search in the table for that log. sine, or log. tangent, which is nearest to the proposed, but less than it, and take out the corresponding degrees and minutes. Find also the difference between this tabular number and the proposed, multiply it by 60 and divide by the tabular difference, the quotient will give the proper number of seconds.

EXAMPLE I.

Required the arc whose log. sine is 9·7634932.

Given log. sine . 9·7634932

log. sine 35° 27′ . 9·7634222 tab. dif. 1774

24″

710

35° 27′ 24″ = req. arc 60

1774)42600(24

3548

7120

7096

To find the arc corresponding to a given log. cosine or log. cotangent.

Proceed, as in last problem, with this exception only, that instead of taking from the table the number next *less*, take the next *greater*; or if we take the next *less*, we must *subtract* the correction, not add it.

EXAMPLE.

Required the arc whose log. cosine is 9·8204851.

Given log. cosine . . 9·8204851
log. cos. 28° 35′ . . 9·8205496 tab. dif. 1433
27″ 645
Req. arc = 28° 35′ 27″ 60
1433)38700(27
2866
10040
10031

TABLE III.

Natural Sines, Tangents, &c.

This table is used like the former, but as the columns of differences are not inserted, when the difference between any two contiguous tabular numbers is required, for the purpose of correcting for seconds, this difference must be found by actual subtraction.

TABLE IV.

Traverse Table to every Quarter point of the Compass.

This table is useful in Navigation, showing, by inspection, the difference of latitude and departure due to any proposed course and distance. If the distance sailed be more than 120 miles it will exceed the limits of the table; but the difference of latitude and departure may still be determined from it by this simple operation: divide the given distance by any number that will give a quotient not exceeding 120; enter the table with this quotient, and multiply the corresponding dif. of lat. and dep. by the assumed divisor, and there will result the dif. of lat. and dep. due to the proposed distance.

The construction of the traverse table is obvious; the given distance and course being always the hypotenuse and adjacent angle from which the dif. of lat. and dep. tabulated are computed.

TABLE V.

Workman's Table for correcting the Middle Latitude.

This table is useful for correcting what in Navigation is called the *middle latitude*. It is usual, in middle latitude sailing, to consider the departure which a ship makes in sailing upon an oblique rhomb from one parallel of latitude to another, to be equal to the distance between the meridians left and come to, measured on the middle parallel (see Trig. p. 74–5); but, as this is not strictly accurate, a correction becomes necessary. This correction is furnished by the present table; the given middle latitude is to be found in the first column to the left; in a horizontal line with which, and under the given difference of latitude, is inserted the proper correction to be *added* to the middle latitude to obtain the latitude in which the meridian distance is accurately equal to the departure. The formula for constructing this table is obtained as follows:

Let $d =$ proper diff. of lat.
$D =$ meridional diff. of lat.
$m =$ middle latitude.
$M = m +$ correction.
$L =$ diff. of longitude.

Then, (Trig. p. 75), $\text{tan. course} = \dfrac{\text{cos. } M \times L}{d}$

But, (Trig. p. 77), $\text{tan. course} = \dfrac{\text{rad.} \times L}{D}$

$$\therefore \frac{\text{cos. } M \times L}{d} = \frac{\text{rad.} \times L}{D} \quad \therefore \text{cos. } M = \frac{\text{rad. } d}{D}$$

$$\therefore \text{correction} = \text{cos.}^{-1}\left(\frac{\text{rad. } d}{D}\right) - m.$$

TABLES VI., VII., VIII., IX., X., XI., AND XII.

These are all tables of corrections to be applied to the observed altitudes of the celestial bodies; the manner of using them must be sufficiently obvious from inspecting them, provided the object of the several corrections is clearly understood; and this is explained at length in the chapter on Nautical Astronomy in the Trigonometry, where several examples of the corrections are given.

LOGARITHMS OF NUMBERS

FROM 1 TO 36,000.

Between $1 = \log.^{-1} 0$, and $600 = \log.^{-1} 2{\cdot}7781513$.

ts.	1	2	3	4	5	6	7	8	9
0	0000000	3010300	4771213	6020600	6989700	7781513	8450980	9030900	9542425
1	0413927	0791812	1139434	1461280	1760913	2041200	2304489	2552725	2787536
2	3222193	3424227	3617278	3802112	3979400	4149733	4313638	4471580	4623980
3	4913617	5051500	5185139	5314789	5440680	5563025	5682017	5797836	5910646
4	6127839	6232493	6334685	6434527	6532125	6627578	6720979	6812412	6901961
5	7075702	7160033	7242759	7323938	7403627	7481880	7558749	7634280	7708520
6	7853298	7923917	7993405	8061800	8129134	8195439	8260748	8325089	8388491
7	8512583	8573325	8633229	8692317	8750613	8808136	8864907	8920946	8976271
8	9084850	9138139	9190781	9242793	9294189	9344985	9395193	9444827	9493900
9	9590414	9637878	9684829	9731279	9777236	9822712	9867717	9912261	9956352
10	0043214	0086002	0128372	0170333	0211893	0253059	0293838	0334238	0374265
11	0453230	0492180	0530784	0569049	0606978	0644580	0681859	0718820	0755470
12	0827854	0863598	0899051	0934217	0969100	1003705	1038037	1072100	1105897
13	1172713	1205739	1238516	1271048	1303338	1335389	1367206	1398791	1430148
14	1492191	1522883	1553360	1583625	1613680	1643529	1673173	1702617	1731863
15	1789769	1818436	1846914	1875207	1903317	1931246	1958997	1986571	2013971
16	2068259	2095150	2121876	2148438	2174839	2201081	2227165	2253093	2278867
17	2329961	2355284	2380461	2405492	2430380	2455127	2479733	2504200	2528530
18	2576786	2600714	2624511	2648178	2671717	2695129	2718416	2741578	2764618
19	2810334	2833012	2855573	2878017	2900346	2922561	2944662	2966652	2988531
20	3031961	3053514	3074960	3096002	3117539	3138672	3159703	3180633	3201463
21	3242825	3263359	3283796	3304138	3324385	3344538	3364597	3384565	3404441
22	3443923	3463530	3483049	3502480	3521825	3541084	3560259	3579348	3598355
23	3636120	3654880	3673559	3692159	3710679	3729120	3747483	3765770	3783979
24	3820170	3838154	3856063	3873898	3891661	3909351	3926970	3944517	3961993
25	3996737	4014005	4031205	4048337	4065402	4082400	4099331	4116197	4132998
26	4166405	4183013	4199557	4216039	4232459	4248816	4265113	4281348	4297523
27	4329693	4345689	4361626	4377506	4393327	4409091	4424798	4440448	4456042
28	4487063	4502491	4517864	4533183	4548449	4563660	4578819	4593925	4608978
29	4638930	4653829	4668676	4683473	4698220	4712917	4727564	4742163	4756712
30	4785665	4800069	4814426	4828736	4842998	4857214	4871384	4885507	4899585
31	4927604	4941546	4955443	4969296	4983106	4996871	5010593	5024271	5037907
32	5065050	5078559	5092025	5105450	5118834	5132176	5145478	5158738	5171959
33	5198280	5211381	5224442	5237465	5250448	5263393	5276299	5289167	5301997
34	5327544	5340261	5352941	5365584	5378191	5390761	5403295	5415792	5428254
35	5453071	5465427	5477747	5490033	5502284	5514500	5526682	5538830	5550944
36	5575072	5587086	5599066	5611014	5622929	5634811	5646661	5658478	5670264
37	5693739	5705429	5717088	5728716	5740313	5751878	5763414	5774918	5786392
38	5809250	5820634	5831988	5843312	5854607	5865873	5877110	5888317	5899496
39	5921768	5932861	5943926	5954962	5965971	5976952	5987905	5998831	6009729
40	6031444	6042261	6053050	6063814	6074550	6085260	6095944	6106602	6117233
41	6138418	6148972	6159501	6170003	6180481	6190933	6201361	6211763	6222140
42	6242821	6253125	6263404	6273659	6283889	6294096	6304279	6314438	6324573
43	6344773	6354837	6364879	6374897	6384893	6394865	6404814	6414741	6424645
44	6444386	6454223	6464037	6473830	6483600	6493349	6503075	6512780	6522463
45	0541765	6551384	6560982	6570559	6580114	6589648	6599162	6608655	6618127
46	6637009	6646420	6655810	6665180	6674530	6683859	6693169	6702459	6711728
47	6730200	6739420	6748611	6757783	6766936	6776070	6785184	6794279	6803355
48	6821451	6830470	6839471	6848454	6857417	6866363	6875290	6884198	6893089
49	6910815	6919651	6928469	6937269	6946052	6954817	6963564	6972293	6981005
50	6998377	7007037	7015680	7024305	7032914	7041505	7050080	7058637	7067178
51	7084209	7092700	7101174	7109631	7118072	7126497	7134905	7143298	7151674
52	7168377	7176705	7185017	7193313	7201593	7209857	7218106	7226339	7234557
53	7250945	7259116	7267272	7275413	7283538	7291648	7299743	7307823	7315888
54	7331973	7339993	7347998	7355989	7363965	7371926	7379873	7387806	7395723
55	7411516	7419391	7427251	7435098	7442930	7450748	7458552	7466342	7474118
56	7489629	7497363	7505084	7512791	7520484	7528164	7535831	7543483	7551123
57	7566361	7573960	7581546	7589119	7596678	7604225	7611758	7619278	7626786
58	7641761	7649230	7656686	7664128	7671559	7678976	7686381	7693773	7701153
59	7715875	7723217	7730547	7737864	7745170	7752463	7759743	7767012	7774268
	1	2	3	4	5	6	7	8	9

Table I.] LOGARITHMS OF NUMBERS FROM 1 TO 36,000.

Between 600 = log.⁻¹ 2.7781513, and 1200 = log.⁻¹ 3·0791812.

tens.	1	2	3	4	5	6	7	8	9
60	7788745	95965	7803173	10369	7817554	24726	7831887	39036	7846173
61	7860412	67514	74605	81684	88751	95807	7902852	09885	7916906
62	7930916	37904	7944880	51846	7958800	65743	72675	79596	86506
63	8000294	07171	8014037	20893	8027737	34571	8041394	48207	8055009
64	68580	75350	82110	88859	95597	/02325	8109043	15750	8122447
65	8135810	42476	8149132	55777	8162413	69038	75654	82259	88854
66	8202015	08580	8215135	21681	8228216	34742	8241258	47765	8254261
67	67225	73693	80151	86599	93038	99467	8305887	12297	8318698
68	8331471	37844	8344207	50561	8356906	63241	69567	75884	82192
69	94780	01061	8407332	13595	8419848	26092	8432328	38554	8444772
70	8457180	/63371	69553	75727	81891	88047	94194	/00333	8506462
71	8518696	24800	8530895	36982	8543060	49130	8555192	61244	67289
72	79353	85372	91383	97386	8603380	09366	8615344	21314	8627275
73	8639174	45111	8651040	56961	62873	68778	74675	80564	86444
74	98182	04039	8709888	15729	8721563	27388	8733206	39016	8744818
75	8756399	/62178	67950	73713	79470	85218	90959	96692	8802418
76	8813847	19550	8825245	30934	8836614	42288	8847954	53612	59263
77	70544	76173	81795	87410	93017	98617	8904210	09796	8915375
78	8926510	32068	8937618	43161	8948697	54225	59747	65262	70770
79	81765	87252	92732	98205	9003671	09131	9014583	20029	9025468
80	9036325	41744	9047155	52560	57959	63350	68735	74114	79485
81	90209	95560	9100905	06244	9111576	16902	9122221	27533	9132839
82	9143432	48718	53998	59272	64539	69800	75055	80303	85545
83	96010	/01233	9206450	11661	9216865	22063	9227255	32440	9237620
84	9247960	53121	58276	63424	68567	73704	78834	83959	89077
85	99296	/04396	9309490	14579	9319661	24738	9329808	34873	9339932
86	9350032	55073	60108	65137	70161	75179	80191	85197	90198
87	9400182	05165	9410142	15114	9420081	25041	9429996	34945	9439889
88	49759	54686	59607	64523	69433	74337	79236	84130	89018
89	98777	/03649	9508515	13375	9518230	23080	9527924	32763	9537597
90	9547248	52065	56878	61684	66486	71282	76073	80856	85639
91	95184	99948	9604708	09462	9614211	18955	9623693	28427	9633155
92	9642596	47309	52017	56720	61417	66110	70797	75480	80157
93	89497	94159	98816	/03469	9708116	12758	9717396	22028	9726656
94	9735896	40509	9745117	49720	54318	58911	63500	68083	72662
95	81805	86369	90929	95484	9800034	04579	9809119	13655	9818186
96	9827234	31751	9836263	40770	45273	49771	54265	58754	63238
97	72192	76663	81128	85590	90046	94498	98946	/03389	9907827
98	9916690	21115	9925535	29951	9934362	38769	9943172	47569	51963
99	60737	65117	69492	73864	78231	82593	86952	91305	95655
100	0004341	08677	0013009	17337	0021661	25980	0030295	34605	0038912
101	47512	51805	56094	60380	64660	68937	73210	77478	81742
102	90257	94509	98756	/03000	0107239	11474	0115704	19931	0124154
103	0132587	36797	0141003	45205	49403	53598	57788	61974	66155
104	74507	78677	82843	87005	91163	95317	99467	/03613	0207755
105	0216027	20157	0224284	28406	0232525	36639	0240750	44857	48960
106	57154	61245	65333	69416	73496	77572	81644	85713	89777
107	97895	/01948	0305997	10043	0314085	18123	0322157	26188	0330214
108	0338257	42273	46285	50293	54297	58298	62295	66289	70279
109	78248	82226	86202	90173	94141	98106	0402066	06023	0409977
110	0417873	21816	0425755	29691	0433623	37551	41476	45398	49315
111	57141	61048	64952	68852	72749	76642	80532	84418	88301
112	96056	99929	0503798	07663	0511525	15384	0519239	23091	0526939
113	0534626	38464	42299	46131	49959	53783	57605	61423	65237
114	72856	76661	80462	84260	88055	91846	95634	99419	0603200
115	0610753	14525	0618293	22058	0625820	29578	0633334	37086	40834
116	48322	52061	55797	59530	63259	66986	70709	74428	78145
117	85569	89276	92980	96681	0700379	04073	0707765	11453	0715138
118	0722499	26175	0729847	33517	37184	40847	44507	48164	51819
119	59118	62763	66404	70043	73679	77312	80942	84568	88192
	1	2	3	4	5	6	7	8	9

Between 1200 = log.$^{-1}$ 3.0791812, and 1800 = log.$^{-1}$ 3·2552725.

tens.	1	2	3	4	5	6	7	8	9
120	0795430	99045	0802656	06265	0809870	13473	0817073	20669	0824263
1	0831441	35026	38608	42187	45763	49336	52906	56473	60037
2	67157	70712	74265	77814	81361	84905	88446	91984	95519
3	0902581	06107	0909631	13152	0916670	20185	0923697	27206	0930713
4	37718	41216	44711	48204	51694	55180	58665	62146	65624
5	72573	76043	79511	82975	86437	89896	93353	96806	1000257
6	1007151	10594	1014034	17471	1020905	24337	1027766	31193	34616
7	41456	44871	48284	51694	55102	58507	61909	65309	68705
8	75491	78880	82267	85650	89031	92410	95785	99159	1102529
9	1109262	12625	1115985	19343	1122698	26050	1129400	32747	36092
130	42773	46110	49444	52776	56105	59432	62756	66077	69396
1	76027	79338	82647	85954	89258	92559	95858	99154	1202448
2	1209028	12316	1215598	18880	1222159	25435	1228709	31981	35250
3	41781	45042	48301	51558	54813	58065	61314	64561	67806
4	74288	77525	80760	83993	87223	90451	93676	96899	1300119
5	1306553	09767	1312978	16187	1319393	22597	1325798	28998	32195
6	38581	41771	44959	48144	51327	54507	57685	60861	64034
7	70375	73541	76705	79867	83027	86184	89339	92492	95643
8	1401937	05080	1408222	11361	1414498	17632	1420765	23895	1427022
9	33271	36392	39511	42628	45742	48854	51964	55072	58177
140	64381	67480	70577	73671	76763	79853	82941	86027	89110
1	95270	98347	1501422	04494	1507564	10633	1513699	16762	1519824
2	1525941	28996	32049	35100	38149	41195	44240	47282	50322
3	56396	59430	62462	65492	68519	71544	74568	77589	80608
4	86640	89653	92663	95672	98678	/01683	1604685	07686	1610684
5	1616674	19666	1622656	25644	1628630	31614	34596	37575	40553
6	46502	49474	52443	55411	58376	61340	64301	67261	70218
7	76127	79078	82027	84975	87920	90864	93805	96744	99682
8	1705551	08482	1711412	14339	1717265	20188	1723110	26029	1728947
9	34776	37688	40598	43506	46412	49316	52218	55118	58016
150	63807	66699	69590	72478	75365	78250	81133	84013	86892
1	92645	95518	98389	/01259	1804126	06992	1809856	12718	1815578
2	1821292	24147	1826999	29850	32698	35545	38390	41234	44075
3	49752	52588	55422	58254	61084	63912	66739	69563	72386
4	78026	80844	83659	86473	89285	92095	94903	97710	1900514
5	1906118	08917	1911715	14510	1917304	20096	1922886	25675	28461
6	34029	36810	39590	42367	45143	47918	50690	53461	56229
7	61762	64525	67287	70047	72806	75562	78317	81070	83821
8	89319	92065	94809	97552	2000293	03032	2005769	08505	2011239
9	2016702	19431	2022158	24883	27607	30329	33049	35768	38485
160	43913	46625	49335	52044	54750	57455	60159	62860	65560
1	70955	73650	76344	79035	81725	84414	87100	89785	92468
2	97830	/00508	2103185	05860	2108534	11205	2113876	16544	2119211
3	2124540	27202	29862	32521	35178	37833	40487	43139	45790
4	51086	53732	56376	59018	61659	64298	66936	69572	72207
5	77471	80100	82729	85355	87980	90603	93225	95845	98464
6	2203696	06310	2208922	11533	2214142	16750	2219356	21960	2224563
7	29764	32363	34959	37555	40148	42740	45331	47920	50507
8	55677	58260	60841	63421	65999	68576	71151	73724	76296
9	81436	84004	86570	89134	91697	94258	96818	99377	2301934
170	2307043	09596	2312146	14696	2317244	19790	2322335	24879	27421
1	32500	35038	37574	40108	42641	45173	47703	50232	52759
2	57809	60331	62853	65373	67891	70408	72923	75437	77950
3	82971	85479	87986	90491	92995	95497	97998	/00498	2402996
4	2407988	10482	2412974	15465	2417954	20442	2422929	25414	27898
5	32861	35341	37819	40296	42771	45245	47718	50189	52658
6	57594	60059	62523	64986	67447	69907	72365	74823	77278
7	82186	84637	87087	89536	91984	94430	96874	99318	2501759
8	2506639	09077	2511513	13949	2516382	18815	2521246	23675	26103
9	30956	33380	35803	38224	40645	43063	45481	47897	50312
	1	2	3	4	5	6	7	8	9

Between 1800 = log.$^{-1}$ 3·2552725, and 2400 = log.$^{-1}$ 3·3802112.

Nos.	1	2	3	4	5	6	7	8	9
180	2555137	57549	2559957	62365	2564772	67177	2569582	71984	2574385
1	79185	81582	83978	86373	88766	91158	93549	95939	98327
2	2603099	05484	2607867	10248	2612629	15008	2617385	19762	2622137
3	26883	29255	31625	33993	36361	38727	41092	43455	45817
4	50538	52896	55253	57609	59964	62317	64669	67020	69369
5	74064	76410	78754	81097	83439	85780	88119	90457	92794
6	97464	99797	2702129	04459	2706788	09116	2711443	13769	2716093
7	2720738	23058	25378	27696	30013	32328	34643	36956	39268
8	43888	46196	48503	50809	53114	55417	57719	60020	62320
9	66915	69211	71506	73800	76092	78383	80673	82962	85250
190	89821	92105	94388	96669	98950	/01229	2803507	05784	2808059
1	2812607	14879	2817150	19419	2821688	23955	26221	28486	30750
2	35274	37534	39793	42051	44307	46563	48817	51070	53322
3	57823	60071	62319	64565	66810	69054	71296	73538	75778
4	80255	82492	84728	86963	89196	91428	93660	95890	98118
5	2902573	04798	2907022	09246	2911468	13689	2915908	18127	2920344
6	24776	26990	29203	31415	33626	35835	38044	40251	42457
7	46866	49069	51271	53471	55671	57869	60067	62263	64458
8	68845	71037	73227	75417	77605	79792	81979	84164	86348
9	90713	92893	95073	97252	99429	/01605	3003781	05955	3008128
200	3012471	14641	3016809	18977	3021144	23309	25474	27637	29799
1	34121	36280	38438	40595	42751	44905	47059	49212	51363
2	55663	57812	59959	62105	64250	66394	68537	70680	72820
3	77099	79237	81374	83509	85644	87778	89910	92042	94172
4	98430	/00557	3102694	04809	3106933	09056	3111178	13300	3115420
5	3119657	21774	23889	26004	28118	30231	32343	34454	36563
6	40780	42887	44992	47097	49201	51303	53405	55505	57605
7	61801	63898	65993	68088	70181	72273	74365	76455	78545
8	82721	84807	86893	88977	91061	93143	95224	97305	99384
9	3203540	05617	3207692	09767	3211840	13913	3215984	18055	3220124
210	24261	26327	28393	30457	32521	34584	36645	38706	40766
1	44882	46939	48995	51050	53104	55157	57209	59260	61310
2	65407	67454	69500	71545	73589	75633	77675	79716	81757
3	85834	87872	89909	91944	93979	96012	98045	/00077	3302108
4	3306167	08195	3310222	12248	3314273	16297	3318320	20343	22364
5	26404	28423	30440	32457	34473	36488	38501	40514	42526
6	46548	48557	50565	52573	54579	56585	58589	60593	62596
7	66598	68598	70597	72595	74593	76589	78584	80579	82572
8	86557	88547	90537	92526	94514	96502	98488	/00473	3402458
9	3406424	08405	3410386	12366	3414345	16323	3418301	20277	22252
220	26200	28173	30145	32116	34086	36055	38023	39991	41957
1	45887	47851	49814	51776	53737	55698	57657	59615	61573
2	65486	67441	69395	71348	73300	75252	77202	79152	81100
3	84996	86942	88887	90832	92775	94718	96660	98601	3500541
4	3504419	06356	3508293	10229	3512163	14098	3516031	17963	19895
5	23755	25684	27612	29539	31465	33391	35316	37239	39162
6	43006	44926	46846	48764	50682	52599	54515	56431	58345
7	62171	64083	65994	67905	69814	71723	73630	75537	77443
8	81253	83156	85059	86961	88862	90762	92662	94560	96458
9	3600251	02146	3604041	05934	3607827	09719	3611610	13500	3615390
230	19166	21053	22939	24825	26709	28593	30476	32358	34239
1	37999	39878	41756	43634	45510	47386	49260	51134	53007
2	56751	58622	60492	62361	64230	66097	67964	69830	71695
3	75423	77285	79147	81009	82869	84728	86587	88445	90302
4	94014	95869	97723	99576	3701428	03280	3705131	06931	3708830
5	3712526	14373	3716219	18065	19909	21753	23596	25438	27279
6	30960	32799	34637	36475	38311	40147	41983	43817	45651
7	49316	51147	52977	54807	56636	58464	60292	62119	63944
8	67594	69418	71240	73063	74884	76704	78524	80343	82161
9	85796	87612	89427	91241	93055	94868	96680	98492	3800302
	1	2	3	4	5	6	7	8	9

Between 2400 = log.$^{-1}$ 3·3802112, and 3000 = log.$^{-1}$ 3·4771213.

tens.	1	2	3	4	5	6	7	8	9
240	3803922	05730	3807538	09345	3811151	12956	3814761	16565	3818368
1	21972	23773	25573	27373	29171	30969	32767	34563	36359
2	39948	41741	43534	45326	47117	48908	50698	52487	54275
3	57850	59636	61421	63206	64990	66773	68555	70337	72118
4	75678	77457	79235	81012	82789	84565	86340	88114	89888
5	93433	95205	96975	98746	3900515	02284	3904052	05819	3907585
6	3911116	12880	3914644	16407	18169	19931	21691	23452	25211
7	28727	30485	32241	33997	35752	37506	39260	41013	42765
8	46268	48018	49767	51516	53264	55011	56758	58504	60249
9	63737	65480	67223	68964	70705	72446	74185	75924	77663
250	81137	82873	84608	86343	88077	89811	91543	93275	95007
1	98467	/00196	4001925	03653	4005380	07106	4008832	10557	4012282
2	4015728	17451	19173	20894	22614	24333	26052	27771	29488
3	32921	34637	36352	38066	39780	41492	43205	44916	46627
4	50047	51755	53464	55171	56878	58584	60289	61994	63698
5	67105	68807	70508	72209	73909	75608	77307	79005	80703
6	84096	85791	87486	89180	90874	92567	94259	95950	97641
7	4101021	02710	4104398	06085	4107772	09459	4111144	12829	4114513
8	17880	19562	21244	22925	24605	26285	27964	29643	31321
9	34674	36350	38025	39700	41374	43047	44719	46391	48063
260	51404	53073	54742	56410	58077	59744	61410	63076	64741
1	68069	69732	71394	73056	74717	76377	78037	79696	81355
2	84670	86327	87983	89638	91293	92947	94601	96254	97906
3	4201208	02859	4204509	06158	4207806	09454	4211101	12748	4214394
4	17684	19328	20972	22615	24257	25898	27539	29180	30820
5	34097	35735	37372	39009	40645	42281	43916	45550	47183
6	50449	52081	53712	55342	56972	58601	60230	61858	63486
7	66739	68365	69990	71614	73238	74861	76484	78106	79727
8	82968	84588	86207	87825	89443	91060	92677	94293	95908
9	99137	/00751	4302364	03976	4305588	07199	4308809	10419	4312029
270	4315246	16853	18460	20067	21673	23278	24883	26487	28090
1	31295	32897	34498	36098	37698	39298	40896	42495	44092
2	47285	48881	50476	52071	53665	55259	56851	58444	60035
3	63217	64807	66396	67985	69573	71161	72748	74334	75920
4	79090	80675	82258	83841	85423	87005	88587	90167	91747
5	94906	96484	98062	99639	4401216	02792	4404368	05943	4407517
6	4410664	12237	4413809	15380	16951	18522	20092	21661	23230
7	26365	27932	29499	31065	32630	34195	35759	37322	38885
8	42010	43571	45132	46692	48252	49811	51370	52928	54485
9	57598	59154	60709	62264	63818	65372	66925	68477	70029
280	73131	74681	76231	77780	79329	80877	82424	83971	85517
1	88608	90153	91697	93241	94784	96327	97868	99410	4500951
2	4504031	05570	4507109	08647	4510185	11722	4513258	14794	16329
3	19399	20932	22466	23998	25531	27062	28593	30124	31654
4	34712	36241	37769	39296	40823	42349	43875	45400	46924
5	49972	51495	53018	54540	56061	57582	59102	60622	62142
6	65179	66696	68213	69730	71246	72762	74277	75791	77305
7	80332	81844	83356	84868	86378	87889	89399	90908	92417
8	95433	96940	98446	99953	4601458	02963	4604468	05972	4607475
9	4610481	11983	4613484	14985	16486	17986	19485	20984	22482
290	25477	26974	28470	29966	31461	32956	34450	35944	37437
1	40422	41914	43405	44895	46386	47875	49364	50853	52341
2	55316	56802	58288	59774	61259	62743	64227	65711	67194
3	70158	71640	73121	74601	76081	77561	79039	80518	81996
4	84950	86427	87903	89378	90853	92327	93801	95275	96748
5	99692	/01164	4702634	04105	4705575	07044	4708513	09982	4711450
6	4714384	15851	17317	18782	20247	21711	23175	24639	26102
7	29027	30488	31949	33410	34870	36329	37788	39247	40705
8	43620	45076	46533	47988	49443	50898	52352	53806	55259
9	58164	59616	61067	62518	63968	65418	66867	68316	69765
	1	2	3	4	5	6	7	8	9

Between 3000 = log.$^{-1}$ 3·4771213, and 3600 = log.$^{-1}$ 3·5563025.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
300	4772660	74107	4775553	76999	4778445	79890	4781334	82778	4784222	1446
1	87108	88550	89991	91432	92873	94313	95753	97192	98631	41
2	4801507	02945	4804381	05818	4807254	08689	4810124	11559	4812993	36
3	15859	17292	18724	20156	21587	23018	24448	25878	27307	31
4	30164	31592	33020	34446	35873	37299	38725	40150	41574	27
5	44422	45845	47268	48690	50112	51533	52954	54375	55795	22
6	58633	60052	61470	62888	64305	65722	67138	68554	69969	17
7	72798	74212	75626	77039	78451	79863	81275	82686	84097	12
8	86917	88326	89735	91144	92552	93959	95366	96773	98179	08
9	4900990	02395	4903799	05203	4906607	08010	4909412	10814	4912216	04
310	15018	16418	17818	19217	20616	22015	23413	24810	26207	1399
1	29000	30396	31791	33186	34581	35974	37368	38761	40154	95
2	42938	44329	45720	47110	48500	49890	51279	52667	54056	90
3	56831	58218	59604	60990	62375	63761	65145	66529	67913	85
4	70679	72062	73444	74825	76206	77587	78967	80347	81727	81
5	84484	85862	87240	88617	89994	91370	92746	94121	95496	77
6	98245	99619	5000992	02365	5003737	05109	5006481	07852	5009222	72
7	5011962	13332	14701	16069	17437	18805	20172	21539	22905	68
8	25637	27002	28366	29731	31094	32458	33821	35183	36545	63
9	39268	40629	41989	43349	44709	46068	47426	48785	50142	59
320	52857	54213	55569	56925	58280	59635	60990	62344	63697	55
1	66403	67755	69107	70459	71810	73160	74511	75860	77210	51
2	79907	81255	82603	83950	85297	86644	87990	89335	90680	47
3	93370	94714	96057	97400	98743	/00085	5101427	02768	5104109	43
4	5106790	08130	5109469	10808	5112147	13485	14823	16160	17497	39
5	20170	21505	22841	24175	25510	26844	28178	29511	30844	35
6	33508	34840	36171	37502	38832	40162	41491	42820	44149	30
7	46805	48133	49460	50787	52113	53439	54764	56089	57114	26
8	60062	61386	62709	64031	65354	66676	67997	69318	70639	22
9	73279	74598	75917	77236	78554	79872	81189	82507	83823	18
330	86455	87771	89086	90400	91715	93028	94342	95655	96968	14
1	99592	/00903	5202214	03525	5204835	06145	5207455	08764	5210073	10
2	5212689	13996	15303	16610	17916	19222	20528	21833	23138	06
3	25746	27050	28353	29656	30958	32260	33562	34863	36164	02
4	38765	40064	41364	42663	43961	45259	46557	47854	49151	1298
5	51744	53040	54336	55631	56925	58220	59513	60807	62100	94
6	64685	65977	67269	68560	69851	71141	72431	73721	75010	91
7	77538	78876	80163	81451	82738	84024	85311	86596	87882	87
8	90452	91736	93020	94304	95587	96870	98152	99434	5300716	83
9	5303278	04558	5305839	07118	5308398	09677	5310955	12234	13512	80
340	16066	17343	18619	19896	21171	22446	23721	24996	26270	76
1	28817	30090	31363	32635	33907	35179	36450	37721	38991	72
2	41531	42800	44069	45338	46606	47874	49141	50408	51675	68
3	54207	55473	56738	58003	59267	60532	61795	63059	64322	64
4	66847	68109	69370	70631	71892	73153	74413	75673	76932	61
5	79450	80708	81966	83223	84481	85737	86994	88250	89506	58
6	92016	93271	94525	95779	97032	98286	99538	/00791	5402043	55
7	5404546	05797	5407048	08298	5409548	10798	5412047	13296	14544	51
8	17040	18288	19535	20781	22028	23274	24519	25765	27010	47
9	29498	30742	31986	33229	34472	35714	36956	38198	39439	43
350	41921	43161	44401	45641	46880	48119	49358	50596	51834	39
1	54308	55545	56781	58018	59253	60489	61724	62958	64193	35
2	66660	67894	69126	70359	71591	72823	74055	75286	76517	32
3	78977	80207	81436	82665	83894	85123	86351	87578	88806	29
4	91259	92486	93712	94937	96162	97387	98612	99836	5501060	25
5	5503507	04730	5505952	07174	5508396	09618	5510839	12059	13280	22
6	15720	16939	18158	19377	20595	21813	23031	24248	25465	18
7	27899	29115	30330	31545	32760	33975	35189	36403	37617	15
8	40043	41256	42468	43680	44892	46103	47314	48524	49735	12
9	52154	53363	54572	55781	56989	58197	59404	60612	61881	08
	1	2	3	4	5	6	7	8	9	

Between 3600 $= \log.^{-1} 3{\cdot}5563025$, and 4200 $= \log.^{-1} 3{\cdot}6232493$.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
360	5564231	65437	5566643	67848	5569053	70257	5571461	72665	5573869	1205
1	76275	77477	78680	79881	81083	82284	83485	84686	85886	2
2	88285	89484	90683	91882	93080	94278	95476	96673	97870	1198
3	5600262	01458	5602654	03849	5605044	06239	5607433	08627	5609821	5
4	12207	13399	14592	15784	16975	18167	19358	20548	21739	1
5	24118	25308	26497	27685	28874	30062	31250	32437	33624	1189
6	35997	37183	38369	39555	40740	41925	43109	44293	45477	5
7	47844	49027	50209	51392	52573	53755	54936	56117	57298	1
8	59658	60838	62017	63196	64375	65553	66731	67909	69087	1178
9	71440	72617	73793	74969	76144	77320	78495	79669	80843	5
370	83191	84364	85537	86710	87882	89054	90226	91397	92568	2
1	94910	96080	97249	98419	99588	/00757	5701926	03094	5704262	1169
2	5706597	07764	5708930	10097	5711263	12429	13594	14759	15924	6
3	18252	19416	20580	21743	22906	24069	25231	26393	27555	3
4	29877	31038	32198	33358	34518	35678	36837	37996	39154	0
5	41471	42628	43786	44943	46099	47256	48412	49568	50723	1156
6	53033	54188	55342	56496	57650	58803	59956	61109	62261	4
7	64565	65717	66868	68019	69170	70320	71470	72620	73769	1
8	76067	77215	78363	79511	80659	81806	82953	84100	85246	1148
9	87538	88683	89828	90973	92118	93262	94406	95550	96693	5
380	98979	/00121	5801263	02405	5803547	04688	5805829	06969	5808110	2
1	5810389	11529	12668	13807	14945	16084	17222	18359	19497	1138
2	21770	22907	24043	25179	26314	27450	28585	29719	30854	5
3	33122	34255	35388	36521	37654	38786	39918	41050	42181	3
4	44443	45574	46704	47834	48963	50093	51222	52351	53479	0
5	55735	56863	57990	59117	60244	61370	62496	63622	64748	1127
6	66998	68123	69247	70371	71495	72618	73742	74865	75987	4
7	78232	79353	80475	81596	82717	83838	84958	86078	87198	1
8	89436	90555	91674	92792	93910	95028	96145	97263	98379	1118
9	5900612	01728	5902844	03959	5905075	06189	5907304	08418	5909532	6
390	11760	12873	13986	15098	16210	17322	18434	19546	20657	2
1	22878	23988	25098	26208	27318	28427	29536	30644	31753	0
2	33968	35076	36183	37290	38397	39503	40609	41715	42820	1107
3	45030	46135	47239	48344	49447	50551	51654	52757	53860	4
4	56064	57166	58268	59369	60470	61571	62671	63771	64871	1
5	67070	68169	69268	70367	71465	72563	73661	74758	75855	1098
6	78048	79145	80241	81336	82432	83527	84622	85717	86811	6
7	88999	90092	91186	92279	93371	94464	95556	96648	97739	3
8	99922	/01013	6002103	03193	6004283	05373	6006462	07551	6008640	0
9	6010817	11905	12993	14081	15168	16255	17341	18428	19514	1087
400	21686	22771	23856	24941	26025	27109	28193	29277	30361	4
1	32527	33609	34692	35774	36855	37937	39018	40099	41180	1
2	43341	44421	45500	46580	47659	48738	49816	50895	51973	1079
3	54128	55205	56282	57359	58435	59512	60587	61663	62739	6
4	64889	65963	67037	68111	69185	70259	71332	72405	73478	4
5	75622	76694	77766	78837	70909	80979	82050	83120	84191	2
6	86330	87399	88468	89537	90605	91674	92742	93809	94877	1069
7	97011	98078	99144	/00210	6101276	02342	6103407	04472	6105537	6
8	6107666	08730	6109794	10857	11921	12984	14046	15109	16171	4
9	18295	19356	20417	21478	22539	23599	24660	25720	26779	1
410	28898	29957	31015	32074	33132	34189	35247	36304	37361	1058
1	39475	40531	41587	42643	43698	44754	45809	46863	47918	5
2	50026	51080	52133	53187	54240	55292	56345	57397	58449	3
3	60552	61603	62654	63705	64755	65805	66855	67905	68954	0
4	71052	72101	73149	74197	75245	76293	77340	78387	79434	1048
5	81527	82573	83619	84665	85710	86755	87800	88845	89889	5
6	91977	93021	94064	95107	96150	97193	98235	99277	6200319	3
7	6202402	03443	6204484	05524	6206565	07605	6208645	09684	10724	1
8	12802	13840	14879	15917	16955	17992	19030	20067	21104	1038
9	23177	24213	25249	26284	27320	28355	29390	30424	31459	5
	1	2	3	4	5	6	7	8	9	

Between 4200 = log.$^{-1}$ 3·6232493, and 4800 = log.$^{-1}$ 3·6812412.

tens.	1	2	3	4	5	6	7	8	9	dif.
420	6233527	34560	6235594	36627	6237660	38693	6239725	40757	6241790	1033
1	43852	44884	45915	46945	47976	49006	50036	51066	52095	0
2	54154	55182	56211	57239	58267	59295	60322	61350	62377	1028
3	64430	65457	66483	67509	68534	69560	70585	71610	72634	5
4	74683	75707	76730	77754	78777	79800	80823	81845	82867	3
5	84911	85933	86954	87975	88996	90016	91037	92057	93076	1
6	95115	96134	97153	98172	99190	/00209	6301225	02244	6303262	1018
7	6305296	06312	6307329	08345	6309361	10377	11393	12408	13423	6
8	15452	16467	17481	18495	19508	20522	21535	22548	23560	3
9	25585	26597	27609	28620	29632	30643	31654	32664	33674	1
430	35694	36704	37713	38723	39732	40740	41749	42757	43765	1009
1	45780	46788	47795	48801	49808	50814	51820	52826	53832	6
2	55843	56848	57852	58857	59861	60865	61869	62873	63876	4
3	65882	66884	67887	68889	69891	70893	71894	72895	73897	2
4	75898	76898	77898	78898	79898	80897	81896	82895	83894	1000
5	85891	86889	87887	88884	89882	90879	91876	92872	93869	997
6	95861	96857	97852	98847	99842	/00837	6401832	02826	6403820	5
7	6405808	06802	6407795	08788	6409781	10773	11765	12758	13749	3
8	15733	16724	17715	18705	19696	20686	21676	22666	23656	0
9	25634	26623	27612	28601	29589	30577	31565	32552	33540	988
440	35514	36500	37487	38473	39459	40445	41431	42416	43401	6
1	45371	46355	47339	48323	49307	50291	51274	52257	53240	4
2	55205	56187	57169	58151	59133	60114	61095	62076	63057	1
3	65018	65998	66977	67957	68936	69915	70894	71873	72851	979
4	74808	75786	76763	77741	78718	79695	80671	81648	82624	7
5	84576	85552	86527	87502	88477	89452	90426	91401	92375	5
6	94322	95296	96269	97242	98215	99187	6500160	01132	6502104	3
7	6504047	05018	6505989	06960	6507930	08901	09871	10841	11811	0
8	13749	14719	15687	16656	17624	18593	19561	20528	21496	968
9	23431	24397	25364	26331	27297	28263	29229	30195	31160	6
450	33090	34055	35019	35984	36948	37912	38876	39839	40802	4
1	42728	43691	44653	45616	46578	47539	48501	49462	50423	2
2	52345	53306	54266	55226	56186	57145	58105	59064	60023	0
3	61941	62899	63857	64815	65773	66730	67688	68645	69602	958
4	71515	72471	73427	74383	75339	76294	77250	78205	79159	5
5	81068	82023	82977	83930	84884	85837	86790	87743	88696	3
6	90601	91553	92505	93456	94408	95359	96310	97261	98212	1
7	6600112	01062	6602012	02962	6603911	04860	6605809	06758	6607706	949
8	09603	10551	11499	12446	13393	14341	15287	16234	17181	7
9	19073	20019	20964	21910	22855	23800	24745	25690	26634	5
460	28522	29466	30410	31353	32296	33239	34182	35125	36067	3
1	37951	38893	39835	40776	41717	42658	43599	44539	45480	1
2	47360	48299	49239	50178	51117	52056	52995	53934	54872	939
3	56748	57686	58623	59560	60497	61434	62371	63307	64244	7
4	66116	67051	67987	68922	69857	70792	71727	72661	73595	5
5	75463	76397	77331	78264	79197	80130	81062	81995	82927	3
6	84791	85723	86654	87585	88516	89447	90378	91308	92239	1
7	94099	95028	95958	96887	97816	98745	99674	/00602	6701530	929
8	6703386	04314	6705242	06169	6707096	08023	6708950	09876	10802	7
9	12654	13580	14506	15431	16356	17281	18205	19130	20054	5
470	21903	22826	23750	24673	25596	26519	27442	28365	29287	3
1	31131	32053	32974	33896	34817	35738	36659	37579	38500	1
2	40340	41260	42179	43099	44018	44937	45856	46775	47693	919
3	49529	50447	51365	52283	53200	54117	55034	55951	56867	7
4	58700	59615	60531	61447	62362	63277	64192	65107	66022	5
5	67850	68764	69678	70592	71505	72418	73332	74244	75157	3
6	76982	77894	78806	79718	80629	81540	82452	83362	84273	1
7	86094	87004	87914	88824	89734	90643	91552	92461	93370	909
8	95187	96096	97004	97912	98819	99727	6800634	01541	6802448	8
9	6804262	05168	6806074	06980	6807886	08792	09697	10602	11507	6
	1	2	3	4	5	6	7	8	9	

Between 4800 = log.⁻¹ 3·6812412, and 5400 = log.⁻¹ 3·7323938.

tens.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	*dif.*
480	6813317	14222	6815126	16030	6816934	17838	6818741	19645	6820548	904
1	22354	23256	24159	25061	25963	26865	27766	28668	29569	2
2	31371	32272	33173	34073	34973	35873	36773	37673	38572	0
3	40370	41269	42168	43066	43965	44863	45761	46659	47556	898
4	49351	50248	51145	52041	52938	53834	54730	55626	56522	6
5	58313	59208	60103	60998	61892	62787	63681	64575	65469	5
6	67256	68150	69043	69936	70828	71721	72613	73506	74398	3
7	76181	77073	77964	78855	79746	80637	81528	82418	83308	1
8	85088	85978	86867	87757	88646	89535	90423	91312	92200	889
9	93977	94864	95752	96640	97527	98414	99301	/00188	6901074	7
490	6902847	03733	6904619	05505	6906390	07275	6908161/	09046	09930	5
1	11699	12584	13468	14352	15235	16119	17002	17885	18768	3
2	20534	21416	22298	23180	24062	24944	25826	26707	27588	2
3	29350	30231	31111	31991	32872	33752	34631	35511	36390	1
4	38149	39027	39906	40785	41663	42541	43419	44297	45175	879
5	46929	47806	48683	49560	50437	51313	52189	53065	53941	7
6	55692	56568	57443	58318	59193	60067	60942	61816	62690	5
7	64438	65311	66185	67058	67931	68804	69676	70549	71421	3
8	73165	74037	74909	75780	76652	77523	78394	79264	80135	2
9	81876	82746	83616	84485	85355	86224	87093	87963	88831	0
500	90569	91437	92305	93173	94041	94908	95776	96643	97510	868
1	99244/	00111	7000977	01843	7002709	03575	7004441	05307	7006172	6
2	7007902/	08767	09632	10496	11361	12225	13089	13953	14816	5
3	16543	17406	18269	19132	19995	20857	21720	22582	23444	3
4	25167	26028	26890	27751	28612	29472	30333	31193	32054	1
5	33774	34633	35493	36352	37212	38071	38930	39788	40647	0
6	42363	43221	44079	44937	45794	46652	47509	48366	49223	858
7	50936	51792	52649	53505	54360	55216	56072	56927	57782	6
8	59492	60347	61201	62055	62910	63764	64617	65471	66325	5
9	68031	68884	69737	70589	71442	72294	73146	73998	74850	3
510	76553	77405	78256	79107	79957	80808	81659	82509	83359	1
1	85059	85908	86758	87607	88456	89305	90154	91003	91851	849
2	93548	94396	95244	96091	96939	97786	98633	99480	7100327	7
3	7102020	02866	7103713	04559	7105404	06250	7107096	07941	08786	6
4	10476	11321	12165	13010	13854	14698	15542	16385	17229	4
5	18915	19759	20601	21444	22287	23129	23971	24813	25655	3
6	27339	28180	29021	29862	30703	31544	32385	33225	34065	1
7	35745	36585	37425	38264	39104	39943	40782	41620	42459	0
8	44136	44974	45812	46650	47488	48325	49162	50000	50837	838
9	52510	53347	54183	55019	55856	56691	57527	58363	59198	7
520	60869	61703	62538	63373	64207	65042	65876	66710	67544	5
1	69211	70044	70877	71710	72543	73376	74208	75041	75873	3
2	77537	78369	79200	80032	80863	81694	82525	83356	84186	1
3	85847	86677	87507	88337	89167	89996	90826	91655	92484	0
4	94142	94970	95799	96627	97455	98283	99111	99938	7200766	828
5	7202420	03247	7204074	04901	7205727	06554	7207380	08206	09032	6
6	10683	11508	12334	13159	13984	14809	15633	16458	17282	5
7	18930	19754	20578	21401	22225	23048	23871	24694	25517	4
8	27162	27984	28806	29628	30450	31272	32093	32914	33736	2
9	35378	36198	37019	37839	38660	39480	40300	41120	41939	1
530	43578	44397	45216	46035	46854	47672	48491	49309	50127	819
1	51763	52581	53398	54216	55033	55850	56667	57483	58300	7
2	59933	60749	61565	62380	63196	64012	64827	65642	66457	6
3	68087	68901	69716	70530	71344	72158	72972	73786	74599	4
4	76226	77039	77852	78664	79477	80290	81102	81914	82726	3
5	84350	85161	85972	86784	87595	88406	89216	90027	90838	1
6	92458	93268	94078	94888	95697	96507	97316	98125	98934	809
7	7300552	01360	7302168	02977	7303785	04593	7305400	06208	7307015	8
8	08630	09437	10244	11051	11857	12663	13470	14276	15082	6
9	16693	17499	18304	19109	19914	20719	21524	22329	23133	5
	1	**2**	**3**	**4**	**5**	**6**		**8**	**9**	

Between 5400 = $\log.^{-1} 3{\cdot}7323938$, and 6000 = $\log.^{-1} 3{\cdot}7781513$.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
540	7324742	25546	7326350	27153	7327957	28760	7329564	30367	7331170	804
1	32775	33578	34380	35183	35985	36787	37588	38390	39192	2
2	40794	41595	42396	43197	43997	44798	45598	46398	47198	0
3	48798	49598	50397	51196	51995	52794	53593	54392	55191	799
4	56787	57585	58383	59181	59979	60776	61574	62371	63168	8
5	64762	65558	66355	67151	67948	68744	69540	70335	71131	7
6	72722	73517	74312	75107	75902	76696	77491	78285	79079	5
7	80667	81461	82254	83048	83841	84634	85427	86220	87013	4
8	88598	89390	90182	90974	91766	92558	93350	94141	94932	2
9	96514	97305	98096	98887	99677	00467	7401257	02047	7402837	0
550	7404416	05206	7405995	06784	7407573	/08362	09151	09939	10728	789
1	12304	13092	13880	14668	15455	16243	17030	17817	18604	8
2	20177	20964	21750	22537	23323	24109	24895	25680	26466	6
3	28037	28822	29607	30392	31176	31961	32745	33530	34314	5
4	35882	36665	37449	38232	39016	39799	40582	41365	42147	4
5	43712	44495	45277	46059	46841	47622	48404	49185	49967	2
6	51529	52310	53091	53871	54652	55432	56212	56992	57772	1
7	59332	60111	60890	61670	62449	63228	64006	64785	65564	779
8	67120	67898	68676	69454	70232	71009	71787	72564	73341	8
9	74895	75672	76448	77225	78001	78777	79553	80329	81105	7
560	82656	83431	84206	84981	85756	86531	87306	88080	88854	5
1	90403	91177	91950	92724	93498	94271	95044	95817	96590	4
2	98136	98908	99681	/00453	7501225	01997	7502769	03541	7504312	2
3	7505855	06626	7507398	08168	08939	09710	10480	11251	12021	1
4	13561	14331	15101	15870	16639	17409	18178	18947	19716	0
5	21253	22022	22790	23558	24326	25094	25862	26629	27397	768
6	28932	29699	30466	31232	31999	32766	33532	34298	35065	7
7	36596	37362	38128	38893	39659	40424	41189	41954	42719	6
8	44248	45012	45777	46541	47305	48069	48832	49596	50359	4
9	51886	52649	53412	54175	54937	55700	56462	57224	57987	3
570	59510	60272	61034	61795	62556	63318	64079	64840	65600	2
1	67122	67882	68642	69402	70162	70922	71682	72442	73201	0
2	74719	75479	76237	76996	77755	78513	79272	80030	80788	759
3	82304	83062	83819	84577	85334	86091	86848	87605	88362	7
4	89875	90632	91388	92144	92900	93656	94412	95168	95923	6
5	97434	98189	98944	99699	7600453	01208	7601962	02717	7603471	4
6	7604979	05733	7606486	07240	07993	08746	09500	10253	11005	3
7	12511	13263	14016	14768	15520	16272	17024	17775	18527	2
8	20030	20781	21532	22283	23034	23784	24535	25285	26035	1
9	27536	28286	29035	29785	30534	31284	32033	32782	33531	749
580	35029	35777	36526	37274	38022	38770	39518	40266	41014	8
1	42509	43256	44003	44750	45497	46244	46991	47737	48484	7
2	49976	50722	51468	52214	52959	53705	54450	55195	55941	6
3	57430	58175	58920	59664	60409	61153	61897	62641	63385	5
4	64872	65616	66359	67102	67845	68588	69331	70074	70816	3
5	72301	73043	73785	74527	75269	76011	76752	77494	78235	2
6	79717	80458	81199	81940	82680	83421	84161	84901	85641	0
7	87121	87860	88600	89339	90079	90818	91557	92296	93035	739
8	94512	95250	95988	96727	97465	98203	98940	99678	7700416	8
9	7701890	02627	7703364	04101	7704838	05575	7706311	07048	07784	7
590	09256	09992	10728	11463	12199	12934	13670	14405	15140	6
1	16610	17344	18079	18813	19547	20282	21016	21750	22483	4
2	23951	24684	25417	26150	26884	27616	28349	29082	29815	3
3	31279	32011	32743	33475	34207	34939	35670	36402	37133	2
4	38596	39326	40057	40788	41519	42249	42979	43710	44440	1
5	45900	46629	47359	48088	48818	49547	50276	51005	51734	0
6	53191	53920	54648	55376	56104	56832	57560	58288	59016	728
7	60471	61198	61925	62652	63379	64106	64833	65559	66286	7
8	67738	68464	69190	69916	70642	71367	72093	72818	73543	6
9	74993	75718	76443	77167	77892	78616	79340	80065	80789	5
	1	2	3	4	5	6	7	8	9	

Between 6000 = $\log.^{-1}$ 3·7781513, and 6600 = $\log.^{-1}$ 3·8195439.

tens.	1	2	3	4	5	6	7	8	9	dif.
600	7782236	82960	7783683	84407	7785130	85853	7786576	87299	7788022	723
1	89467	90190	90912	91634	92356	93078	93800	94522	95243	2
2	96686	97408	98129	98850	99571	00291	7801012	01732	7802453	1
3	7803893	04613	7805333	06053	7806773	07492	08212	08931	09650	0
4	11088	11807	12526	13245	13963	14681	15400	16118	16836	718
5	18272	18989	19707	20424	21141	21859	22576	23293	24010	7
6	25443	26159	26876	27592	28308	29024	29740	30456	31171	6
7	32602	33318	34033	34748	35463	36178	36892	37607	38321	5
8	39750	40464	41178	41892	42606	43319	44033	44746	45460	4
9	46886	47599	48312	49024	49737	50450	51162	51874	52586	3
610	54010	54722	55434	56145	56857	57568	58279	58990	59701	2
1	61123	61833	62544	63254	63965	64675	65385	66095	66805	1
2	68224	68933	69643	70352	71061	71770	72479	73188	73896	709
3	75313	76021	76730	77438	78146	78854	79561	80269	80976	•8
4	82391	83098	83805	84512	85219	85926	86632	87339	88045	7
5	89457	90163	90869	91575	92281	92986	93692	94397	95102	6
6	96512	97217	97922	98626	99331	00035	7900739	01444	7902148	5
7	7903555	04259	7904963	05666	7906370	07073	07776	08479	09182	4
8	10587	11290	11992	12695	13397	14099	14801	15503	16205	2
9	17608	18309	19011	19712	20413	21114	21815	22516	23216	1
620	24617	25318	26018	26718	27418	28118	28817	29517	30217	0
1	31615	32314	33014	33712	34411	35110	35809	36507	37206	699
2	38602	39300	39998	40696	41394	42091	42789	43486	44183	8
3	45578	46274	46971	47668	48365	49061	49757	50454	51150	7
4	52542	53238	53933	54629	55324	56020	56715	57410	58105	6
5	59495	60190	60884	61579	62273	62967	63662	64356	65050	5
6	66437	67131	67824	68517	69211	69904	70597	71290	71983	4
7	73368	74060	74753	75445	76137	76829	77521	78213	78905	2
8	80288	80979	81671	82362	83053	83744	84435	85125	85816	1
9	87197	87887	88577	89267	89957	90647	91337	92027	92716	0
630	94095	94784	95473	96162	96851	97540	98228	98917	99605	689
1	8000982	01670	8002358	03046	8003734	04421	8005109	05796	8006484	8
2	07858	08545	09232	09919	10605	11292	11978	12665	13351	7
3	14723	15409	16095	16781	17466	18152	18837	19522	20208	5
4	21578	22262	22947	23632	24316	25001	25685	26369	27053	4
5	28421	29105	29789	30472	31156	31839	32522	33205	33888	3
6	35254	35937	36619	37302	37984	38666	39348	40031	40712	2
7	42076	42758	43439	44121	44802	45483	46164	46845	47526	1
8	48887	49568	50248	50929	51609	52289	52969	53649	54329	0
9	55688	56368	57047	57726	58405	59085	59764	60442	61121	679
640	62478	63157	63835	64513	65191	65869	66547	67225	67903	8
1	69258	69935	70612	71290	71967	72644	73320	73997	74674	7
2	76027	76703	77379	78055	78731	79407	80083	80759	81434	6
3	82785	83460	84136	84811	85486	86160	86835	87510	88184	5
4	89533	90207	90881	91555	92229	92903	93577	94250	94924	4
5	96270	96944	97617	98290	98962	99635	8100308	00980	8101653	3
6	8102997	03670	8104342	05013	8105685	06357	07029	07700	08372	2
7	09714	10385	11056	11727	12398	13068	13739	14409	15081	1
8	16420	17090	17760	18430	19100	19769	20439	21108	21778	0
9	23116	23785	24454	25123	25792	26460	27129	27797	28465	669
650	29802	30470	31138	31805	32473	33141	33808	34475	35143	8
1	36477	37144	37811	38478	39144	39811	40477	41144	41817	7
2	43142	43808	44474	45140	45805	46471	47136	47801	48463	6
3	49797	50462	51127	51791	52456	53120	53785	54449	55110	5
4	56441	57105	57769	58433	59097	59760	60423	61087	61750	4
5	63076	63739	64402	65064	65727	66389	67052	67714	68376	3
6	69700	70362	71024	71686	72347	73009	73670	74331	74993	2
7	76315	76976	77636	78297	78958	79618	80278	80939	81599	1
8	82919	83579	84239	84898	85558	86217	86877	87536	88195	0
9	89513	90172	90831	91489	92148	92806	93465	94123	94781	659
	1	2	3	4	5	6	7	8	9	

Between 6600 = log.⁻¹ 3·8195439, and 7200 = log.⁻¹ 3·8573325.

Tens.	1	2	3	4	5	6	7	8	9	dif.
660	8196097	96755	8197413	98071	8198728	99386	8200043	00700	8201358	658
1	8202572	03328	8203985	04642	8205298	05955	06611	07268	07924	7
2	09236	09892	10548	11203	11859	12514	13170	13825	14480	6
3	15790	16445	17100	17755	18409	19064	19718	20372	21027	5
4	22335	22989	23643	24296	24950	25603	26257	26910	27563	4
5	28869	29522	30175	30828	31481	32133	32786	33438	34090	3
6	35394	36046	36698	37350	38002	38653	39305	39956	40607	2
7	41909	42560	43211	43862	44513	45163	45814	46464	47114	1
8	48415	49065	49715	50364	51014	51664	52313	52963	53612	0
9	54910	55559	56208	56857	57506	58154	58803	59451	60100	649
670	61396	62044	62692	63340	63988	64635	65283	65931	66578	8
1	67872	68519	69166	69813	70460	71107	71753	72400	73046	7
2	74339	74985	75631	76277	76923	77569	78214	78860	79505	6
3	80796	81441	82086	82731	83376	84021	84665	85310	85955	5
4	87243	87887	88532	89176	89820	90463	91107	91751	92394	4
5	93681	94324	94967	95611	96254	96896	97539	98182	98824	3
6	8300109	00752	8301394	02036	8302678	03320	8303962	04604	8305245	2
7	06528	07159	07811	08452	09093	09734	10375	11016	11656	1
8	12937	13578	14218	14858	15499	16139	16778	17418	18058	0
9	19337	19977	20616	21255	21895	22534	23173	23812	24450	0
680	25728	26366	27005	27643	28281	28919	29558	30195	30833	639
1	32109	32746	33384	34021	34659	35296	35933	36570	37207	8
2	38480	39117	39754	40390	41027	41663	42299	42935	43571	7
3	44843	45479	46114	46750	47385	48021	48656	49291	49926	6
4	51196	51831	52465	53100	53735	54369	55003	55638	56272	5
5	57540	58174	58807	59441	60075	60708	61341	61975	62608	4
6	63874	64507	65140	65773	66405	67038	67670	68303	68935	3
7	70199	70832	71463	72095	72727	73359	73990	74622	75253	2
8	76516	77147	77778	78409	79039	79670	80301	80931	81562	1
9	82822	83453	84083	84713	85343	85973	86602	87232	87861	0
690	89120	89750	90379	91008	91637	92266	92895	93523	94152	629
1	95409	96037	96666	97294	97922	98550	99178	99806	8400433	8
2	8401683	02316	8402943	03571	8404198	04825	8405452	06079	06706	7
3	07959	08586	09212	09838	10465	11091	11717	12343	12969	7
4	14220	14846	15472	16097	16723	17348	17973	18598	19223	6
5	20473	21098	21722	22347	22971	23596	24220	24844	25468	5
6	26716	27340	27964	28588	29211	29835	30458	31081	31705	4
7	32951	33574	34197	34819	35442	36065	36687	37310	37932	3
8	39176	39798	40420	41042	41664	42286	42907	43529	44150	2
9	45393	46014	46635	47256	47877	48498	49119	49739	50360	1
700	51601	52221	52841	53461	54081	54701	55321	55941	56561	0
1	57800	58419	59038	59658	60277	60896	61515	62134	62752	619
2	63990	64608	65227	65845	66463	67081	67700	68318	68935	8
3	70171	70789	71406	72024	72641	73258	73876	74493	75110	7
4	76343	76960	77577	78193	78810	79426	80043	80659	81275	6
5	82507	83123	83739	84355	84970	85586	86201	86817	87432	5
6	88662	89277	89892	90507	91122	91736	92351	92965	93580	5
7	94808	95423	96037	96651	97264	97878	98492	99106	99719	4
8	8500946	01559	8502172	02786	8503399	04011	8504624	05237	8505850	3
9	07075	07687	08300	08912	09524	10136	10748	11360	11972	2
710	13195	13807	14418	15030	15641	16252	16863	17474	18085	1
1	19307	19917	20528	21139	21749	22359	22970	23580	24190	0
2	25410	26020	26629	27239	27849	28458	29068	29677	30286	600
3	31504	32113	32722	33331	33940	34548	35157	35765	36374	9
4	37590	38198	38807	39414	40022	40630	41238	41845	42453	8
5	43668	44275	44882	45489	46096	46703	47310	47917	48524	7
6	49737	50343	50950	51556	52162	52768	53374	53980	54586	6
7	55797	56403	57008	57614	58219	58824	59429	60035	60640	5
8	61849	62454	63059	63663	64268	64872	65476	66081	66685	5
9	67893	68497	69101	69704	70308	70912	71515	72118	72722	4
	1	2	3	4	5	6	7	8	9	

Between 7200 = log.$^{-1}$ 3·8573325, and 7800 = log.$^{-1}$ 3·8920946.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
720	8573928	74531	8575134	75737	8576340	76943	8577545	78148	8578750	603
1	79955	80557	81159	81761	82363	82965	83567	84169	84770	2
2	85973	86575	87176	87777	88379	88980	89581	90181	90782	2
3	91984	92584	93185	93785	94385	94986	95586	96186	96786	1
4	97985	98585	99185	99784	8600384	00983	8601583	02182	8602781	0
5	8603979	04578	8605177	05776	06374	06973	07571	08170	08768	599
6	09964	10562	11160	11758	12356	12954	13552	14149	14747	8
7	15941	16539	17136	17733	18330	18927	19524	20121	20717	7
8	21910	22507	23103	23699	24296	24892	25488	26084	26680	7
9	27871	28467	29062	29658	30253	30848	31443	32039	32634	6
730	33823	34418	35013	35608	36202	36797	37391	37985	38580	5
1	39768	40362	40956	41550	42143	42737	43331	43924	44517	4
2	45704	46297	46890	47483	48076	48669	49262	49855	50447	3
3	51632	52225	52817	53409	54001	54593	55185	55777	56369	2
4	57552	58144	58735	59327	59918	60509	61100	61691	62282	1
5	63464	64055	64646	65236	65827	66417	67008	67598	68188	1
6	69368	69958	70548	71138	71728	72317	72907	73496	74086	0
7	75264	75853	76442	77031	77620	78209	78798	79387	79975	589
8	81152	81740	82329	82917	83505	84093	84681	85269	85857	8
9	87032	87620	88207	88794	89382	89969	90556	91143	91730	7
740	92904	93491	94077	94664	95251	95837	96423	97010	97596	7
1	98763	99354	99940	/00526	8701112	01697	8702283	02868	8703454	6
2	8704624	05210	8705795	06380	06965	07549	08134	08719	09304	5
3	10473	11057	11641	12226	12810	13394	13978	14562	15146	4
4	16313	16897	17480	18064	18647	19230	19814	20397	20980	3
5	22146	22728	23311	23894	24476	25059	25641	26224	26806	2
6	27970	28552	29134	29716	30298	30880	31462	32043	32625	2
7	33787	34369	34950	35531	36112	36693	37274	37855	38435	1
8	39597	40177	40757	41338	41918	42498	43078	43658	44238	0
9	45398	45978	46557	47137	47716	48296	48875	49454	50034	579
750	51192	51771	52349	52928	53507	54086	54664	55243	55821	9
1	56978	57556	58134	58712	59290	59868	60446	61023	61601	8
2	62756	63333	63911	64488	65065	65642	66219	66796	67373	7
3	68526	69103	69680	70256	70833	71409	71985	72561	73137	7
4	74289	74865	75441	76017	76592	77168	77743	78319	78894	6
5	80045	80620	81195	81770	82345	82919	83494	84069	84643	5
6	85792	86367	86941	87515	88089	88663	89237	89811	90385	4
7	91532	92106	92680	93253	93826	94400	94973	95546	96119	3
8	97265	97838	98411	98983	99556	/00128	8800701	01273	8801846	3
9	8802990	03562	8804134	04706	8805278	05850	06421	06993	07564	2
760	08707	09279	09850	10421	10992	11563	12134	12705	13276	1
1	14417	14983	15553	16129	16699	17269	17840	18410	18980	0
2	20120	20689	21259	21829	22398	22968	23537	24107	24676	569
3	25815	26384	26953	27522	28090	28659	29228	29797	30365	8
4	31502	32070	32639	33207	33775	34343	34911	35479	36047	8
5	37182	37750	38317	38885	39452	40019	40586	41154	41721	7
6	42855	43421	43988	44555	45122	45688	46255	46821	47387	7
7	48520	49086	49652	50218	50784	51350	51915	52481	53047	6
8	54178	54743	55308	55874	56439	57004	57569	58134	58699	5
9	59828	60393	60957	61522	62086	62651	63215	63779	64343	4
770	65471	66035	66599	67163	67726	68290	68854	69417	69980	4
1	71107	71670	72233	72796	73359	73922	74485	75048	75610	3
2	76736	77298	77860	78423	78985	79547	80109	80671	81233	2
3	82357	82918	83480	84042	84603	85165	85726	86287	86848	2
4	87971	88532	89093	89653	90214	90775	91336	91896	92457	1
5	93577	94138	94698	95258	95818	96378	96938	97498	98058	0
6	99177	99736	8900296	00855	8901415	01974	8902533	03092	8903651	0
7	8904769	05328	05887	06445	07004	07563	08121	08679	09238	559
8	10354	10912	11470	12028	12586	13144	13702	14259	14817	8
9	15932	16489	17047	17604	18161	18718	19275	19832	20389	7
	1	2	3	4	5	6	7	8	9	

Between 7800 = log.⁻¹ 3·8920946, and 8400 = log.⁻¹ 3·9242793.

tens.	1	2	3	4	5	6	7	8	9	dif.
780	8921503	22059	8922616	23173	8923729	24285	8924842	25398	8925954	556
1	27066	27622	28178	28734	29290	29846	30401	30957	31512	6
2	32623	33178	33733	34288	34843	35398	35953	36508	37063	5
3	38172	38727	39281	39836	40390	40944	41498	42053	42607	4
4	43715	44268	44822	45376	45929	46483	47037	47590	48143	4
5	49250	49803	50356	50909	51462	52015	52568	53120	53673	3
6	54778	55330	55883	56435	56987	57539	58092	58644	59195	2
7	60299	60851	61403	61954	62506	63057	63608	64160	64711	2
8	65813	66364	66915	67466	68017	68568	69118	69669	70220	1
9	71320	71871	72421	72971	73521	74071	74621	75171	75721	0
790	76821	77370	77920	78469	79019	79568	80117	80667	81216	0
1	82314	82863	83412	83960	84509	85058	85606	86155	86703	549
2	87800	88348	88897	89445	89993	90541	91089	91636	92184	8
3	93279	93827	94375	94922	95469	96017	96564	97111	97658	7
4	98752	99299	99846	/00392	9000939	01486	9002032	02579	9003125	7
5	9004218	04764	9005310	05856	06402	06948	07494	08039	08585	6
6	09676	10222	10767	11313	11858	12403	12948	13493	14038	5
7	15128	15673	16218	16762	17307	17851	18396	18940	19485	5
8	20573	21117	21661	22205	22749	23293	23837	24381	24924	4
9	26011	26555	27098	27641	28185	28728	29271	29814	30357	4
800	31443	31985	32528	33071	33613	34156	34698	35241	35783	3
1	36867	37409	37951	38493	39035	39577	40119	40661	41202	2
2	42285	42827	43368	43909	44450	44992	45533	46074	46615	1
3	47696	48237	48778	49318	49859	50399	50940	51480	52020	1
4	53101	53641	54181	54721	55260	55800	56340	56880	57419	0
5	58498	59038	59577	60116	60655	61195	61734	62273	62812	539
6	63889	64428	64967	65505	66044	66582	67121	67659	68197	9
7	69273	69812	70350	70887	71425	71963	72501	73038	73576	8
8	74651	75188	75726	76263	76800	77337	77874	78411	78948	7
9	80022	80559	81095	81632	82169	82705	83241	83778	84314	6
810	85386	85922	86458	86994	87530	88066	88602	89137	89673	6
1	90744	91279	91815	92350	92885	93420	93955	94490	95025	5
2	96095	96630	97165	97699	98234	98768	99303	99837	9100371	5
3	9101440	01974	9102508	03042	9103576	04109	9104643	05177	05710	4
4	06778	07311	07844	08378	08911	09444	09977	10510	11043	3
5	12109	12642	13174	13707	14240	14772	15305	15837	16369	2
6	17434	17966	18498	19030	19562	20094	20626	21157	21689	2
7	22752	23284	23815	24346	24878	25409	25940	26471	27002	1
8	28064	28595	29126	29656	30187	30717	31248	31778	32309	1
9	33369	33899	34430	34960	35490	36019	36549	37079	37609	0
820	38668	39198	39727	40257	40786	41315	41844	42373	42903	529
1	43961	44489	45018	45547	46076	46604	47133	47661	48190	9
2	49246	49775	50303	50831	51359	51887	52415	52943	53471	8
3	54526	55054	55581	56109	56636	57163	57691	58218	58745	7
4	59799	60326	60853	61380	61907	62433	62960	63487	64013	7
5	65066	65592	66118	66645	67171	67697	68223	68749	69275	6
6	70326	70852	71378	71903	72429	72954	73479	74005	74530	5
7	75580	76105	76630	77155	77680	78205	78730	79254	79779	5
8	80828	81352	81877	82401	82925	83449	83973	84497	85021	4
9	86069	86593	87117	87640	88164	88687	89211	89734	90258	4
830	91304	91827	92350	92873	93396	93919	94442	94965	95488	3
1	96533	97055	97578	98100	98623	99145	99667	/00189	9200711	2
2	9201755	02277	9202799	03321	9203842	04364	9204886	05407	05929	2
3	06971	07493	08014	08535	09056	09577	10098	10619	11140	1
4	12181	12702	13222	13743	14263	14784	15304	15824	16345	0
5	17385	17905	18425	18945	19465	19984	20504	21024	21543	0
6	22582	23102	23621	24140	24659	25179	25698	26217	26736	519
7	27773	28292	28811	29330	29848	30367	30885	31404	31922	9
8	32958	33477	33995	34513	35031	35549	36066	36584	37102	8
9	38137	38655	39172	39690	40207	40724	41242	41759	42276	7
	1	2	3	4	5	6	7	8	9	

Between 8400 = $\log.^{-1}$ 3·9242793, and 9000 = $\log.^{-1}$ 3·9542425.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
840	9243310	43827	9244344	44860	9245377	45894	9246410	46927	9247444	517
1	48476	48993	49509	50025	50541	51057	51573	52089	52605	6
2	53637	54152	54668	55184	55699	56215	56730	57245	57761	6
3	58791	59306	59821	60336	60851	61366	61880	62395	62910	5
4	63939	64453	64968	65482	65997	66511	67025	67539	68053	4
5	69081	69595	70109	70622	71136	71650	72163	72677	73190	4
6	74217	74730	75243	75757	76270	76783	77296	77808	78321	3
7	79347	79859	80372	80885	81397	81909	82422	82934	83446	2
8	84471	84983	85495	86007	86518	87030	87542	88054	88565	2
9	89588	90100	90611	91123	91634	92145	92656	93167	93678	1
850	94700	95211	95722	96233	96743	97254	97764	98275	98785	1
1	99806	/00316	9300826	01336	9301847	02357	9302866	03376	9303886	0
2	9304906	05415	05925	06434	06944	07453	07963	08472	08981	0
3	09999	10508	11017	11526	12035	12544	13053	13562	14070	509
4	15087	15596	16104	16612	17121	17629	18137	18645	19153	8
5	20169	20677	21185	21692	22200	22708	23215	23723	24230	8
6	25245	25752	26259	26767	27274	27781	28288	28795	29301	7
7	30315	30822	31328	31835	32341	32848	33354	33860	34367	7
8	35379	35885	36391	36897	37403	37909	38415	38920	39426	6
9	40437	40943	41448	41953	42459	42964	43469	43974	44479	5
860	45489	45994	46499	47004	47509	48013	48518	49023	49527	5
1	50536	51040	51544	52049	52553	53057	53561	54065	54569	4
2	55576	56080	56584	57087	57591	58095	58598	59101	59605	4
3	60611	61114	61617	62120	62623	63126	63629	64132	64635	3
4	65640	66143	66645	67148	67650	68152	68655	69157	69659	3
5	70663	71165	71667	72169	72671	73172	73674	74176	74677	2
6	75680	76182	76683	77184	77686	78187	78688	79189	79690	1
7	80692	81193	81693	82194	82695	83195	83696	84196	84697	1
8	85698	86198	86698	87198	87698	88198	88698	89198	89698	0
9	90697	91197	91697	92196	92696	93195	93695	94194	94693	0
870	95692	96191	96690	97189	97688	98187	98685	99184	99683	499
1	9400680	01179	9401677	02176	9402674	03172	9403670	04169	9404667	8
2	05663	06161	06659	07157	07654	08152	08650	09147	09645	8
3	10640	11137	11635	12132	12629	13126	13623	14120	14617	7
4	15611	16108	16605	17101	17598	18095	18591	19088	19584	7
5	20577	21073	21569	22065	22562	23058	23553	24049	24545	6
6	25537	26032	26528	27024	27519	28015	28510	29005	29501	6
7	30491	30986	31481	31976	32471	32966	33461	33956	34450	5
8	35440	35934	36429	36923	37418	37912	38406	38900	39395	5
9	40383	40877	41371	41865	42358	42852	43346	43840	44333	4
880	45320	45814	46307	46800	47294	47787	48280	48773	49266	3
1	50252	50745	51238	51730	52223	52716	53208	53701	54193	3
2	55178	55671	56163	56655	57147	57639	58131	58623	59115	2
3	60099	60591	61082	61574	62066	62557	63049	63540	64031	2
4	65014	65505	65996	66487	66978	67469	67960	68451	68942	1
5	69923	70414	70905	71395	71886	72376	72866	73357	73847	1
6	74827	75317	75807	76297	76787	77277	77767	78257	78747	0
7	79726	80215	80705	81194	81684	82173	82662	83151	83641	489
8	84619	85108	85597	86085	86574	87063	87552	88040	88529	9
9	89506	89995	90483	90971	91460	91948	92436	92924	93412	8
890	94388	94876	95364	95852	96339	96827	97315	97802	98290	8
1	99264	99752	9500239	00726	9501213	01701	9502188	02675	9503162	7
2	9504135	04622	05109	05596	06082	06569	07055	07542	08028	7
3	09001	09487	09973	10459	10946	11432	11918	12404	12889	6
4	13861	14347	14832	15318	15803	16289	16774	17260	17745	6
5	18716	19201	19686	20171	20656	21141	21626	22111	22595	5
6	23565	24049	24534	25018	25503	25987	26472	26956	27440	5
7	28409	28893	29377	29861	30345	30828	31312	31796	32280	4
8	33247	33731	34214	34697	35181	35664	36147	36631	37114	4
9	38080	38563	39046	39529	40012	40494	40977	41460	41943	3
	1	2	3	4	5	6	7	8	9	

Between 9000 = $\log.^{-1}$ 3·9542425, and 9600 = $\log.^{-1}$ 3·9822712.

tens.	1	2	3	4	5	6	7	8	9	dif.
900	9542908	43390	9543873	44355	9544837	45319	9545802	46284	9546766	482
1	47730	48212	48694	49176	49657	50139	50621	51102	51584	2
2	52547	53028	53510	53991	54472	54953	55434	55916	56397	1
3	57358	57839	58320	58801	59282	59762	60246	60723	61204	1
4	62165	62645	63125	63606	64086	64566	65046	65526	66006	0
5	66966	67445	67925	68405	68885	69364	69844	70323	70803	0
6	71761	72241	72720	73199	73678	74157	74636	75115	75594	479
7	76552	77030	77509	77988	78466	78945	79423	79902	80380	9
8	81337	81815	82293	82771	83249	83727	84205	84683	85161	8
9	86117	86594	87072	87549	88027	88505	88982	89459	89937	8
910	90891	91368	91845	92322	92800	93276	93753	94230	94707	7
1	95660	96137	96614	97090	97567	98043	98520	98996	99472	7
2	9600425	00901	9601377	01853	9602329	02805	9603281	03756	9604232	6
3	05183	05659	06135	06610	07086	07561	08036	08512	08987	6
4	09937	10412	10887	11362	11837	12312	12787	13262	13736	5
5	14686	15160	15635	16109	16583	17058	17532	18006	18481	5
6	19429	19903	20377	20851	21325	21799	22272	22746	23220	4
7	24167	24640	25114	25587	26061	26534	27007	27481	27954	4
8	28900	29373	29846	30319	30792	31264	31737	32210	32683	3
9	33628	34100	34573	35045	35517	35990	36462	36934	37406	2
920	38350	38822	39294	39766	40238	40710	41181	41653	42125	1
1	43068	43539	44011	44482	44953	45425	45896	46367	46838	2
2	47780	48251	48722	49193	49664	50135	50605	51076	51546	1
3	52488	52958	53428	53899	54369	54839	55309	55780	56250	0
4	57190	57660	58130	58599	59069	59539	60009	60478	60948	0
5	61887	62356	62826	63295	63764	64233	64703	65172	65641	469
6	66579	67048	67517	67985	68454	68923	69392	69860	70329	9
7	71266	71734	72203	72671	73139	73607	74076	74544	75012	8
8	75948	76416	76884	77351	77819	78287	78754	79222	79690	8
9	80625	81092	81559	82027	82494	82961	83428	83895	84362	7
930	85296	85763	86230	86697	87164	87630	88097	88564	89030	7
1	89963	90430	90896	91362	91829	92295	92761	93227	93693	6
2	94625	95091	95557	96023	96488	96954	97420	97885	98351	6
3	99282	99747	9700213	00678	9701143	01608	9702074	02539	9703004	5
4	9703934	04399	04863	05328	05793	06258	06722	07187	07652	5
5	08581	09045	09509	09974	10438	10902	11366	11830	12294	4
6	13222	13686	14150	14614	15078	15542	16005	16469	16932	4
7	17859	18323	18786	19249	19713	20176	20639	21102	21565	3
8	22491	22954	23417	23880	24343	24805	25268	25731	26193	3
9	27118	27581	28043	28506	28968	29430	29892	30354	30816	2
940	31741	32202	32664	33126	33588	34050	34511	34973	35435	2
1	36358	36819	37281	37742	38203	38664	39126	39587	40048	1
2	40970	41431	41892	42353	42814	43274	43735	44196	44656	1
3	45577	46038	46498	46959	47419	47879	48340	48800	49260	0
4	50180	50640	51100	51560	52020	52479	52939	53399	53858	0
5	54778	55237	55697	56156	56615	57075	57534	57993	58452	459
6	59370	59829	60288	60747	61206	61665	62124	62582	63041	9
7	63958	64417	64875	65334	65792	66251	66709	67167	67625	8
8	68541	69000	69458	69915	70373	70831	71289	71747	72204	8
9	73120	73577	74035	74492	74950	75407	75864	76322	76779	7
950	77693	78150	78607	79064	79521	79978	80435	80892	81348	7
1	82262	82718	83175	83631	84088	84544	85001	85457	85913	6
2	86826	87282	87738	88194	88650	89106	89562	90017	90473	6
3	91385	91840	92296	92751	93207	93662	94118	94573	95028	6
4	95939	96394	96849	97304	97759	98214	98669	99124	99579	5
5	9800488	00943	9801398	01852	9802307	02761	9803216	03670	9804125	5
6	05033	05487	05942	06396	06850	07304	07758	08212	08666	4
7	09573	10027	10481	10934	11388	11841	12295	12748	13202	4
8	14108	14562	15015	15468	15921	16374	16827	17280	17733	3
9	18639	19092	19544	19997	20450	20902	21355	21807	22260	3
	1	2	3	4	5	6	7	8	9	

Between 9600 = log.$^{-1}$ 3·9822712, and 10200 = log.$^{-1}$ 4·0086002.

tens.	1	2	3	4	5	6	7	8	9	dif.
960	9823165	23617	9824069	24522	9824974	25426	9825878	26330	9826782	452
1	27686	28138	28589	29041	29493	29945	30396	30848	31299	2
2	32202	32654	33105	33556	34007	34459	34910	35361	35812	1
3	36714	37165	37616	38066	38517	38968	39419	39869	40320	1
4	41221	41671	42122	42572	43022	43473	43923	44373	44823	0
5	45723	46173	46623	47073	47523	47973	48422	48872	49322	0
6	50221	50670	51120	51569	52019	52468	52917	53366	53816	449
7	54714	55163	55612	56061	56510	56959	57407	57856	58305	9
8	59202	59651	60099	60548	60996	61445	61893	62341	62790	8
9	63686	64134	64582	65030	65478	65926	66374	66822	67270	8
970	68165	68613	69060	69508	69955	70403	70850	71298	71745	8
1	72640	73087	73534	73981	74428	74875	75322	75769	76216	7
2	77109	77556	78003	78450	78896	79343	79789	80236	80682	7
3	81575	82021	82467	82913	83360	83806	84252	84698	85144	6
4	86035	86481	86927	87373	87818	88264	88710	89155	89601	6
5	90492	90937	91382	91828	92273	92718	93163	93608	94053	5
6	94943	95388	95833	96278	96722	97167	97612	98057	98501	5
7	99390	99835	9900279	00723	9901168	01612	9902056	02500	9902944	4
8	9903833	04277	04721	05164	05608	06052	06496	06940	07383	4
9	08271	08714	09158	09601	10044	10488	10931	11374	11818	3
980	12704	13147	13590	14033	14476	14919	15362	15805	16247	3
1	17133	17575	18018	18461	18903	19345	19788	20230	20673	3
2	21557	21999	22441	22884	23326	23768	24210	24651	25093	2
3	25977	26419	26860	27302	27744	28185	28627	29068	29510	2
4	30392	30834	31275	31716	32157	32598	33039	33480	33921	1
5	34803	35244	35685	36126	36566	37007	37448	37888	38329	1
6	39210	39650	40090	40531	40971	41411	41851	42291	42731	0
7	43612	44051	44491	44931	45371	45811	46251	46690	47130	0
8	48009	48448	48888	49327	49767	50206	50645	51085	51524	439
9	52402	52841	53280	53719	54158	54597	55036	55474	55913	9
990	56791	57229	57668	58106	58545	58983	59422	59860	60298	8
1	61175	61613	62051	62489	62927	63365	63803	64241	64679	8
2	65544	65992	66430	66868	67305	67743	68180	68618	69055	8
3	69930	70367	70804	71242	71679	72116	72553	72990	73427	7
4	74301	74738	75174	75611	76048	76485	76921	77358	77794	7
5	78667	79104	79540	79976	80413	80849	81285	81721	82157	6
6	83029	83465	83901	84337	84773	85209	85645	86080	86516	6
7	87387	87823	88258	88694	89129	89564	90000	90435	90870	5
8	91741	92176	92611	93046	93481	93916	94350	94785	95220	5
9	96090	96524	96959	97393	97828	98262	98697	99131	99566	4
1000	0000434	00869	0001303	01737	0002171	02605	0003039	03473	0003907	4
1	04775	05208	05642	06076	06510	06943	07377	07810	08244	4
2	09111	09544	09977	10411	10844	11277	11710	12143	12576	3
3	13442	13875	14308	14741	15174	15607	16039	16472	16905	3
4	17770	18202	18635	19067	19499	19932	20364	20796	21228	2
5	22093	22525	22957	23389	23821	24253	24685	25116	25548	2
6	26411	26843	27275	27706	28138	28569	29001	29432	29863	1
7	30726	31157	31588	32019	32451	32882	33313	33744	34174	1
8	35036	35467	35898	36328	36759	37190	37620	38051	38481	1
9	39342	39772	40203	40633	41063	41493	41924	42354	42784	0
1010	43644	44074	44504	44933	45363	45793	46223	46652	47082	0
1	47941	48371	48800	49229	49659	50088	50517	50947	51376	429
2	52234	52663	53092	53521	53950	54379	54808	55237	55666	9
3	56523	56952	57380	57809	58238	58666	59094	59523	59951	8
4	60808	61236	61664	62092	62521	62949	63377	63805	64233	8
5	65088	65516	65944	66372	66799	67227	67655	68082	68510	8
6	69365	69792	70219	70647	71074	71501	71928	72355	72782	7
7	73637	74064	74490	74917	75344	75771	76198	76624	77051	7
8	77904	78331	78757	79184	79610	80037	80463	80889	81316	6
9	82168	82594	83020	83446	83872	84298	84724	85150	85576	6
	1	2	3	4	5	6	7	8	9	

Between 10200 = $\log.^{-1}$ 4·0086002, and 10800 = $\log.^{-1}$ 4·0334238.

tens.	1	2	3	4	5	6	7	8	9	*dif*
1020	0086427	86853	0087279	87704	0088130	88556	0088981	89407	0089832	426
1	90683	91108	91533	91959	92384	92809	93234	93659	94084	5
2	94934	95359	95784	96208	96633	97058	97483	97907	98332	5
3	99181	99605	0100030	00454	0100878	01303	0101727	02151	0102575	4
4	0103424	03848	04272	04696	05120	05544	05967	06391	06815	4
5	07662	08086	08510	08933	09357	09780	10204	10627	11050	4
6	11897	12320	12743	13166	13590	14013	14436	14859	15282	3
7	16127	16550	16973	17396	17818	18241	18664	19086	19509	3
8	20354	20776	21198	21621	22043	22465	22887	23310	23732	2
9	24576	24998	25420	25842	26264	26685	27107	27529	27951	2
1030	28794	29215	29637	30059	30480	30901	31323	31744	32165	2
1	33008	33429	33850	34271	34692	35113	35534	35955	36376	1
2	37218	37639	38059	38480	38901	39321	39742	40162	40583	1
3	41424	41844	42264	42685	43105	43525	43945	44365	44785	0
4	45625	46045	46465	46885	47305	47725	48144	48564	48984	0
5	49823	50243	50662	51082	51501	51920	52340	52759	53178	419
6	54017	54436	54855	55274	55693	56112	56531	56950	57369	8
7	58206	58625	59044	59462	59881	60300	60718	61137	61555	9
8	62392	62810	63229	63647	64065	64483	64901	65319	65737	8
9	66573	66991	67409	67827	68245	68663	69080	69498	69916	8
1040	70751	71168	71586	72003	72421	72838	73256	73673	74090	7
1	74924	75342	75759	76176	76593	77010	77427	77844	78260	7
2	79094	79511	79927	80344	80761	81177	81594	82010	82427	7
3	83259	83676	84092	84508	84925	85341	85757	86173	86589	6
4	87421	87837	88253	88669	89084	89500	89916	90332	90747	6
5	91578	91994	92410	92825	93240	93656	94071	94486	94902	5
6	95732	96147	96562	96977	97392	97807	98222	98637	99052	5
7	99882	/00296	0200711	01126	0201540	01955	0202369	02784	0203198	5
8	0204027	04442	04856	05270	05684	06099	06513	06927	07341	4
9	08169	08583	08997	09411	09824	10238	10652	11066	11479	4
1050	12307	12720	13134	13547	13961	14374	14787	15201	15614	3
1	16440	16854	17267	17680	18093	18506	18919	19332	19745	3
2	20570	20983	21396	21808	22221	22634	23046	23459	23871	3
3	24696	25109	25521	25933	26345	26758	27170	27582	27994	2
4	28818	29230	29642	30054	30466	30878	31289	31701	32113	2
5	32936	33348	33759	34171	34582	34994	35405	35817	36228	2
6	37050	37462	37873	38284	38695	39106	39517	39928	40339	1
7	41161	41572	41982	42393	42804	43214	43625	44036	44446	1
8	45267	45678	46088	46498	46909	47319	47729	48139	48549	0
9	49370	49780	50190	50600	51010	51419	51829	52239	52649	0
1060	53468	53878	54288	54697	55107	55516	55926	56335	56744	409
1	57563	57972	58382	58791	59200	59609	60018	60427	60836	9
2	61654	62063	62472	62881	63289	63698	64107	64515	64924	9
3	65741	66150	66558	66967	67375	67783	68192	68600	69008	8
4	69824	70233	70641	71049	71457	71865	72273	72680	73088	8
5	73904	74312	74719	75127	75535	75942	76350	76757	77165	8
6	77979	78387	78794	79201	79609	80016	80423	80830	81237	7
7	82051	82458	82865	83272	83679	84086	84492	84899	85306	7
8	86119	86526	86932	87339	87745	88152	88558	88964	89371	7
9	90183	90590	90996	91402	91808	92214	92620	93026	93432	6
1070	94244	94649	95055	95461	95867	96272	96678	97084	97489	6
1	98300	98706	99111	99516	99922	/00327	0300732	01138	0301543	6
2	0302353	02758	0303163	03568	0303973	04378	04783	05188	05592	5
3	06402	06807	07211	07616	08020	08425	08830	09234	09638	5
4	10447	10851	11256	11660	12064	12468	12872	13277	13681	4
5	14489	14893	15296	15700	16104	16508	16912	17315	17719	4
6	18526	18930	19333	19737	20140	20544	20947	21350	21754	4
7	22560	22963	23367	23770	24173	24576	24979	25382	25785	3
8	26590	26993	27396	27799	28201	18604	29007	29409	29812	3
9	30617	31019	31422	31824	32226	32629	33031	33433	33835	2
	1	2	3	4	5	6	7	8	9	

Between 10800 = log.⁻¹ 4·0334236, and 11400 = log.⁻¹ 4·0569049.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
1080	0334640	35042	0335444	35846	0336248	36650	0337052	37453	0337855	402
1	38659	39060	39462	39864	40265	40667	41068	41470	41871	2
2	42674	43075	43477	43878	44279	44680	45081	45482	45884	1
3	46686	47087	47487	47888	48289	48690	49091	49491	49892	1
4	50693	51094	51495	51895	52296	52696	53096	53497	53897	0
5	54698	55098	55498	55898	56298	56698	57098	57498	57898	0
6	58698	59098	59498	59898	60297	60697	61097	61496	61896	0
7	62695	63094	63494	63893	64293	64692	65091	65491	65890	399
8	66688	67087	67486	67885	68284	68683	69082	69481	69880	9
9	70678	71076	71475	71874	72272	72671	73070	73468	73867	9
1090	74663	75062	75460	75858	76257	76655	77053	77451	77849	8
1	78646	79044	79442	79839	80237	80635	81033	81431	81829	8
2	82624	83022	83419	83817	84214	84612	85009	85407	85804	7
3	86599	86996	87393	87791	88188	88585	88982	89379	89776	7
4	90570	90967	91364	91761	92158	92554	92951	93348	93745	7
5	94538	94934	95331	95727	96124	96520	96917	97313	97709	6
6	98502	98898	99294	99690	0400086	00482	0400878	01274	0401670	6
7	0402462	02858	0403254	03650	04045	04441	04837	05232	05628	6
8	06419	06814	07210	07605	08001	08396	08791	09187	09582	5
9	10372	10767	11162	11557	11952	12347	12742	13137	13532	5
1100	14322	14716	15111	15506	15900	16295	16690	17084	17479	5
1	18268	18662	19056	19451	19845	20239	20633	21028	21422	4
2	22210	22604	22998	23392	23786	24180	24574	24968	25361	4
3	26149	26543	26936	27330	27723	28117	28510	28904	29297	4
4	30084	30477	30871	31264	31657	32050	32444	32837	33230	3
5	34016	34409	34802	35195	35587	35980	36373	36766	37159	3
6	37944	38337	38729	39122	39514	39907	40299	40692	41084	3
7	41869	42261	42653	43045	43437	43829	44222	44614	45006	2
8	45790	46181	46573	46965	47357	47749	48140	48532	48924	2
9	49707	50099	50490	50882	51273	51664	52056	52447	52839	2
1110	53621	54012	54403	54795	55186	55577	55968	56359	56750	1
1	57531	57922	58313	58704	59095	59485	59876	60267	60657	1
2	61438	61829	62219	62610	63000	63391	63781	64171	64561	0
3	65342	65732	66122	66512	66902	67292	67682	68072	68462	0
4	69242	69632	70021	70411	70801	71190	71580	71970	72359	0
5	73138	73528	73917	74306	74696	75085	75474	75864	76253	389
6	77031	77420	77809	78198	78587	78976	79365	79754	80143	9
7	80921	81309	81698	82087	82475	82864	83253	83641	84030	9
8	84806	85195	85583	85972	86360	86748	87136	87525	87913	8
9	88689	89077	89465	89853	90241	90629	91017	91405	91792	8
1120	92568	92956	93343	93731	94119	94506	94894	95281	95669	8
1	96444	96831	97218	97606	97993	98380	98767	99154	99541	7
2	0500316	00703	0501090	01477	0501863	02250	0502637	03024	0503411	7
3	04184	04571	04958	05344	05731	06117	06504	06890	07277	7
4	08049	08436	08822	09208	09595	09981	10367	10753	11139	6
5	11911	12297	12683	13069	13455	13841	14227	14612	14998	6
6	15770	16155	16541	16926	17312	17697	18083	18468	18854	6
7	19624	20010	20395	20780	21166	21551	21036	22321	22706	5
8	23476	23861	24246	24631	25016	25400	25785	26170	26555	5
9	27324	27709	28093	28478	28862	29247	29631	30016	30400	5
1130	31169	31553	31937	32321	32706	33090	33474	33858	34242	4
1	35010	35394	35778	36162	36546	36929	37313	37697	38081	4
2	38848	39232	39615	39999	40382	40766	41149	41532	41916	4
3	42682	43066	43449	43832	44215	44598	44981	45365	45748	3
4	46514	46896	47279	47662	48045	48428	48811	49193	49576	3
5	50341	50724	51106	51489	51871	52254	52636	53019	53401	3
6	54166	54548	54930	55312	55694	56077	56459	56841	57223	2
7	57987	58369	58750	59132	59514	59896	60278	60659	61041	2
8	61804	62186	62567	62949	63330	63712	64093	64475	64856	1
9	65619	66000	66381	66762	67143	67524	67905	68287	68668	1
	1	2	3	4	5	6	7	8	9	

Between 11400 = log.$^{-1}$ 4·0569049, and 12000 = log.$^{-1}$ 4·0791812.

tens.	1	2	3	4	5	6	7	8	9	dif.
1140	0569429	69810	0570191	70572	0570953	71334	0571714	72095	0572476	381
1	73237	73618	73998	74379	74759	75140	75520	75900	76281	1
2	77041	77422	77802	78182	78562	78942	79322	79702	80082	0
3	80842	81222	81602	81982	82362	82741	83121	83501	83881	0
4	84640	85019	85399	85778	86158	86537	86917	87296	87676	0
5	88434	88813	89193	89572	89951	90330	90709	91088	91467	379
6	92225	92604	92983	93362	93741	94119	94498	94877	95256	9
7	96013	96391	96770	97148	97527	97905	98284	98662	99041	9
8	99797	00175	0600554	00932	0601310	01688	0602066	02444	0602822	8
9	0603578	03956	04334	04712	05090	05468	05845	06223	06601	8
1150	07356	07734	08111	08489	08866	09244	09621	09999	10376	8
1	11131	11508	11885	12262	12639	13017	13394	13771	14148	7
2	14902	15279	15656	16032	16409	16786	17163	17540	17916	7
3	18670	19046	19423	19799	20176	20552	20929	21305	21682	7
4	22434	22811	23187	23563	23939	24316	24692	25068	25444	6
5	26196	26572	26948	27324	27699	28075	28451	28827	29203	6
6	29954	30330	30705	31081	31456	31832	32207	32583	32958	6
7	33709	34084	34460	34835	35210	35585	35960	36335	36711	5
8	37461	37836	38211	38585	38960	39335	39710	40085	40460	5
9	41209	41584	41958	42333	42708	43082	43457	43831	44205	5
1160	44954	45329	45703	46077	46451	46826	47200	47574	47948	4
1	48696	49070	49444	49818	50192	50566	50940	51314	51688	4
2	52435	52809	53182	53556	53930	54303	54677	55050	55424	4
3	56171	56544	56917	57291	57664	58037	58410	58784	59157	3
4	59903	60276	60649	61022	61395	61768	62141	62514	62886	3
5	63632	64005	64377	64750	65123	65495	65868	66241	66613	3
6	67358	67730	68103	68475	68847	69220	69592	69964	70336	2
7	71081	71453	71825	72197	72569	72941	73313	73685	74057	2
8	74800	75172	75544	75915	76287	76659	77030	77402	77774	2
9	78517	78888	79259	79631	80002	80374	80745	81116	81487	1
1170	82230	82601	82972	83343	83714	84085	84456	84827	85198	1
1	85940	86311	86681	87052	87423	87794	88164	88535	88906	1
2	89647	90017	90388	90758	91129	91499	91869	92240	92610	0
3	93350	93721	94091	94461	94831	95201	95571	95941	96311	0
4	97051	97421	97791	98160	98530	98900	99270	99639	0700009	0
5	0700748	01118	0701487	01857	0702226	02596	0702965	03335	03704	369
6	04442	04812	05181	05550	05919	06288	06658	07027	07396	9
7	08134	08503	08871	09240	09609	09978	10347	10715	11084	9
8	11822	12190	12559	12927	13296	13664	14033	14401	14770	9
9	15506	15875	16243	16611	16979	17348	17716	18084	18452	8
1180	19188	19556	19924	20292	20660	21028	21396	21763	22131	8
1	22867	23234	23602	23970	24337	24705	25072	25440	25807	8
2	26542	26910	27277	27644	28011	28379	28746	29113	29480	7
3	30215	30582	30949	31316	31683	32050	32416	32783	33150	7
4	33884	34251	34617	34984	35351	35717	36084	36450	36817	7
5	37550	37916	38283	38649	39016	39382	39748	40114	40481	6
6	41213	41579	41945	42311	42677	43043	43409	43775	44141	6
7	44873	45239	45605	45970	46336	46702	47068	47433	47799	6
8	48530	48895	49261	49626	49992	50357	50723	51088	51453	5
9	52184	52549	52914	53279	53644	54010	54375	54740	55105	5
1190	55835	56199	56564	56929	57294	57659	58024	58388	58753	5
1	59482	59847	60211	60576	60940	61305	61669	62034	62398	4
2	63127	63491	63855	64220	64584	64948	65312	65676	66040	4
3	65768	67132	67496	67860	68224	68588	68952	69316	69680	4
4	70407	70771	71134	71498	71862	72225	72589	72952	73316	4
5	74042	74406	74769	75133	75496	75859	76222	76585	76949	3
6	77675	78038	78401	78764	79127	79490	79853	80216	80579	3
7	81304	81667	82030	82393	82755	83118	83480	83843	84206	3
8	84931	85293	85656	86018	86380	86743	87105	87467	87830	2
9	88554	88916	89278	89640	90003	90365	90727	91089	91451	2
	1	2	3	4	5	6	7	8	9	

Between 12000 = $\log.^{-1}$ 4·0791812, and 12600 = $\log.^{-1}$ 4·1003705.

tens.	1	2	3	4	5	6	7	8	9	*dif*
1200	0792174	92536	0792898	93260	0793622	93983	0794345	94707	0795068	362
1	95792	96153	96515	96876	97238	97599	97961	98322	98683	1
2	99406	99767	0800128	00490	0800851	01212	0801573	01934	0802295	1
3	0803017	03378	03739	04100	04461	04822	05183	05543	05904	1
4	06626	06986	07347	07707	08068	08429	08789	09150	09510	1
5	10231	10591	10952	11312	11672	12032	12393	12753	13113	0
6	13833	14193	14553	14913	15273	15633	15993	16353	16713	0
7	17432	17792	18152	18512	18871	19231	19591	19950	20310	0
8	21029	21388	21748	22107	22467	22826	23185	23545	23904	359
9	24622	24981	25341	25700	26059	26418	26777	27136	27495	9
1210	28213	28571	28930	29289	29648	30007	30365	30724	31083	9
1	31800	32159	32517	32876	33234	33593	33951	34309	34668	8
2	35385	35743	36101	36459	36817	37176	37534	37892	38250	8
3	38966	39324	39682	40040	40398	40756	41114	41471	41829	8
4	42545	42902	43260	43618	43975	44333	44690	45048	45405	8
5	46120	46478	46835	47192	47550	47907	48264	48621	48979	7
6	49693	50050	50407	50764	51121	51478	51835	52192	52549	7
7	53263	53619	53976	54333	54690	55046	55403	55760	56116	7
8	56829	57186	57542	57899	58255	58612	58968	59324	59681	6
9	60393	60750	61106	61462	61818	62174	62530	62886	63242	6
1220	63954	64310	64666	65022	65378	65734	66089	66445	66801	6
1	67512	67868	68224	68579	68935	69290	69646	70001	70357	6
2	71067	71423	71778	72133	72489	72844	73199	73554	73909	5
3	74620	74975	75330	75685	76040	76395	76750	77104	77459	5
4	78169	78524	78878	79233	79588	79943	80297	80652	81006	5
5	81715	82070	82424	82779	83133	83488	83842	84196	84550	4
6	85259	85613	85967	86321	86676	87030	87384	87738	88092	4
7	88800	89153	89507	89861	90215	90569	90923	91276	91630	4
8	92337	92691	93045	93398	93752	94105	94459	94812	95165	4
9	95872	96226	96579	96932	97285	97639	97992	98345	98698	3
1230	99404	99757	0900110	00463	0900816	01169	0901522	01875	0902228	3
1	0902933	03286	03639	03991	04344	04697	05049	05402	05755	3
2	06460	06812	07164	07517	07869	08222	08574	08926	09279	2
3	09983	10335	10687	11039	11392	11744	12096	12448	12800	2
4	13504	13855	14207	14559	14911	15263	15614	15966	16318	2
5	17021	17373	17724	18076	18427	18779	19130	19482	19833	2
6	20536	20887	21239	21590	21941	22292	22644	22995	23346	1
7	24048	24399	24750	25101	25452	25803	26154	26505	26856	1
8	27557	27908	28259	28609	28960	29311	29661	30012	30363	1
9	31064	31414	31764	32115	32465	32816	33166	33516	33867	0
1240	34567	34917	35267	35618	35968	36318	36668	37018	37368	0
1	38068	38418	38768	39117	39467	39817	40167	40517	40866	0
2	41566	41915	42265	42614	42964	43313	43663	44012	44362	349
3	45061	45410	45759	46109	46458	46807	47156	47506	47855	9
4	48553	48902	49251	49600	49949	50298	50647	50996	51345	9
5	52042	52391	52740	53089	53437	53786	54135	54483	54832	9
6	55529	55877	56226	56574	56923	57271	57620	57968	58316	8
7	59013	59361	59709	60057	60406	60754	61102	61450	61798	8
8	62494	62842	63190	63538	63885	64233	64581	64929	65277	8
9	65972	66320	66667	67015	67363	67710	68058	68405	68753	8
1250	69448	69795	70142	70490	70837	71184	71531	71879	72226	7
1	72920	73267	73614	73962	74309	74656	75003	75349	75696	7
2	76390	76737	77084	77431	77777	78124	78471	78817	79164	7
3	79857	80204	80550	80897	81243	81590	81936	82283	82629	7
4	83322	83668	84014	84360	84707	85053	85399	85745	86091	6
5	86783	87129	87475	87821	88167	88513	88859	89205	89551	6
6	90242	90588	90934	91279	91625	91971	92316	92662	93007	6
7	93698	94044	94389	94735	95080	95425	95771	96116	96461	5
8	97152	97497	97842	98187	98532	98877	99222	99567	99912	5
9	1000602	00947	1001292	01637	1001982	02327	1002671	03016	1003361	5
	1	2	3	4	5	6	7	8	9	

Between 12600 = log.$^{-1}$ 4·1003705, and 13200 = log.$^{-1}$ 4·1205739.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
1260	1004050	04395	1004739	05084	1005429	05773	1006118	06462	1006806	345
1	07495	07840	08184	08528	08873	09217	09561	09905	10249	4
2	10938	11282	11626	11970	12314	12658	13002	13346	13690	4
3	14377	14721	15065	15409	15752	16096	16440	16784	17127	4
4	17814	18158	18501	18845	19188	19532	19875	20219	20562	3
5	21249	21592	21935	22278	22621	22965	23308	23651	23994	3
6	24680	25023	25366	25709	26052	26395	26738	27081	27423	3
7	28109	28452	28794	29137	29480	29822	30165	30507	30850	3
8	31535	31877	32220	32562	32905	33247	33589	33932	34274	2
9	34958	35301	35643	35985	36327	36669	37011	37353	37695	2
1270	38379	38721	39063	39405	39747	40089	40430	40772	41114	2
1	41797	42139	42480	42822	43164	43505	43847	44188	44530	2
2	45213	45554	45895	46237	46578	46919	47260	47602	47943	1
3	48625	48966	49307	49648	49989	50331	50671	51012	51353	1
4	52035	52376	52717	53058	53398	53739	54080	54421	54761	1
5	55442	55783	56124	56464	56805	57145	57486	57826	58166	0
6	58847	59187	59528	59868	60208	60548	60889	61229	61569	0
7	62249	62589	62929	63269	63609	63949	64289	64629	64969	0
8	65648	65988	66328	66668	67007	67347	67687	68026	68366	0
9	69045	69385	69724	70063	70403	70742	71082	71421	71760	339
1280	72439	72778	73117	73457	73796	74135	74474	74813	75152	9
1	75830	76169	76508	76847	77186	77525	77864	78203	78541	9
2	79219	79558	79896	80235	80574	80912	81251	81590	81928	9
3	82605	82944	83282	83620	83959	84297	84635	84974	85312	8
4	85988	86327	86665	87003	87341	87679	88017	88355	88693	8
5	89369	89707	90045	90883	90721	91059	91396	91734	92072	8
6	92747	93085	93423	93760	94098	94435	94773	95111	95488	8
7	96123	96460	96798	97135	97472	97810	98147	98484	98821	7
8	99496	99833	1100170	00507	1100844	01181	1101518	01855	1102192	7
9	1102866	03203	03540	03877	04213	04550	04887	05224	05560	7
1290	06234	06570	06907	07244	07580	07917	08253	08590	08926	7
1	09599	99935	10272	10608	10944	11280	11617	11953	12289	6
2	12961	13297	13633	13969	14306	14642	14977	15313	15649	6
3	16321	16657	16993	17329	17664	18000	18336	18671	19007	6
4	19678	20014	20350	20685	21021	21356	21691	22027	22362	6
5	23033	23368	23704	24039	24374	24709	25045	25380	25715	5
6	26385	26720	27055	27390	27725	28060	28395	28730	29065	5
7	29735	30069	30404	30739	31074	31408	31743	32078	32412	5
8	33081	33416	33751	34085	34420	34754	35088	35423	35757	4
9	36426	36760	37094	37429	37763	38097	38431	38765	39099	4
1300	39768	40102	50436	40770	41104	41437	41771	42105	42439	4
1	43107	43441	43774	44108	44442	44775	45109	45443	45776	4
2	46443	46777	47110	47444	47777	84111	48444	48777	49111	3
3	49777	50111	50444	50777	51110	51444	51777	52110	52443	3
4	53109	53442	53775	54108	54441	54774	55107	55439	55772	3
5	56438	56771	57103	57436	57769	58101	58434	58767	59099	3
6	59764	60097	60429	60762	61094	61427	61759	62091	62424	2
7	63088	63420	63753	64085	64417	64749	65081	65413	65745	2
8	66409	66741	67073	67405	67737	68069	68401	68733	69065	2
9	69728	70060	70392	70723	71055	71387	71718	72050	72381	2
1310	73044	73376	73707	74039	74370	74702	75033	75364	75696	1
1	76358	76689	77021	77352	77683	78014	78345	78676	79007	1
2	79669	80000	80331	80662	80993	81324	81655	81986	82316	1
3	82978	83309	83639	83970	84301	84631	84962	85293	85623	1
4	86284	86615	86945	87276	87606	87936	88267	88597	88927	0
5	89588	89918	90248	90578	90909	91239	91561	91899	92229	0
6	92889	93219	93549	93879	94209	94539	94868	95198	95528	0
7	96187	96517	96847	97177	97506	97836	98165	98495	98825	0
8	99484	99813	1200143	00472	1200801	01131	1201460	01789	1202119	329
9	1202777	03106	03436	03765	04094	04423	04752	05081	05410	9
	1	2	3	4	5	6	7	8	9	

Between 13200 = log.$^{-1}$ 4·1205739, and 13800 = log.$^{-1}$ 4·1398791.

tens.	1	2	3	4	5	6	7	8	9	*dif*
1320	1206068	06397	1206726	07055	1207384	07713	1208042	08371	1208699	329
1	09357	09686	10014	10343	10672	11000	11329	11657	11986	9
2	12643	12972	13300	13628	13957	14285	14614	14942	15270	8
3	15927	16255	16583	16911	17239	17568	17896	18224	18552	8
4	19208	19536	19864	20192	20520	20848	21175	21503	21831	8
5	22487	22814	23142	23470	23797	24125	24453	24780	25108	8
6	25763	26090	26418	26745	27073	27400	27727	28055	28382	7
7	29036	29364	29691	30018	30345	30672	31000	31327	31654	7
8	32308	32635	32962	33289	33616	33942	34269	34596	34923	7
9	35577	35903	36230	36557	36883	37210	37537	37863	38190	7
1330	38843	39169	39496	39822	40149	40475	40802	41128	41454	6
1	42107	42433	42759	43086	43412	43738	44064	44390	44716	6
2	45368	45694	46020	46346	46672	46998	47324	47650	47976	6
3	48627	48953	49279	49605	49930	50256	50582	50907	51233	6
4	51884	52209	52535	52860	53186	53511	53837	54162	54487	5
5	55138	55463	55788	56114	56439	56764	57089	57414	57739	5
6	58390	58715	59040	59365	59690	60015	60339	60664	60989	5
7	61639	61964	62288	62613	62938	63263	63587	63912	64237	5
8	64886	65210	65535	65859	66184	66508	66833	67157	67481	4
9	68130	68454	68779	69103	69427	69751	70076	70400	70724	4
1340	71372	71696	72020	72344	72668	72992	73316	73640	73964	4
1	74612	74935	75259	75583	75907	76230	76554	76878	77202	4
2	77849	78172	78496	78819	79143	79466	79790	80113	80437	4
3	81083	81407	81730	82053	82377	82700	83023	83346	83670	3
4	84316	84639	84962	85285	85608	85931	86254	86577	86900	3
5	87546	87869	88191	88514	88837	89160	89483	89805	90128	3
6	90773	91096	91418	91741	92064	92386	92709	93031	93354	3
7	93998	94321	94643	94965	95288	95610	95932	96255	96577	2
8	97221	97543	97865	98187	98510	98832	99154	99476	99798	2
9	1300441	00763	1301085	01407	1301729	02051	1302372	02694	1303016	2
1350	03659	03981	04303	04624	04946	05267	05589	05911	06232	2
1	06875	07196	07518	07839	08161	08482	08803	09124	09446	1
2	10088	10409	10730	11052	11373	11694	12015	12336	12657	1
3	13299	13620	13941	14262	14583	14903	15224	15545	15866	1
4	16507	16828	17149	17469	17790	18111	18431	18752	19072	1
5	19713	20034	20354	20675	20995	21316	21636	21956	22277	0
6	22917	23237	23558	23878	24198	24518	24838	25158	25478	0
7	26119	26439	26758	27078	27398	27718	28038	28358	28678	0
8	29317	29637	29957	30277	30596	30916	31236	31555	31875	0
9	32514	32834	33153	33473	33792	34112	34431	34750	35070	319
1360	35708	36028	36347	36666	36985	37305	37624	37943	38262	9
1	38900	39219	39538	39857	40176	40495	40814	41133	41452	9
2	42090	42409	42728	43046	43365	43684	44003	44321	44640	9
3	45277	45596	45914	46233	46551	46870	47188	47507	47825	8
4	48462	48780	49099	49417	49735	50054	50372	50690	51008	8
5	51645	51963	52281	52599	52917	53235	53553	53871	54189	8
6	54825	55143	55461	55779	56096	56414	56732	57050	57367	8
7	58003	58320	58638	58956	59273	59591	59908	60226	60543	8
8	61178	61496	61813	62131	62448	62765	63083	63400	63717	7
9	64352	64669	64986	65303	65620	65937	66255	66572	66889	7
1370	67523	67840	68157	68473	68790	69107	69424	69741	70058	7
1	70691	71008	71325	71641	71958	72275	72591	72908	73225	7
2	73858	74174	74491	74807	75124	75440	75756	76073	76389	6
3	77022	77338	77654	77970	78287	78603	78919	79235	79551	6
4	80183	80499	80815	81131	81447	81763	82079	82395	82711	6
5	83343	83659	83974	84290	84606	84922	85237	85553	85869	6
6	86500	86816	87131	87447	87762	88078	88393	88709	89024	5
7	89655	89970	90285	90601	90916	91231	91547	91862	92177	5
8	92807	93122	93438	93753	94068	94383	94698	95013	95328	5
9	95958	96272	96587	96902	97217	97532	97847	98161	98476	5
	1	2	3	4	5	6	7	8	9	

Between 13800 = log.⁻¹ 4·1398791, and 14400 = log.⁻¹ 4·1583625.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
1380	1399106	99420	1399735	/00050	1400364	00679	1400993	01308	1401622	315
1	1402251	02566	1402880	03195	03509	03823	04138	04452	04766	4
2	05395	05709	06023	06337	06651	06966	07280	07594	07908	4
3	08536	08850	09164	09478	09792	10106	10419	10733	11047	4
4	11675	11988	12302	12616	12930	13243	13557	13871	14184	4
5	14811	15125	15438	15752	16065	16379	16692	17006	17319	4
6	17946	18259	18572	18885	19199	19512	19825	20138	20451	3
7	21078	21391	21704	22017	22330	22643	22956	23269	23582	3
8	24208	24520	24833	25146	25459	25772	26084	26397	26710	3
9	27335	27648	27960	28273	28586	28898	29211	29523	29836	3
1390	30460	30773	31085	31396	31710	32022	32335	32647	32959	2
1	33584	33896	34208	34520	34832	35144	35456	35768	36080	2
2	36704	37016	37328	37640	37952	38264	38576	38888	39199	2
3	39823	40135	40446	40758	41070	41381	41693	42005	42316	2
4	42939	43251	43562	43874	44185	44497	44808	45119	45431	1
5	46053	46365	46676	46987	47298	47610	47921	48232	48543	1
6	49165	49476	49787	50098	50409	50720	51031	51342	51653	1
7	52275	52586	52897	53207	53518	53829	54140	54450	54761	1
8	55382	55693	56004	56314	56625	56935	57246	57556	57867	0
9	58488	58798	59108	59419	59729	60039	60350	60660	60970	0
1400	61591	61901	62211	62521	62831	63141	63451	63761	64071	0
1	64691	65001	65311	65621	65931	66241	66551	66861	67170	0
2	67790	68100	68409	68719	69029	69338	69648	69958	70267	0
3	70886	71196	71505	71815	72124	72434	72743	73052	73362	309
4	73980	74290	74599	74908	75217	75527	75836	76145	76454	9
5	77072	77381	77690	77999	78308	78617	78826	79235	79544	9
6	80162	80471	80780	81089	81397	81706	82015	82324	82632	9
7	83250	83558	83867	84175	84484	84793	85101	85410	85718	9
8	86335	86643	86952	87260	87569	87877	88185	88493	88802	8
9	89418	89726	90035	90343	90651	90959	91267	91575	91883	8
1410	92499	92807	93115	93423	93731	94039	94347	94655	94962	8
1	95578	95886	96193	96501	96809	97116	97424	97732	98039	8
2	98655	98962	99270	99577	99885	/00192	1500499	00807	1501114	7
3	1501729	02036	1502344	02651	1502958	03265	03573	03880	04187	7
4	04801	05108	05415	05722	06030	06337	06644	06951	07257	7
5	07871	08178	08485	08792	09099	09406	09712	10019	10326	7
6	10939	11246	11553	11859	12166	12472	12779	13085	13392	7
7	14005	14311	14618	14924	15231	15537	15843	16150	16456	6
8	17069	17375	17681	17987	18293	18600	18906	19212	19518	6
9	20130	20436	20742	21048	21354	21660	21966	22272	22578	6
1420	23189	23495	23801	24107	24412	24718	25024	25329	25635	6
1	26246	26552	26858	27163	27469	27774	28080	28385	28691	6
2	29301	29607	29912	30217	30523	30828	31133	31439	31744	5
3	32354	32659	32964	33270	33575	33880	34185	34490	34795	5
4	35405	35710	36015	36320	36625	36929	37234	37539	37844	5
5	38453	38758	39063	39368	39672	39977	40281	40586	40891	5
6	41500	41804	42109	42413	42718	43022	43327	43631	43935	5
7	44544	44848	45153	45457	45761	46065	46370	46674	46978	4
8	47586	47890	48194	48498	48802	49106	49410	49714	50018	4
9	50626	50930	51234	51538	51842	52145	52449	52753	53057	4
1430	53664	53968	54271	54575	54879	55182	55486	55789	56093	4
1	56700	57003	57307	57610	57914	58217	58520	58824	59127	3
2	59733	60037	60340	60643	60946	61249	61553	61856	62159	3
3	62765	63068	63371	63674	63977	64280	64583	64886	65189	3
4	65794	66097	66400	66703	67006	67308	67611	67914	68216	3
5	68822	69124	69427	69729	70032	70334	70637	70939	71242	3
6	71847	72149	72452	72754	73056	73359	73661	73963	74265	2
7	74870	75172	75474	75776	76079	76381	76683	76985	77287	2
8	77891	78193	78495	78797	79099	79401	79702	80004	80306	2
9	80910	81212	81513	81815	82117	82418	82720	83022	83323	2
	1	2	3	4	5	6	7	8	9	

Between 14400 = log.$^{-1}$ 4·1583625, and 15000 = log.$^{-1}$ 4·1760913.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
1440	1583927	4223	1584530	4831	1585133	5434	1585736	6037	1586338	301
1	6941	7243	7544	7845	8146	8448	8749	9050	9351	1
2	9954	/0255	1590556	0857	1591158	1459	1591760	2061	1592362	1
3	1592964	3265	3566	3867	4168	4469	4770	5070	5371	1
4	5973	6273	6574	6875	7175	7476	7777	8077	8378	1
5	8979	9280	9580	9881	1600181	0481	1600782	1082	1601383	1
6	1601983	2284	1602584	2884	3184	3485	3785	4085	4385	0
7	4985	5286	5586	5886	6186	6486	6786	7086	7386	0
8	7986	8285	8585	8885	9185	9485	9785	/0084	1610384	0
9	1610984	1283	1611583	1883	1612182	2482	1612781	3081	3380	0
1450	3980	4279	4578	4878	5177	5477	5776	6075	6375	0
1	6973	7273	7572	7871	8170	8470	8769	9068	9367	299
2	9965	/0264	1620563	0862	1621161	1460	1621759	2058	1622357	9
3	1622955	3254	3553	3852	4150	4449	4748	5047	5345	9
4	5943	6241	6540	6839	7137	7436	7734	8033	8331	9
5	8928	9227	9525	9824	1630122	0420	1630719	1017	1631315	8
6	1631912	2210	1632508	2807	3105	3403	3701	3999	4297	8
7	4894	5192	5490	5788	6086	6384	6682	6979	7277	8
8	7873	8171	8469	8767	9064	9362	9660	9958	1640255	8
9	1640851	1148	1641446	1743	1642041	2339	1642636	2934	3231	8
1460	3826	4123	4421	4718	5016	5313	5610	5908	6205	7
1	6799	7097	7394	7691	7988	8285	8582	8880	9177	7
2	9771	/0068	1650365	0662	1650959	1256	1651553	1850	1652146	7
3	1652740	3037	3334	3631	3927	4224	4521	4817	5114	7
4	5707	6004	6301	6597	6894	7190	7487	7783	8080	7
5	8673	8969	9265	9562	9858	/0155	1660451	0747	1661043	6
6	1661636	1932	1662228	2525	1662821	3117	3413	3709	4005	6
7	4597	4893	5189	5485	5781	6077	6373	6669	6965	6
8	7556	7852	8148	8444	8740	9035	9331	9627	9922	6
9	1670514	0809	1671105	1400	1671696	1991	1672287	2582	1672878	6
1470	3469	3764	4060	4355	4650	4946	5241	5536	5831	5
1	6422	6717	7012	7308	7603	7898	8193	8488	8783	5
2	9373	9668	9963	/0258	1680553	0848	1681143	1438	1681733	5
3	1682322	2617	1682912	3207	3501	3796	4091	4386	4680	5
4	5269	5564	5859	6153	6448	6742	7037	7331	7626	5
5	8215	8509	8803	9098	9392	9686	9981	/0275	1690569	4
6	1691158	1452	1691746	2040	1692335	2629	1692923	3217	3511	4
7	4099	4393	4687	4981	5275	5569	5863	6157	6450	4
8	7038	7332	7626	7920	8213	8507	8801	9094	9388	4
9	9975	/0269	1700563	0856	1701150	1443	1701737	2030	1702324	4
1480	1702911	3204	3497	3791	4084	4377	4671	4964	5257	3
1	5844	6137	6430	6723	7017	7310	7603	7896	8189	3
2	8775	9068	9361	9654	9947	/0240	1710533	0826	1711119	3
3	1711704	1997	1712290	2583	1712876	3168	3461	3754	4046	3
4	4632	4924	5217	5509	5802	6095	6387	6690	6972	3
5	7557	7849	8142	8434	8727	9019	9311	9604	9896	2
6	1720480	0773	1721065	1357	1721649	1941	1722233	2526	1722818	2
7	3402	3694	3986	4278	4570	4862	5154	5446	5737	2
8	6321	6613	6905	7197	7488	7780	8072	8364	8655	2
9	9239	9530	9822	/0113	1730405	0697	1730988	1280	1731571	1
1490	1732154	2446	1732737	3028	3320	3611	3903	4194	4485	1
1	5068	5359	5650	5941	6233	6524	6815	7106	7397	1
2	7979	8270	8561	8852	9143	9434	9725	/0016	1740307	1
3	1740889	1180	1741471	1761	1742052	2343	1742634	2925	3215	1
4	3797	4087	4378	4669	4959	5250	5540	5831	6121	0
5	6702	6993	7283	7574	7864	8155	8445	8735	9026	0
6	9606	9897	1750187	0477	1750767	1057	1751348	1638	1751928	0
7	1752508	2798	3088	3378	3668	3958	4248	4538	4828	0
8	5408	5698	5988	6278	6567	6857	7147	7437	7727	0
9	8306	8596	8885	9175	9465	9754	1760044	0333	1760623	0
	1	2	3	4	5	6	7	8	9	

Between 15000 = $\log.^{-1}$ 4·1760913, and 15600 = $\log.^{-1}$ 4·1931246.

tens.	1	2	3	4	5	6	7	8	9	dif.
1500	1761202	1492	1761781	2071	1762360	2649	1762939	3228	1763518	289
1	4096	4386	4675	4964	5253	5543	5832	6121	6410	9
2	6988	7278	7567	7856	8145	8434	8723	9012	9301	9
3	9879	/0168	1770457	0745	1771034	1323	1771612	1901	1772190	9
4	1772767	3056	3345	3633	3922	4211	4499	4788	5076	9
5	5654	5942	6231	6519	6808	7096	7385	7673	7961	8
6	8538	8826	9115	9403	9691	9960	1780268	0556	1780844	8
7	1781421	1709	1781997	2285	1782573	2861	3149	3437	3725	8
8	4301	4589	4877	5165	5453	5741	6029	6317	6605	8
9	7180	7468	7756	8043	8331	8619	8907	9194	9482	8
1510	1790057	0345	1790632	0920	1791207	1495	1791782	2070	1792357	8
1	2932	3219	3507	3794	4082	4369	4656	4943	5231	7
2	5805	6092	6380	6667	6954	7241	7528	7815	8102	7
3	8676	8963	9250	9537	9824	/0111	1800398	0685	1800972	7
4	1801546	1832	1802119	2406	1802693	2980	3266	3553	3840	7
5	4413	4700	4986	5273	5559	5846	6133	6419	6706	7
6	7278	7565	7851	8138	8424	8711	8997	9283	9570	7
7	1810142	0428	1810715	1001	1811287	1573	1811859	2145	1812432	6
8	3004	3290	3576	3862	4148	4434	4720	5006	5292	6
9	5864	6150	6435	6721	7007	7293	7579	7864	8150	6
1520	8722	9007	9293	9579	9864	/0150	1820435	0721	1821007	6
1	1821578	1863	1822149	2434	1822720	3005	3290	3576	3861	6
2	4432	4717	5002	5288	5573	5858	6143	6429	6714	5
3	7284	7569	7854	8140	8425	8710	8995	9280	9565	5
4	1830135	0420	1830704	0989	1831274	1559	1831844	2129	1832414	5
5	2983	3268	3553	3837	4122	4407	4691	4976	5261	5
6	5830	6114	6399	6684	6968	7253	7537	7822	8106	5
7	8675	8959	9244	9528	9812	/0096	1840381	0665	1840949	4
8	1841518	1802	1842086	2370	1842654	2939	3223	3507	3791	4
9	4359	4643	4927	5211	5495	5779	6063	6347	6630	4
1530	7198	7482	7766	8050	8333	8617	8901	9185	9468	4
1	1850036	0319	1850603	0886	1851170	1454	1851737	2021	1852304	4
2	2871	3155	3438	3721	4005	4288	4572	4855	5138	3
3	5705	5988	6271	6555	6838	7121	7404	7687	7970	3
4	8537	8820	9103	9386	9669	9952	1860235	0518	1860801	3
5	1861367	1650	1861932	2215	1862498	2781	3064	3347	3629	3
6	4195	4478	4760	5043	5326	5608	5891	6174	6456	3
7	7021	7304	7586	7869	8151	8434	8716	8999	9281	2
8	9846	/0128	1870410	0693	1870975	1257	1871540	1822	1872104	2
9	1872668	2951	3233	3515	3797	4079	4361	4643	4925	2
1540	5489	5771	6053	6335	6617	6899	7181	7463	7745	2
1	8308	8590	8872	9154	9435	6717	9999	/0280	1880562	2
2	1881125	1407	1881689	1970	1882252	2533	1882815	3096	3378	2
3	3941	4222	4504	4785	5066	5348	5629	5910	6192	1
4	6754	7035	7317	7598	7879	8160	8441	8723	9004	1
5	9566	9847	1890128	0409	1890690	0971	1891252	1533	1891814	1
3	1892376	2657	2938	3218	3499	3780	4061	4342	4622	1
7	5184	5465	5745	6026	6307	6587	6868	7148	7429	1
8	7990	8271	8551	8832	9112	9393	9673	9953	1900234	0
9	1900795	1075	1901355	1636	1901916	2196	1902476	2757	3037	0
1550	3597	3877	4157	4438	4718	4998	5278	5558	5838	0
1	6398	6678	6958	7238	7518	7798	8078	8357	8637	0
2	9197	9477	9757	/0036	1910316	0596	1910876	1155	1911435	0
3	1911994	2274	1912553	2833	3113	3392	3672	3951	4231	0
4	4790	5060	5348	5628	5907	6187	6466	6745	7025	279
5	7583	7862	8142	8421	8700	8979	9259	9538	9817	9
6	1920375	0654	1920933	1212	1921491	1770	1922049	2328	1922607	9
7	3165	3444	3723	4002	4281	4559	4838	5117	5396	9
8	5953	6232	6511	6789	7068	7347	7625	7904	8183	9
9	8740	9018	9297	9575	9854	/0132	1930411	0689	1930968	9
	1	2	3	4	5	6	7	8	9	

Between 15600 = log.$^{-1}$ 4·1931246, and 16200 = log.$^{-1}$ 4·2095150.

tens.	1	2	3	4	5	6	7	8	9	dif.
1560	1931524	1803	1932081	2359	1932638	2916	1933194	3473	1933751	278
1	4307	4585	4864	5142	5420	5698	5976	6254	6532	8
2	7088	7366	7644	7922	8200	8478	8756	9034	9312	8
3	9868	/ 0145	1940423	0701	1940979	1257	1941534	1812	1942090	8
4	1942645	2923	3200	3478	3756	4033	4311	4588	4866	8
5	5421	5698	5976	6253	6531	6808	7086	7363	7640	7
6	8195	8472	8749	9027	9304	9581	9858	/ 0136	1950413	7
7	1950967	1244	1951521	1798	1952075	2353	1952630	2907	3184	7
8	3738	4014	4291	4568	4845	5122	5399	5676	5953	7
9	6506	6783	7060	7336	7613	7890	8167	8443	8720	7
1570	9273	9550	9826	/ 0103	1960379	0656	1960932	1209	1961485	7
1	1962038	2315	1962591	2867	3144	3420	3697	3973	4249	6
2	4802	5078	5354	5630	5907	6183	6459	6735	7011	6
3	7563	7839	8115	8391	8667	8943	9219	9495	9771	6
4	1970323	0599	1970875	1151	1971427	1702	1971978	2254	1972530	6
5	3081	3357	3633	3908	4184	4460	4735	5011	5287	6
6	5838	6113	6389	6664	6940	7215	7491	7766	8042	6
7	8592	8868	9143	9418	9694	9969	1980244	0520	1980795	5
8	1981345	1620	1981896	2171	1982446	2721	2996	3271	3546	5
9	4096	4371	4646	4921	5196	5471	5746	6021	6296	5
1580	6846	7121	7395	7670	7945	8220	8495	8769	9044	5
1	9593	9868	1990143	0417	1990692	0967	1991241	1516	1991790	5
2	1992339	2614	2888	3163	3437	3712	3986	4260	4535	4
3	5083	5358	5632	5906	6181	6455	6729	7003	7278	4
4	7826	8100	8374	8648	8922	9197	9471	9745	2000019	4
5	2000567	0841	2001115	1389	2001662	1936	2002210	2484	2758	4
6	3306	3579	3853	4127	4401	4674	4948	5222	5496	4
7	6043	6317	6590	6864	7137	7411	7684	7958	8231	4
8	8778	9052	9325	9599	9872	0146	2010419	0692	2010966	3
9	2011512	1786	2012059	2332	2012605	/ 2879	3152	3425	3698	3
1590	4244	4517	4791	5064	5337	5610	5883	6156	6429	3
1	6975	7248	7521	7794	8066	8339	8612	8885	9158	3
2	9703	9976	2020249	0522	2020794	1067	2021340	1612	2021885	3
3	2022430	2703	2976	3248	3521	3793	4066	4338	4611	3
4	5156	5428	5700	5973	6245	6518	6790	7062	7335	3
5	7879	8151	8424	8696	8968	9240	9512	9785	2030057	2
6	2030601	0873	2031145	1417	2031689	1961	2032233	2505	2777	2
7	3321	3593	3865	4137	4409	4681	4952	5224	5496	2
8	6040	6311	6583	6855	7126	7398	7670	7941	8213	2
9	8756	9028	9299	9571	9842	/ 0114	2040385	0657	2040928	2
1600	2041471	1743	2042014	2285	2042557	2828	3099	3371	3642	1
1	4185	4456	4727	4998	5269	5541	5812	6083	6354	1
2	6896	7167	7438	7709	7980	8251	8522	8793	9064	1
3	9606	9877	2050148	0419	2050690	0960	2051231	1502	2051773	1
4	2052314	2585	2856	3127	3397	3668	3939	4209	4480	1
5	5021	5292	5562	5833	6103	6374	6644	6915	7185	1
6	7726	7996	8267	8537	8807	9078	9348	9618	9889	0
7	2060429	0699	2060969	1240	2061510	1780	2062050	2320	2062590	0
8	3131	3401	3671	3941	4211	4481	4751	5021	5291	0
9	5830	6100	6370	6640	6910	7180	7449	7719	7989	0
1610	8529	8798	9068	9338	9607	9877	2070147	0416	2070686	0
1	2071225	1495	2071764	2034	2072303	2573	2842	3112	3381	0
2	3920	4189	4459	4728	4997	5267	5536	5805	6074	269
3	6613	6882	7151	7421	7690	7959	8228	8497	8766	9
4	9304	9573	9842	/ 0111	2080380	0649	2080918	1187	2081456	9
5	2081994	2263	2082532	2801	3070	3338	3607	3876	4145	9
6	4682	4951	5220	5488	5757	6026	6294	6563	6832	9
7	7369	7637	7906	8174	8443	8711	8980	9248	9517	9
8	2090054	0322	2090590	0859	2091127	1395	2091664	1932	2092200	8
9	2737	3005	3273	3541	3810	4078	4346	4614	4882	8
	1	2	3	4	5	6	7	8	9	

Between 16200 = $\log.^{-1}$ 4·2095150, and 16800 = $\log.^{-1}$ 4·2253093.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
1620	2095418	5686	2095954	6222	2096490	6758	2097026	7294	2097562	268
1	8098	8366	8634	8902	9170	9437	9705	9973	2100241	8
2	2100776	1044	2101312	1579	2101847	2115	2102382	2650	2918	8
3	3453	3720	3988	4255	4523	4790	5058	5325	5593	8
4	6128	6395	6662	6930	7197	7464	7732	7999	8266	7
5	8801	9068	9335	9603	9870	/0137	2110404	0671	2110938	7
6	2111472	1740	2112007	2274	2112541	2808	3075	3342	3609	7
7	4142	4409	4676	4943	5210	5477	5744	6010	6277	7
8	6811	7078	7344	7611	7878	8144	8411	8678	8944	7
9	9477	9744	2120011	0277	2120544	0810	2121077	1343	2121610	7
1630	2122142	2409	2675	2942	3208	3474	3741	4007	4273	6
1	4806	5072	5338	5605	5871	6137	6403	6669	6935	6
2	7468	7734	8000	8266	8532	8798	9064	9330	9596	6
3	2130128	0394	2130660	0926	2131191	1457	2131723	1989	2132255	6
4	2786	3052	3318	3584	3849	4115	4381	4646	4912	6
5	5443	5709	5974	6240	6505	6771	7037	7302	7568	6
6	8098	8364	8629	8895	9160	9425	9691	9956	2140221	5
7	2140752	1017	2141283	1548	2141813	2078	2142343	2609	2874	5
8	3404	3669	3934	4199	4464	4730	4995	5260	5525	5
9	6055	6319	6584	6849	7114	7379	7644	7909	8174	5
1640	8703	8668	9233	9498	9762	/0027	2150292	0556	2150821	5
1	2151350	1615	2151880	2144	2152409	2673	2938	3203	3467	5
2	3996	4260	4525	4789	5054	5318	5583	5847	6111	4
3	6640	6904	7169	7433	7697	7961	8226	8490	8754	4
4	9282	9546	9811	/0075	2160339	0603	2160867	1131	2161395	4
5	2161923	2187	2162421	2715	2979	3243	3507	3771	4034	4
6	4562	4826	5090	5354	5617	5881	6145	6409	6672	4
7	7200	7463	7727	7991	8254	8518	8781	9045	9309	4
8	9836	/0099	2170363	0626	2170890	1153	2171416	1680	2171943	3
9	2172470	2733	2997	3260	3523	3786	4050	4313	4576	3
1650	5103	5366	5629	5892	6155	6418	6682	6945	7208	3
1	7734	7997	8260	8523	8786	9049	9312	9575	9838	3
2	2180463	0626	2180889	1152	2181415	1677	2181940	2203	2182466	3
3	2991	3254	3517	3779	4042	4305	4567	4830	5092	3
4	5618	5880	6143	6405	6668	6930	7193	7455	7718	2
5	8242	8505	8767	9030	9292	9554	9816	/0079	2190341	2
6	2190866	1128	2191390	1652	2191914	2177	2192439	2701	2963	2
7	3487	3749	4011	4273	4535	4797	5059	5351	5583	2
8	6107	6369	6631	6893	7155	7417	7678	7940	8202	2
9	8726	8987	9249	9511	9773	/0034	2200296	0558	2200819	2
1660	2201342	1604	2201866	2127	2202389	2650	2912	3173	3435	2
1	3958	4219	4481	4742	5003	5265	5526	5788	6049	1
2	6571	6833	7094	7355	7617	7878	8139	8400	8666	1
3	9184	9445	9706	9967	2210228	0489	2210750	1011	2211272	1
4	2211794	2055	2212316	2577	2838	3099	3360	3621	3882	1
5	4403	4664	4925	5186	5446	5707	5968	6229	6489	1
6	7011	7271	7532	7793	8053	8314	8574	8835	9095	1
7	9617	9877	2220138	0398	2220658	0919	2221179	1440	2221700	0
8	2222221	2481	2741	3002	3262	3522	3783	4043	4303	0
9	4824	5084	5344	5604	5864	6124	6384	6645	6905	0
1670	7425	7685	7945	8205	8465	8725	8985	9245	9505	0
1	2230024	0284	2230544	0804	2231064	1324	2231583	1843	2232103	0
2	2622	2882	3142	3402	3661	3921	4181	4440	4700	0
3	5219	5479	5738	5998	6257	6517	6776	7036	7295	0
4	7814	8073	8333	8592	8852	9111	9370	9630	9889	259
5	2240407	0667	2240926	1185	2241444	1704	2241963	2222	2242481	9
6	2999	3258	3517	3777	4036	4295	4554	4813	5072	9
7	5590	5849	6107	6366	6625	6884	7143	7402	7661	9
8	8178	8437	8696	8955	9213	9472	9731	9990	2250248	9
9	2250766	1024	2251283	1541	2251800	2059	2252317	2576	2834	9
	1	2	3	4	5	6	7	8	9	

Between 16800 = log.$^{-1}$ 4·2253093, and 17400 = log.$^{-1}$ 4·2405492.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
1680	2253351	3610	2253868	4127	2254385	4644	2254902	5160	2255419	258
1	5935	6194	6452	6710	6969	7227	7485	7743	8002	8
2	8518	8776	9034	9293	9551	9809	2260067	0325	2260583	8
3	2261099	1357	2261615	1873	2262131	2389	2647	2905	3163	8
4	3679	3937	4194	4452	4710	4968	5226	5484	5741	8
5	6257	6515	6772	7030	7288	7545	7803	8060	8318	8
6	8833	9091	9348	9606	9863	/0121	2270378	0636	2270893	8
7	2271408	1666	2271923	2180	2272438	2695	2953	3210	3467	7
8	3982	4239	4496	4753	5011	5268	5525	5782	6039	7
9	6554	6811	7068	7325	7582	7839	8096	8353	8610	7
1690	9124	9381	9638	9895	2280152	0409	2280666	0922	2281179	7
1	2281693	1950	2282206	2463	2720	2977	3233	3490	3747	7
2	4260	4517	4774	5030	5287	5543	5800	6057	6313	7
3	6826	7083	7339	7596	7852	8108	8365	8621	8878	6
4	9390	9647	9903	/0159	2290416	0672	2290928	1185	2291441	6
5	2291953	2209	2292466	2722	2978	3234	3490	3746	4002	6
6	4515	4771	5027	5283	5539	5795	6051	6307	6562	6
7	7074	7330	7586	7842	8098	8354	8609	8865	9121	6
8	9633	9888	2300144	0400	2300656	0911	2301167	1423	2301678	6
9	2302189	2445	2701	2956	3212	3467	3723	3978	4234	6
1700	4745	5000	5256	5511	5766	6022	6277	6532	6788	5
1	7298	7554	7809	8064	8320	8575	8830	9085	9340	5
2	9851	/0106	2310361	0616	2310871	1126	2311381	1636	2311891	5
3	2312401	2656	2911	3166	3421	3676	3931	4186	4441	5
4	4951	5206	5460	5715	5970	6225	6480	6734	6989	5
5	7499	7753	8008	8263	8517	8772	9026	9281	9536	5
6	2320045	0299	2320554	0808	2321063	1317	2321572	1826	2322081	5
7	2590	2844	3098	3353	3607	3861	4116	4370	4624	4
8	5133	5387	5641	5896	6150	6404	6658	6912	7166	4
9	7675	7929	8183	8437	8691	8945	9199	9453	9707	4
1710	2330215	0469	2330723	0977	2331231	1485	2331739	1992	2332246	4
1	2754	3008	3262	3515	3769	4023	4277	4530	4784	4
2	5291	5545	5799	6052	6306	6559	6813	7067	7320	4
3	7827	8081	8334	8588	8841	9095	9348	9601	9855	4
4	2340362	0615	2340868	1122	2341375	1628	2341881	2135	2342388	3
5	2894	3148	3401	3654	3907	4160	4414	4667	4920	3
6	5426	5679	5932	6185	6438	6691	6944	7197	7450	3
7	7956	8209	8462	8715	8967	9220	9473	9726	9979	3
8	2350484	0737	2350990	1243	2351495	1748	2352001	2253	2352506	3
9	3011	3264	3517	3769	4022	4274	4527	4779	5032	3
1720	5537	5789	6042	6294	6547	6799	7052	7304	7556	2
1	8061	8313	8566	8818	9070	9323	9575	9827	2360079	2
2	2360584	0836	2361088	1340	2361592	1844	2362097	2349	2601	2
3	3105	3357	3609	3861	4113	4365	4617	4869	5121	2
4	5625	5876	6128	6380	6632	6884	7136	7387	7639	2
5	8143	8394	8646	8898	9150	9401	9653	9905	2370156	2
6	2370660	0911	2371163	1414	2371666	1917	2372169	2420	2672	2
7	3175	3426	3678	3929	4181	4432	4683	4935	5186	1
8	5689	5940	6191	6443	6694	6945	7196	7448	7699	1
9	8201	8452	8703	8955	9206	9457	9708	9959	2380210	1
1730	2380712	0963	2381214	1465	2381716	1967	2382218	2469	2720	1
1	3222	3472	3723	3974	4225	4476	4727	4977	5228	1
2	5730	5980	6231	6482	6732	6983	7234	7484	7735	1
3	8236	8487	8737	8988	9238	9489	9739	9990	2390240	0
4	2390741	0992	2391242	1493	2391743	1993	2392244	2494	2744	0
5	3245	3495	3746	3996	4246	4496	4747	4997	5247	0
6	5747	5998	6248	6498	6748	6998	7248	7498	7748	0
7	8248	8498	8748	8998	9248	9498	9748	9998	2400248	0
8	2400748	0997	2401247	1497	2401747	1997	2402247	2496	2746	0
9	3246	3495	3745	3995	4244	4494	4744	4993	5243	0
	1	2	3	4	5	6	7	8	9	

Between 17400 = log.$^{-1}$ 4·2405492, and 18000 = log.$^{-1}$ 4·2552725.

tens.	1	2	3	4	5	6	7	8	9	*dif*
1740	2405742	5992	2406241	6491	2406740	6990	2407239	7489	2407738	250
1	8237	8487	8736	8985	9235	9484	9734	9983	2410232	249
2	2410731	0980	2411229	1479	2411728	1977	2412226	2476	2725	9
3	3223	3472	3721	3970	4220	4469	4718	4967	5216	9
4	5714	5963	6212	6461	6710	6959	7208	7457	7705	9
5	8203	8452	8701	8950	9199	9447	9696	9945	2420194	9
6	2420691	0940	2421189	1437	2421686	1935	2422183	2432	2680	9
7	3178	3426	3675	3923	4172	4420	4669	4917	5166	9
8	5663	5911	6160	6408	6656	6905	7153	7401	7650	8
9	8146	8395	8643	8891	9139	9388	9636	9884	2430132	8
1750	2430629	0877	2431125	1373	2431621	1869	2432117	2365	2613	8
1	3109	3357	3605	3853	4101	4349	4597	4845	5093	8
2	5589	5837	6085	6332	6580	6828	7076	7324	7571	8
3	8067	8315	8562	8810	9058	9305	9553	9801	2440048	8
4	2440543	0791	2441039	1286	2441534	1781	2442029	2276	2524	8
5	3019	3266	3514	3761	4008	4256	4503	4750	4998	7
6	5492	5740	5987	6234	6483	6729	6976	7223	7470	7
7	7965	8212	8459	8706	8953	9200	9448	9695	9942	7
8	2450436	0683	2450930	1177	2451424	1671	2451918	2165	2452411	7
9	2905	3152	3399	3646	3893	4140	4386	4633	4880	7
1760	5373	5620	5867	6114	6360	6607	6854	7100	7347	7
1	7840	8087	8333	8580	8826	9073	9320	9566	9813	7
2	2460306	0552	2460798	1045	2461291	1538	2461784	2030	2462277	6
3	2769	3016	3262	3508	3755	4001	4247	4493	4740	6
4	5232	5478	5724	5970	6217	6463	6709	6955	7201	6
5	7693	7939	8185	8431	8677	8923	9169	9415	9661	6
6	2470153	0399	2470645	0891	2471136	1382	2471628	1874	2472120	6
7	2611	2857	3103	3349	3594	3840	4086	4331	4577	6
8	5068	5314	5559	5805	6051	6296	6542	6787	7033	6
9	7524	7769	8015	8260	8506	8751	8997	9242	9487	5
1770	9978	/0223	2480469	0714	2480959	1205	2481450	1695	2481940	5
1	2482431	2676	2921	3166	3412	3657	3902	4147	4392	5
2	4882	5127	5372	5617	5862	6107	6352	6597	6842	5
3	7332	7577	7822	8067	8312	8557	8802	9047	9291	5
4	9781	/0026	2490271	0515	2490760	1005	2491249	1494	2491739	5
5	2492228	2473	2718	2962	3207	3451	3696	3941	4185	5
6	4674	4919	5163	5408	5652	5897	6141	6385	6630	4
7	7119	7363	7607	7852	8096	8340	8585	8829	9073	4
8	9562	9806	2500050	0294	2500539	0783	2501027	1271	2501515	4
9	2502004	2248	2492	2736	2980	3224	3468	3712	3956	4
1780	4444	4688	4932	5176	5420	5664	5908	6151	6395	4
1	6883	7127	7371	7614	7858	8102	8346	8590	8833	4
2	9321	9564	9808	/0052	2510295	0539	2510783	1026	2511270	4
3	2511757	2001	2512244	2488	2713	2975	3218	3462	3705	3
4	4192	4435	4679	4922	5166	5409	5652	5896	6139	3
5	6625	6869	7112	7355	7599	7842	8085	8328	8571	3
6	9058	9301	9544	9787	2520030	0273	2520516	0759	2521002	3
7	2521489	1732	2521975	2218	2461	2703	2946	3189	3432	3
8	3918	4161	4404	4647	4889	5132	5375	5618	5861	3
9	6346	6589	6832	7074	7317	7560	7802	8045	8288	3
1790	8773	9016	9258	9501	9743	9986	2530228	0471	2530713	3
1	2531198	1441	2531683	1926	2532168	2411	2653	2895	3138	2
2	3622	3865	4107	4349	4592	4834	5076	5318	5561	2
3	6045	6287	6529	6772	7014	7256	7498	7740	7982	2
4	8466	8709	8951	9193	9435	9677	9919	/0161	2540403	2
5	2540886	1128	2541370	1612	2541854	2096	2542338	2580	2822	2
6	3305	3547	3789	4030	4272	4514	4756	4997	5239	2
7	5722	5964	6206	6447	6689	6931	7172	7414	7655	2
8	8138	8380	8621	8863	9104	9346	9587	9829	2550070	2
9	2550553	0794	2551036	1277	2551519	1760	2552001	2242	2484	1
	1	2	3	4	5	6	7	8	9	

Between 18000 = $\log.^{-1}$ 4·2552725, and 18600 = $\log.^{-1}$ 4·2695129.

tens.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	*dif.*
1800	2552966	3208	2553449	3690	2553931	4172	2554414	4655	2554896	241
1	5378	5619	5860	6102	6343	6584	6825	7066	7307	1
2	7789	8030	8271	8512	8753	8994	9235	9475	9716	1
3	2560198	0439	2560680	0921	2561161	1402	2561643	1884	2562125	1
4	2606	2847	3087	3328	3569	3810	4050	4291	4531	1
5	5013	5253	5494	5734	5975	6215	6456	6696	6937	1
6	7418	7658	7899	8139	8380	8620	8860	9101	9341	0
7	9822	/0062	2570302	0543	2570783	1023	2571264	1504	2571744	0
8	2572224	2465	2705	2945	3185	3425	3665	3905	4146	0
9	4626	4866	5106	5346	5586	5826	6066	6306	6546	0
1810	7026	7266	7506	7745	7985	8225	8465	8705	8945	0
1	9424	9664	9904	/0144	2580383	0623	2580863	1103	2581342	0
2	2581822	2061	2582301	2541	2780	3020	3259	3499	3738	0
3	4218	4457	4697	4936	5176	5415	5655	5894	6133	0
4	6612	6852	7091	7330	7570	7809	8048	8288	8527	239
5	9006	9245	9484	9723	9963	0202	2590441	0680	2590919	9
6	2591398	1637	2591876	2115	2592354	2593	2832	3071	3310	9
7	3788	4027	4266	4505	4744	4983	5222	5461	5700	9
8	6178	6417	6655	6894	7133	7372	7611	7849	8088	9
9	8566	8804	9043	9282	9521	9759	9998	/0237	2600475	9
1820	2600952	1191	2601430	1668	2601907	2145	2002384	2622	2861	9
1	3338	3576	3815	4053	4292	4530	4769	5007	5245	8
2	5722	5960	6199	6437	6675	6914	7152	7390	7628	8
3	8105	8343	8581	8820	9058	9296	9534	9772	2610010	8
4	2610486	0725	2610963	1201	2611439	1677	3611915	2153	2391	8
5	2867	3105	3343	3580	3818	4056	4294	4532	4770	8
6	5246	5483	5721	5959	6197	6435	6672	6910	7148	8
7	7623	7861	8099	8336	8574	8811	9049	9287	9524	8
8	9999	/0237	2620475	0712	2620950	1187	2621425	1662	2621900	8
9	2622374	2612	2849	3087	3324	3562	3799	4036	4274	7
1830	4748	4986	5223	5460	5697	5935	6172	6409	6646	7
1	7121	4358	7595	7832	8069	8306	8543	8781	9018	7
2	9492	9729	9966	/0203	2630440	0677	2630914	1151	2631388	7
3	2631862	2098	2632335	2572	2809	3046	3283	3520	3757	7
4	4230	4467	4704	4940	5177	5414	5651	5887	6124	7
5	6597	6834	7071	7307	7544	7780	8017	8254	8490	7
6	8963	9200	9436	9673	9909	/0146	2640382	0619	2640855	6
7	2641328	1564	2641801	2037	2642273	2510	2746	2982	3219	6
8	3691	3928	4164	4400	4636	4873	5109	5345	5581	6
9	6053	6290	6526	6762	6998	7234	7470	7706	7944	6
1840	8414	8650	8886	9122	9358	9594	9830	/0066	2650302	6
1	2650774	1010	2651246	1481	2651717	1953	3652189	2425	2660	6
2	3132	3368	3604	3839	4075	4311	4546	4782	5018	6
3	5489	5725	5960	6196	6431	6667	6903	7138	7374	6
4	7845	8080	8316	8551	8787	9022	9257	9493	9728	5
5	2660199	0434	2660670	0905	2661140	1376	2661611	1846	2662082	5
6	2552	2787	3023	3258	3493	3728	3963	4199	4434	5
7	4904	5139	5374	5609	5844	6080	6315	6550	6785	5
8	7255	7490	7725	7960	8195	8429	8664	8899	9134	5
9	9604	9839	2670074	0309	2670543	0778	2671013	1248	2671483	5
1850	2671952	2187	2421	2656	2891	3126	3360	3595	3830	5
1	4299	4533	4768	5003	5237	5472	5706	5941	6175	5
2	6644	6879	7113	7348	7582	7817	8051	8285	8520	4
3	8969	9223	9457	9692	9926	/0160	2680394	0629	2680863	4
4	2681332	1566	2681800	2034	2682268	2503	2737	2971	3205	4
5	3673	3907	4141	4376	4610	4844	5078	5312	5546	4
6	6014	6248	6482	6716	6950	7183	7417	7651	7885	4
7	8353	8587	8821	9054	9288	9522	9756	9990	2690223	4
8	2690691	0925	2691158	1392	2691626	1859	2692093	2327	2560	4
9	3028	3261	3495	3728	3962	4195	4429	4662	4896	4
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	

Between 18600 = log.⁻¹ 4·2695129, and 19200 = log.⁻¹ 4·2833012.

Tens.	1	2	3	4	5	6	7	8	9	dif.
1860	2695363	5596	2695830	6063	2696297	6530	2696764	6997	2697230	233
1	7697	7930	8164	8397	8630	8864	9097	9330	9564	3
2	2700030	0263	2700496	0730	2700963	1196	2701429	1662	2701895	3
3	2362	2595	2828	3061	3294	3527	3760	3993	4226	3
4	4692	4925	5158	5391	5624	5857	6090	6323	6555	3
5	7021	7254	7487	7720	7953	8185	8418	8651	8884	3
6	9349	9582	9815	/0047	2710280	0513	2710745	0978	2711211	3
7	2711676	1908	2712141	/2374	2606	2839	3071	3304	3536	3
8	4001	4234	4466	4699	4931	5163	5396	5628	5861	2
9	6325	6558	6790	7022	7255	7487	7719	7952	8184	2
1870	8648	8881	9113	9345	9577	9809	2720041	0274	2720506	2
1	2720970	1202	2721434	1666	2721898	2130	2362	2594	2826	2
2	3290	3522	3754	3986	4218	4450	4682	4914	5146	2
3	5610	5841	6073	6305	6537	6769	7001	7232	7464	2
4	7928	8159	8391	8623	8854	9086	9318	9549	9781	2
5	2730244	0476	2730708	0939	2731171	1402	2731634	1865	2732097	2
6	2560	2791	3023	3254	3486	3717	3949	4180	4411	1
7	4874	5106	5337	5568	5799	6031	6262	6493	6725	1
8	7187	7418	7650	7881	8112	8343	8574	8806	9037	1
9	9499	9730	9961	/0192	2740423	0654	2740885	1116	2741347	1
1880	2741809	2040	2742271	/2502	2733	2964	3195	3426	3657	1
1	4119	4350	4581	4811	5042	5273	5504	5735	5965	1
2	6427	6658	6888	7119	7350	7581	7811	8042	8273	1
3	8734	8964	9195	9426	9656	9887	2750117	0348	2750578	1
4	2751039	1270	2751500	1731	2751961	2192	2422	2653	2883	0
5	3344	3574	3805	4035	4265	4496	4726	4956	5187	0
6	5647	5877	6108	6338	6568	6798	7028	7259	7489	0
7	7949	8179	8409	8640	8870	9100	9330	9560	9790	0
8	2760250	0480	2760710	0940	2761170	1400	2761630	1860	2762090	0
9	2549	2779	3009	3239	3469	3699	3929	4158	4388	0
1890	4848	5078	5307	5537	5767	5997	6226	6456	6686	0
1	7145	7375	7604	7834	8063	8293	8523	8752	8982	0
2	9441	9670	9900	/0129	2770359	0588	2770818	1047	2771277	0
3	2771736	1965	2772194	/2424	2653	2882	3112	3341	3570	229
4	4029	4258	4488	4717	4946	5175	5405	5634	5863	9
5	6321	6550	6780	7009	7238	7467	7696	7925	8154	9
6	8612	8841	9070	9299	9528	9757	9986	/0215	2780444	9
7	2780902	1131	2781360	1589	2781818	2047	2782276	/2504	2733	9
8	3191	3420	3648	3877	4106	4335	4564	4792	5021	9
9	5478	5707	5936	6164	6393	6622	6850	7079	7307	9
1900	7765	7993	8222	8450	8679	8907	9136	9364	9593	9
1	2790050	0278	2790506	0735	2790963	1192	2791420	1648	2791877	8
2	2333	2562	2790	3018	3247	3475	3703	3931	4160	8
3	4616	4844	5072	5301	5529	5757	5985	6213	6441	8
4	6898	7126	7354	7582	7810	8038	8266	8494	8722	8
5	9178	9406	9634	9862	2800090	0317	2800545	0773	2801001	8
6	2801457	1685	2801912	2140	2368	2596	2824	3051	3279	8
7	3735	3962	4190	4418	4645	4873	5101	5328	5556	8
8	6011	6239	6467	6694	6922	7149	7377	7604	7832	8
9	8287	8514	8742	8969	9197	9424	9651	9879	2810106	7
1910	2810561	0788	2811016	1243	2811470	1698	2811925	2152	2380	7
1	2834	3061	3289	3516	3743	3970	4197	4425	4652	7
2	5106	5333	5560	5787	6014	6242	6469	6696	6923	7
3	7377	7604	7831	8058	8285	8512	8739	8966	9192	7
4	9646	9873	2820100	0327	2820554	0781	2821007	1234	2821461	7
5	2821915	2141	2368	2595	2822	3048	3275	3502	3728	7
6	4182	4408	4635	4862	5088	5315	5541	5768	5995	7
7	6448	6674	6901	7127	7354	7580	7807	8033	8260	7
8	8712	8939	9165	9392	9618	9844	2830071	0297	2830523	6
9	2830976	1202	2831429	1655	2831881	2107	2334	2560	2786	6
	1	2	3	4	5	6	7	8	9	

Between 19200 = log.⁻¹ 4·2833012, and 19800 = log.⁻¹ 4·2966652.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
1920	2833238	3465	2833691	3917	2834143	4369	2834595	4821	2835048	226
1	5500	5726	5952	6178	6404	6630	6856	7082	7308	6
2	7760	7986	8212	8438	8663	8889	9115	9341	9567	6
3	2840019	0245	2840470	0696	2840922	1148	2841373	1599	2841825	6
4	2276	2502	2728	2953	3179	3405	3630	3856	4082	6
5	4533	4759	4984	5210	5435	5661	5886	6112	6337	6
6	6788	7014	7239	7465	7690	7916	8141	8366	8592	6
7	9043	9268	9493	9719	9944	/0169	2850394	0620	2850845	5
8	2851296	1521	2851746	1971	2852196	2422	2647	2872	3097	5
9	3547	3773	3998	4223	4448	4673	4898	5123	5348	5
1930	5798	6023	6248	6473	6698	6923	7148	7373	7598	5
1	8048	8273	8497	8722	8947	9172	9397	9622	9846	5
2	2860296	0521	2860746	0970	2861195	1420	2861644	1869	2862094	5
3	2543	2768	2993	3217	3442	3666	3891	4116	4340	5
4	4789	5014	5238	5463	5687	5912	6136	6361	6585	5
5	7034	7259	7483	7707	7932	8156	8381	8605	8829	4
6	9278	9502	9726	9951	2870175	0399	2870624	0848	2871072	4
7	2871520	1745	2871969	2193	2417	2641	2865	3090	3314	4
8	3762	3986	4210	4434	4658	4882	5106	5330	5554	4
9	6002	6226	6450	6674	6898	7122	7346	7570	7793	4
1940	8241	8465	8689	8913	9136	9360	9584	9808	2880032	4
1	2880479	0703	2880927	1150	2881374	1598	2881821	2045	2269	4
2	2716	2939	3163	3387	3610	3834	4057	4281	4504	4
3	4952	5175	5399	5622	5845	6069	6292	6516	6739	3
4	7186	7409	7633	7856	8079	8303	8526	8749	8973	3
5	9419	9643	9866	/0089	2890312	0536	2890759	0982	2891205	3
6	2891652	1875	2892098	2321	2544	2767	2990	3213	3436	3
7	3883	4106	4329	4552	4775	4998	5221	5444	5667	3
8	6112	6335	6558	6781	7004	7227	7450	7673	7896	3
9	8341	8564	8787	9010	9232	9455	9678	9901	2900123	3
1950	2900569	0792	2901014	1237	2901460	1682	2901905	2127	2350	3
1	2795	3018	3240	3463	3686	3908	4131	4353	4576	3
2	5021	5243	5466	5688	5910	6133	6355	6578	6800	2
3	7245	7467	7690	7912	8134	8356	8579	8801	9023	2
4	9468	9690	9912	/0135	2910357	0579	2910801	1023	2911245	2
5	2911690	1912	2912134	2356	2578	2800	3022	3244	3466	2
6	3911	4133	4355	4577	4799	5020	5242	5464	5686	2
7	6130	6352	6574	6796	7018	7240	7461	7683	7905	2
8	8349	8570	8792	9014	9236	9458	9679	9901	2920123	2
9	2920566	0788	2921009	1231	2921453	1674	2921896	2118	2339	2
1960	2782	3004	3225	3447	3668	3890	4111	4333	4554	2
1	4997	5219	5440	5662	5883	6105	6326	6547	6769	1
2	7211	7433	7654	7875	8097	8318	8539	8760	8982	1
3	9424	9645	9867	/0088	2930309	0530	2930751	0973	2931194	1
4	2931636	1857	2932078	2299	2520	2741	2962	3183	3405	1
5	3847	4068	4289	4510	4730	4951	5172	5393	5614	1
6	6056	6277	6498	6719	6940	7160	7381	7602	7823	1
7	8264	8485	8706	8927	9147	9368	9589	9810	2940030	1
8	2940472	0692	2940913	1134	2941354	1575	2941795	2016	2237	1
9	2678	2898	3119	3339	3560	3780	4001	4221	4442	1
1970	4883	5103	5324	5544	5764	5985	6205	6426	6646	0
1	7087	7307	7527	7748	7968	8188	8408	8629	8849	0
2	9289	9510	9730	9950	2950170	0390	2950610	0831	2951051	0
3	2951491	1711	2951931	2151	2371	2591	2811	3031	3251	0
4	3691	3911	4131	4351	4571	4791	5011	5231	5451	0
5	5891	6111	6331	6550	6770	6990	7210	7430	7650	0
6	8089	8309	8529	8748	8968	9188	9408	9627	9847	0
7	2960286	0506	2960726	0945	2961165	1385	2961604	1824	2962043	0
8	2482	2702	2922	3141	3361	3580	3800	4019	4238	219
9	4677	4897	5116	5336	5555	5774	5994	6213	6433	9
	1	2	3	4	5	6	7	8	9	

Between 19800 = log.$^{-1}$ 4·2966652, and 20400 = log.$^{-1}$ 4·3096302.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
1980	2966871	7091	2967310	7529	2967748	7968	2968187	8406	2968626	219
1	9064	9283	9502	9722	9941	/0160	2970379	0598	2970817	9
2	2971256	1475	2971694	1913	2972132	2351	2570	2789	3008	9
3	3446	3665	3884	3103	4322	4541	4760	4979	5198	9
4	3636	5854	6073	6292	6511	6730	6949	7168	7386	9
5	7824	8043	8261	8480	8699	8918	9136	9355	9574	9
6	2980011	0230	2980448	0667	2980886	1104	2981323	1542	2981760	9
7	2197	2416	2634	2853	3071	3290	3508	3727	3945	8
8	4382	4601	4819	5038	5256	5474	5693	5911	6129	8
9	6566	6785	7003	7221	7439	7658	7876	8094	8313	8
1990	8749	8967	9185	9404	9622	9840	2990058	0276	2990494	8
1	2990931	1149	2991367	1585	2991803	2021	2239	2457	2675	8
2	3111	3329	3547	3765	3983	4201	4419	4637	4855	8
3	5291	5509	5727	5945	6162	6380	6598	6816	7034	8
4	7469	7687	7905	8123	8340	8558	8776	8994	9211	8
5	9647	9864	3000082	0300	3000517	0735	3000953	1170	3001388	8
6	3001823	2041	2258	2476	2693	2911	3128	3346	3563	8
7	3998	4216	4433	4650	4868	5085	5303	5520	5737	7
8	6172	6390	6607	6824	7042	7259	7476	7693	7911	7
9	8345	8562	8780	8997	9214	9431	9648	9866	3010083	7
2000	3010517	0734	3010951	1168	3011386	1603	3011820	2037	2254	7
1	2688	2905	3122	3339	3556	3773	3990	4207	4424	7
2	4858	5075	5291	5508	5725	5942	6159	6376	6593	7
3	7026	7243	7460	7677	7893	8110	8327	8544	8760	7
4	9194	9411	9627	9844	3020061	0277	3020494	0711	3020927	7
5	3021360	1577	3021794	2010	2227	2443	2660	2876	3093	7
6	3526	3742	3959	4175	4392	4608	4825	5014	5257	6
7	5690	5906	6123	6339	6556	6772	6988	7204	7421	6
8	7853	8070	8286	8502	8718	8935	9151	9367	9583	6
9	3030016	0232	3030448	0664	3030880	1096	3031312	1528	3031745	6
2010	2177	2393	2609	2825	3041	3257	3473	3689	3905	6
1	4337	4553	4769	4984	5200	5416	5632	5848	6064	6
2	6496	6711	6927	7143	7359	7575	7790	8006	8222	6
3	8653	8869	9085	9301	9516	9732	9948	/0163	3040379	6
4	3040810	1026	3041242	1457	3041673	1888	3042104	/2319	2535	6
5	2966	3182	3397	3613	3828	4043	4259	4474	4690	5
6	5121	5336	5552	5767	5982	6198	6413	6628	6844	5
7	7274	7490	7705	7920	8135	8351	8566	8781	8996	5
8	9427	9642	9857	/0072	3050288	0503	3050718	0933	3051148	5
9	3051578	1793	3052008	/2224	2439	2654	2869	3084	3299	5
2020	3729	3944	4159	4374	4589	4803	5018	5233	5448	5
1	5878	6093	6308	6523	6737	6952	7167	7382	7597	5
2	8026	8241	8456	8671	8885	9100	9315	9529	9744	5
3	3060174	0388	3060603	0817	3061032	1247	3061461	1676	3061891	5
4	2320	2534	2749	2963	3178	3392	3607	3821	4036	5
5	4465	4679	4894	5108	5322	5537	5751	5966	6180	4
6	6609	6823	7037	7252	7466	7680	7895	8109	8323	4
7	8752	8966	9180	9394	9609	9823	3070037	0251	3070465	4
8	3070894	1108	3071322	1536	3071750	1964	2178	2392	2606	4
9	3035	3249	3463	3677	3891	4105	4319	4532	4746	4
2030	5174	5388	5602	5816	6030	6244	6458	6672	6885	4
1	7313	7527	7741	7954	8168	8382	8596	8810	9023	4
2	9451	9664	9878	/0092	3080306	0519	3080733	0947	3081160	4
3	3081587	1801	3082015	/2228	2442	2655	2869	3082	3296	4
4	3723	3936	4150	4363	4577	4790	5004	5217	5431	4
5	5858	6071	6284	6498	6711	6924	7138	7351	7564	3
6	7991	8204	8418	8631	8844	9057	9271	9484	9697	3
7	3090123	0337	3090550	0763	3090976	1189	3091402	1616	3091829	3
8	2255	2468	2681	2894	3107	3320	3533	3746	3959	3
9	4385	4598	4811	5024	5237	5450	5663	5876	6089	3
	1	2	3	4	5	6	7	8	9	

Between 20400 = $\log.^{-1}$ 4·3096302, and 21000 = $\log.^{-1}$ 4·3222193.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
2040	3096515	6727	3096940	7153	3097366	7579	3097792	8004	3098217	213
1	8643	8856	9068	9281	9494	9707	9919	0132	3100345	3
2	3100770	0983	3101195	1408	3101621	1833	3102046	/2258	2471	3
3	2896	3109	3321	3534	3746	3959	4171	4384	4596	3
4	5021	5234	5446	5659	5871	6084	6296	6508	6721	3
5	7145	7358	7570	7783	7995	8207	8419	8632	8844	2
6	9269	9481	9693	9905	3110117	0330	3110542	0754	3110966	2
7	3111391	1603	3111815	2027	2239	2451	2663	2875	3087	2
8	3512	3724	3936	4148	4360	4572	4784	4996	5208	2
9	5632	5843	6055	6267	6479	6691	6903	7115	7327	2
2050	7750	7962	8174	8386	8598	8810	9021	9233	9445	2
1	9868	/0080	3120292	0504	3120715	0927	3121139	1350	3121562	2
2	3121985	2197	2408	2620	2832	3043	3255	3466	3678	2
3	4101	4313	4524	4736	4947	5159	5370	5581	5793	2
4	6216	6427	6639	6850	7061	7273	7484	7696	7907	1
5	8330	8541	8752	8964	9175	9386	9597	9809	3130020	1
6	3130442	0654	3130865	1076	3131287	1498	3131709	1921	2132	1
7	2554	2765	2976	3187	3398	3610	3821	4032	4243	1
8	4665	4876	5087	5298	5509	5720	5931	6142	6353	1
9	6774	6985	7196	7407	7618	7829	8040	8251	8461	1
2060	8883	9094	9305	9515	9726	9937	3140148	0358	3140569	1
1	3140991	1201	3141412	1623	3141833	2044	2255	2465	2676	1
2	3097	3308	3518	3729	3940	4150	4361	4571	4782	1
3	5203	5413	5624	5834	6045	6255	6466	6676	6887	1
4	7307	7518	7728	7939	8149	8359	8570	8780	8990	0
5	9411	9621	9831	/0042	3150252	0462	3150672	0883	3151093	0
6	3151513	1724	3151934	2144	2354	2564	2774	2985	3195	0
7	3615	3825	4035	4245	4455	4665	4875	5085	5295	0
8	5715	5925	6135	6345	6555	6765	6975	7185	7395	0
9	7815	8025	8235	8444	8654	8864	9074	9284	9494	0
2070	9913	/0123	3160333	0543	3160752	0962	3161172	1382	3161591	0
1	3162011	2220	2430	2640	2849	3059	3269	3478	3688	0
2	4107	4317	4526	4736	4945	5155	5364	5574	5784	0
3	6203	6412	6621	6831	7040	7250	7459	7669	7878	209
4	8297	8506	8716	8925	9134	9344	9553	9762	9972	9
5	3170390	0600	3170809	1018	3171227	1437	3171646	1855	3172064	9
6	2483	2692	2901	3110	3319	3528	3738	3947	4156	9
7	4574	4783	4992	5201	5410	5619	5828	6037	6246	9
8	6664	6873	7082	7291	7500	7709	7918	8127	8336	9
9	8754	8963	9172	9380	9589	9798	3180007	0216	3180425	9
2080	3180842	1051	3181260	1468	3181677	1886	2095	2303	2512	9
1	2929	3138	3347	3556	3764	3973	4181	4390	4599	9
2	5016	5224	5433	5642	5850	6059	6267	6476	6684	9
3	7101	7310	7518	7727	7935	8143	8352	8560	8769	8
4	9186	9394	9602	9811	3190019	0227	3190436	0644	3190852	8
5	3191269	1477	3191685	1894	2102	2310	2518	2727	2935	8
6	3351	3559	3768	3976	4184	4392	4600	4808	5016	8
7	5433	5641	5849	6057	6265	6473	6681	6889	7097	8
8	7513	7721	7929	8137	8345	8553	8761	8969	9176	8
9	9592	9800	3200008	0216	3200424	0632	3200839	1047	3201255	8
2090	3201671	1878	2086	2294	2502	2709	2917	3125	3333	8
1	3748	3956	4163	4371	4579	4786	4994	5202	5409	8
2	5824	6032	6240	6447	6655	6862	7070	7277	7485	8
3	7900	8107	8315	8522	8730	8937	9145	9352	9559	7
4	9974	0182	3210389	0596	3210804	1011	3211218	1426	3211633	7
5	3212048	/2255	2462	2669	2877	3084	3291	3498	3706	7
6	4120	4327	4534	4742	4949	5156	5363	5570	5777	7
7	6191	6398	6606	6813	7020	7227	7434	7641	7848	7
8	8262	8469	8676	8883	9090	9297	9504	9711	9917	7
9	3220331	0538	3220745	0952	3221159	1366	3221572	1779	3221986	7
	1	2	3	4	5	6	7	8	9	

Between 21000 = log.$^{-1}$ 4·3222193, and 21600 = log.$^{-1}$ 4·3344538.

ens.	1	2	3	4	5	6	7	8	9	dif
100	3222400	2607	3222813	3020	3223227	3434	3223640	3847	3224054	207
1	4467	4674	4881	5087	5294	5501	5707	5914	6121	7
2	6534	6740	6947	7153	7360	7567	7773	7980	8186	7
3	8599	8806	9012	9219	9425	9632	9838	0045	3230251	6
4	3230664	0870	3231077	1283	3231489	1696	3231902	2108	2315	6
5	2727	2934	3140	3346	3552	3759	3965	4171	4377	6
6	4790	4996	5202	5408	5615	5821	6027	6233	6439	6
7	6851	7058	7264	7470	7676	7882	8088	8294	8500	6
8	8912	9118	9324	9530	9736	9942	3240148	0354	3240560	6
9	3240972	1178	3241384	1589	3241795	2001	2207	2413	2619	6
2110	3030	3236	3442	3648	3854	4059	4265	4471	4677	6
1	5088	5294	5499	5705	5911	6117	6322	6528	6734	6
2	7145	7350	7556	7762	7967	8173	8378	8584	8789	6
3	9201	9406	9612	9817	3250023	0228	3250433	0639	3250844	5
4	3251255	1461	3251666	1872	2077	2282	2488	2693	2898	5
5	3309	3514	3720	3925	4130	4336	4541	4746	4951	5
6	5362	5567	5772	5978	6183	6388	6593	6798	7003	5
7	7414	7619	7824	8029	8234	8439	8644	8849	9055	5
8	9465	9670	9875	0080	3260285	0490	3260695	0900	3261105	5
9	3261515	1719	3261924	2129	2334	2539	2744	2949	3154	5
2120	3563	3768	3973	4178	4383	4588	4792	4997	5202	5
1	5611	5816	6021	6226	6430	6635	6840	7044	7249	5
2	7658	7863	8068	8272	8477	8682	8886	9091	9295	5
3	9705	9909	3270114	0318	3270523	0727	3270932	1136	3271341	5
4	3271750	1954	2158	2363	2567	2772	2976	3181	3385	4
5	3794	3998	4202	4407	4611	4815	5020	5224	5428	4
6	5837	6041	6245	6450	6654	6858	7062	7267	7471	4
7	7879	8083	8287	8492	8696	8900	9104	9308	9512	4
8	9920	0124	3280328	0533	3280737	0941	3281145	1349	3281553	4
9	3281961	2165	2369	2572	2776	2980	3184	3388	3592	4
2130	4000	4204	4408	4612	4815	5019	5223	5427	5631	4
1	6038	6242	6446	3650	6853	7057	7261	7465	7668	4
2	8076	8279	8483	8687	8890	9094	9298	9501	9705	4
3	3290112	0316	3290519	0723	3290926	1130	3291334	1537	3291741	4
4	2148	2351	2555	2758	2962	3165	3369	3572	3775	3
5	4182	4386	4589	4792	4996	5199	5402	5606	5809	3
6	6216	6419	6622	6826	7029	7232	7436	7639	7842	3
7	8248	8452	8655	8858	9061	9264	9468	9671	9874	3
8	3300280	0483	3300686	0889	3301093	1296	3301499	1702	3301905	3
9	2311	2514	2717	2920	3123	3326	3529	3732	3935	3
2140	4341	4544	4747	4949	5152	5355	5558	5761	5964	3
1	6370	6572	6775	6978	7181	7384	7586	7789	7992	3
2	8397	8600	8803	9006	9208	9411	9614	9816	3310019	3
3	3310424	0627	3310830	1032	3311235	1437	3311640	1843	2045	3
4	2450	2653	2855	3058	3261	3463	3666	3868	4070	2
5	4475	4678	4880	5083	5285	5488	5690	5892	6095	2
6	6500	6702	6904	7107	7309	7511	7714	7916	8118	2
7	8523	8725	8927	9129	9332	9534	9736	9938	3320141	2
8	3320545	0747	3320949	1151	3321354	1556	3321758	1960	2162	2
9	2566	2768	2970	3172	3374	3577	3779	3981	4183	2
2150	4587	4789	4991	5193	5394	5596	5798	6000	6202	3
1	6606	6808	7010	7212	7414	7615	7817	8019	8221	3
2	8624	8826	9028	9230	9432	9633	9835	0037	3330239	3
3	3330642	0844	3331045	1247	3331449	1650	3331852	2054	2255	2
4	2659	2860	3062	3263	3465	3667	3868	4070	4271	1
5	4674	4876	5077	5279	5480	5682	5883	6085	6286	1
6	6689	6890	7092	7293	7495	7696	7897	8099	8300	1
7	8703	8904	9105	9307	9508	9709	9911	0112	3340313	1
8	3340716	0917	3341118	1319	3341521	1722	3341923	2124	2325	1
9	2728	2929	3130	3331	3532	3733	3934	4135	4336	1
	1	2	3	4	5	6	7	8	9	

Between 21600 = log.$^{-1}$ 4·3344538, and 22200 = log.$^{-1}$ 4·3463530.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
2160	3344739	4940	3345141	5342	3345543	5744	3345945	6146	3346347	201
1	6749	6950	7151	7351	7552	7753	7954	8155	8356	1
2	8758	8959	9159	9360	9561	9762	9963	/0164	3350364	1
3	3350766	0967	3351168	1368	3351569	1770	3351970	2171	2372	1
4	2773	2974	3175	3375	3576	3777	3977	4178	4378	1
5	4780	4980	5181	5381	5582	5782	5983	6183	6384	1
6	6785	6986	7186	7386	7587	7787	7988	8188	8389	200
7	8790	8990	9190	9391	9591	9791	9992	/0192	3360392	0
8	3360793	0993	3361194	1394	3361594	1795	3361995	2195	2395	0
9	2796	2996	3196	3396	3597	3797	3997	4197	4397	0
2170	4797	4998	5198	5398	5598	5798	5998	6198	6398	0
1	6798	6998	7198	7398	7598	7798	7998	8198	8398	0
2	8798	8998	9198	9398	9598	9798	9998	/0198	3370397	0
3	3370797	0997	3371197	1397	3371596	1796	3371996	2196	2396	0
4	2795	2995	3195	3394	3594	3794	3994	4193	4393	0
5	4792	4992	5192	5391	5591	5791	5990	6190	6389	0
6	6788	6988	7188	7387	7587	7786	7986	8185	8385	0
7	8784	8983	9183	9382	9582	9781	9981	/0180	3380379	199
8	3380778	0978	3381177	1376	3381576	1775	3381974	2174	2373	9
9	2772	2971	3170	3369	3569	3768	3967	4166	4366	9
2180	4764	4963	5163	5362	5561	5760	5959	6158	6358	9
1	6756	6955	7154	7353	7552	7751	7950	8149	8348	9
2	8746	8946	9145	9344	9543	9742	9940	/0139	3390338	9
3	3390736	0935	3391134	1333	3391532	1731	3391930	2129	2327	9
4	2725	2924	3123	3322	3520	3719	3918	4117	4316	9
5	4713	4912	5111	5309	5508	5707	5906	6104	6303	9
6	6700	6899	7098	7296	7495	7693	7892	8091	8289	9
7	8686	8885	9084	9282	9481	9679	9878	/0076	3400275	9
8	3400672	0870	3401069	1267	3401466	1664	3401862	2061	2259	198
9	2656	2854	3053	3251	3449	3648	3846	4045	4243	8
2190	4639	4838	5036	5234	5433	5631	5829	6027	6226	8
1	6622	6820	7018	7217	7415	7613	7811	8009	8207	8
2	8604	8802	9000	9198	9396	9594	9792	9990	3410188	8
3	3410584	0782	3410980	1178	3411376	1574	3411772	1970	2168	8
4	2564	2762	2960	3158	3356	3554	3752	3950	4147	8
5	4543	4741	4939	5137	5334	5532	5730	5928	6126	8
6	6521	6719	6917	7114	7312	7510	7708	7905	8103	8
7	8498	8696	8894	9091	9289	9486	9684	9882	3420079	8
8	3420474	0672	3420870	1067	3421265	1462	3421660	1857	2055	8
9	2450	2647	2845	3042	3240	3437	3635	3832	4029	197
2200	4424	4622	4819	5016	5214	5411	5608	5806	6003	7
1	6398	6595	6792	6990	7187	7384	7581	7779	7976	7
2	8370	8568	8765	8962	9159	9356	9554	9751	9948	7
3	3430342	0539	3430736	0933	3431131	1328	3431525	1722	3431919	7
4	2313	2510	2707	2904	3101	3298	3495	3692	3889	7
5	4283	4480	4677	4874	5071	5268	5464	5661	5858	7
6	6252	6449	6646	6842	7039	7236	7433	7630	7827	7
7	8220	8417	8614	8810	9007	9204	9401	9597	9794	7
8	3440187	0384	3440581	0777	3440974	1171	3441367	1564	3441761	7
9	2134	2350	2547	2743	2940	3137	3333	3530	3726	7
2210	4119	4316	4512	4709	4905	5102	5298	5495	5691	196
1	6084	6280	6477	6673	6869	7066	7262	7459	7655	6
2	8064	8244	8440	8636	8833	9029	9225	9422	9618	6
3	3450010	0207	3450403	0599	3450795	0991	3451188	1384	3451580	6
4	1972	2168	2365	2561	2757	2953	3149	3345	3541	6
5	3933	4129	4325	4522	4718	4914	5110	5306	5502	6
6	5894	6090	6285	6481	6677	6873	7069	7265	7461	6
7	7853	8049	8245	8440	8636	8832	9028	9224	9420	6
8	9811	/0007	3460203	0399	3460594	0790	3460986	1182	3461377	6
9	3461769	1964	2160	2356	2551	2747	2943	3138	3334	6
	1	2	3	4	5	6	7	8	9	

Between 22200 = log.$^{-1}$ 4·3463530, and 22800 = log.$^{-1}$ 4·3579348.

Tens.	1	2	3	4	5	6	7	8	9	dif.
2220	3463725	3921	3464117	4312	3464508	4703	3464899	5094	3465290	196
1	5681	5877	6072	6268	6463	6659	6854	7050	7245	195
2	7636	7831	8027	8222	8418	8613	8808	9004	9199	5
3	9590	9785	9981	0176	3470371	0567	3470762	0957	3471153	5
4	3471543	1738	3471934	/2129	2324	2519	2715	2910	3105	5
5	3495	3691	3886	4081	4276	4471	4666	4861	5056	5
6	5447	5642	5837	6032	6227	6422	6617	6812	7007	5
7	7397	7592	7787	7982	8177	8372	8567	8762	8957	5
8	9347	9542	9737	9931	3480126	0321	3480516	0711	3480906	5
9	3481296	1490	3481685	1880	2075	2270	2464	2659	2854	5
2230	3243	3438	3633	3828	4022	4217	4412	4606	4801	5
1	5190	5385	5580	5774	5969	6164	6358	6553	6747	5
2	7136	7331	7526	7720	7915	8109	8304	8498	8693	5
3	9082	9276	9471	9665	9860	0054	3490248	0443	3490637	194
4	3491026	1220	3491415	1609	3491804	/1998	2192	2387	2581	4
5	2970	3164	3358	3552	3747	3941	4135	4330	4524	4
6	4912	5106	5301	5495	5689	5883	6077	6272	6466	4
7	6854	7048	7242	7436	7630	7825	8019	8213	8407	4
8	8795	8989	9183	9377	9571	9765	9959	0153	3500347	4
9	3500735	0929	3501123	1317	3501511	1705	3501898	/2092	2286	4
2240	2674	2868	3062	3256	3449	3643	3837	4031	4225	4
1	4612	4806	5000	5194	5387	5581	5775	5969	6162	4
2	6550	6743	6937	7131	7325	7518	7712	7905	8099	4
3	8486	8680	8874	9067	9261	9454	9648	9841	3510035	4
4	3510422	0616	3510809	1003	3511196	1390	3511583	1777	1970	193
5	2357	2550	2744	2937	3131	3324	3517	3711	3905	3
6	4291	4484	4678	4871	5064	5258	5451	5644	5837	3
7	6224	6417	6611	6804	6997	7190	7383	7577	7770	3
8	8156	8349	8543	8736	8929	9122	9315	9508	9701	3
9	3520088	0281	3520474	0667	3520860	1053	3521246	1439	3521632	3
2250	2018	2211	2404	2597	2790	2983	3176	3369	3562	3
1	3948	4141	4334	4527	4720	4912	5105	5298	5491	3
2	5877	6070	6262	6455	6648	6841	7034	7226	7419	3
3	7805	7997	8190	8383	8576	8768	8961	9154	9346	3
4	9732	9924	3530117	0310	3530502	0695	3530888	1080	3531273	3
5	3531658	1851	2043	2236	2428	2621	2813	3006	3198	3
6	3583	3776	3968	4161	4353	4546	4738	4931	5123	192
7	5508	5700	5893	6085	6278	6470	6662	6855	7047	2
8	7432	7624	7816	8009	8201	8393	8586	8778	8970	2
9	9355	9547	9739	9931	3540123	0316	3540508	0700	3540892	2
2260	3541277	1469	3541661	1853	2045	2237	2429	2621	2814	2
1	3198	3390	3582	3774	3966	4158	4350	4542	4734	2
2	5118	5310	5502	5694	5886	6078	6270	6462	6654	2
3	7037	7229	7421	7613	7805	7997	8189	8381	8572	2
4	8956	9148	9340	9531	9723	9915	3550107	0299	3550490	2
5	3550874	1066	3551257	1449	3551641	1832	2024	2216	2407	2
6	2791	2982	3174	3366	3557	3749	3940	4132	4324	2
7	4707	4898	5090	5281	5473	5664	5856	6048	6239	2
8	6622	6813	7005	7196	7388	7579	7771	7962	8154	191
9	8536	8728	8919	9111	9302	9493	9685	9876	3560067	1
2270	3560450	0641	3560832	1024	3561215	1406	3561598	1789	1980	1
1	2363	2554	2745	2936	3127	3319	3510	3701	3892	1
2	4274	4466	4657	4848	5039	5230	5421	5612	5803	1
3	6185	6376	6568	6759	6950	7141	7332	7523	7714	1
4	8096	8287	8478	8668	8859	9050	9241	9432	9623	1
5	3570005	0196	3570387	0578	3570768	0959	3571150	1341	3571532	1
6	1913	2104	2295	2486	2677	2867	3058	3249	3440	1
7	3821	4012	4202	4393	4584	4775	4965	5156	5347	1
8	5728	5918	6109	6300	6490	6681	6872	7062	7253	1
9	7634	7824	8015	8205	8396	8586	8777	8967	9158	1
	1	2	3	4	5	6	7	8	9	

Between 22800 = log.$^{-1}$ 4·3579348, and 23400 = log.$^{-1}$ 4·3692159.

tens.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	*dif.*
2280	3579539	9729	3579920	/0110	3580301	0491	3580682	0872	3581062	190
1	3581443	1634	3581824	2014	2205	2395	2585	2776	2966	0
2	3347	3537	3727	3918	4108	4298	4488	4679	4869	0
3	5249	5440	5630	5820	6010	6200	6391	6581	6771	0
4	7151	7341	7531	7722	7912	8102	8292	8482	8672	0
5	9052	9242	9432	9622	9812	/0002	3590192	0382	3590572	0
6	3590952	1142	3591332	1522	3591712	1902	2092	2282	2472	0
7	2852	3041	3231	3421	3611	3801	3991	4181	4370	0
8	4750	4940	5130	5319	5509	5699	5889	6078	6268	0
9	6648	6837	7027	7217	7406	7596	7786	7976	8165	0
2290	8544	8734	8924	9113	9303	9493	9682	9872	3600061	0
1	3600440	0630	3600820	1009	3601199	1388	3601578	1767	1957	0
2	2336	2525	2715	2904	3093	3283	3472	3662	3851	189
3	4230	4419	4609	4798	4987	5177	5366	5555	5745	9
4	6123	6313	6502	6691	6881	7070	7259	7448	7638	9
5	8016	8205	8395	8584	8773	8962	9151	9341	9530	9
6	9908	/0097	3610286	0475	3610664	0854	3611043	1232	3611421	9
7	3611799	1988	2177	2366	2555	2744	2933	3122	3311	9
8	3689	3878	4067	4256	4445	4634	4823	5012	5201	9
9	5579	5768	5956	6145	6334	6523	6712	6901	7090	9
2300	7467	7656	7845	8034	8222	8411	8600	8789	8977	9
1	9355	9544	9732	9921	3620110	0298	3620487	0676	3620865	9
2	3621242	1430	3621619	1808	1996	2185	2374	2562	2751	9
3	3128	3317	3505	3694	3882	4071	4259	4448	4636	9
4	5013	5202	5390	5579	5767	5956	6144	6332	6521	188
5	6898	7086	7275	7463	7651	7840	8028	8216	8405	8
6	8781	8970	9158	9346	9535	9723	9911	/0099	3630288	8
7	3630664	0852	3631041	1229	3631417	1605	3631794	1982	2170	8
8	2546	2734	2923	3111	3299	3487	3675	3863	4051	8
9	4427	4615	4804	4992	5180	5368	5556	5744	5932	8
2310	6308	6496	6684	6872	7060	7248	7436	7624	7812	8
1	8187	8375	8563	8751	8939	9127	9315	9503	9690	8
2	3640066	0254	3640442	0630	3640817	1005	3641193	1381	3641569	8
3	1944	2132	2320	2507	2695	2883	3070	3258	3446	8
4	3821	4009	4197	4384	4572	4759	4947	5135	5322	8
5	5698	5885	6073	6260	6448	6635	6823	7010	7198	8
6	7573	7761	7948	8136	8323	8511	8698	8885	9073	187
7	9448	9635	9823	/0010	3650197	0385	3650572	0760	3650947	7
8	3651322	1509	3651696	1884	2071	2258	2446	2633	2820	7
9	3195	3382	3569	3757	3944	4131	4318	4505	4693	7
2320	5067	5254	5441	5629	5816	6003	6190	6377	6564	7
1	6939	7126	7313	7500	7687	7874	8061	8248	8435	7
2	8809	8996	9183	9370	9557	9744	9931	/0118	3660305	7
3	3660679	0866	3661053	1240	3661427	1614	3661801	1987	2174	7
4	2548	2735	2922	3109	3296	3482	3669	3856	4043	7
5	4416	4603	4790	4977	5163	5350	5537	5724	5910	7
6	6284	6471	6657	6844	7031	7217	7404	7591	7777	7
7	8150	8337	8524	8710	8897	9083	9270	9457	9643	7
8	3670016	0203	3670389	0576	3670762	0949	3671135	1322	3671508	7
9	1881	2068	2254	2441	2627	2814	3000	3186	3373	186
2330	3746	3932	4118	4305	4491	4677	4864	5050	5236	6
1	5609	5795	5982	6168	6354	6540	6727	6913	7099	6
2	7472	7658	7844	8030	8217	8403	8589	8775	8961	6
3	9334	9520	9706	9892	3680078	0264	3680450	0636	3680822	6
4	3681195	1381	3681567	1753	1939	2125	2311	2497	2683	6
5	3055	3241	3427	3613	3799	3985	4171	4357	4542	6
6	4914	5100	5286	5472	5658	5844	6030	6215	6401	6
7	6773	6959	7145	7330	7516	7702	7888	8074	8259	6
8	8631	8817	9002	9188	9374	9559	9745	9931	3690117	6
9	3690498	0674	3690859	1045	3691230	1416	3691602	1787	1973	6
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	

Between 23400 = log.$^{-1}$ 4·3692159, and 24000 = log.$^{-1}$ 4·3802112.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
2340	3692344	2530	3692715	2901	3693086	3272	3693458	3643	3693829	186
1	4200	4385	4571	4756	4942	5127	5313	5498	5683	5
2	6054	6240	6425	6611	6796	6981	7167	7352	7538	5
3	7908	8094	8279	8464	8650	8835	9020	9205	9391	5
4	9761	9947	3700132	0317	3700502	0688	3700873	1058	3701243	5
5	3701614	1799	1984	2169	2354	2540	2725	2910	3095	5
6	3456	3650	3835	4020	4206	4391	4576	4761	4946	5
7	5316	5501	5686	5871	6056	6241	6426	6611	6796	5
8	7166	7351	7536	7721	7906	8091	8275	8460	8645	5
9	9015	9200	9385	9570	9754	9939	3710124	0309	3710494	5
2350	3710863	1048	3711233	1418	3711603	1787	1972	2157	2342	5
1	2711	2896	3080	3265	3450	3635	3819	4004	4189	5
2	4558	4742	4927	5112	5296	5481	5666	5850	6035	5
3	6404	6588	6773	6957	7142	7327	7511	7696	7880	5
4	8249	8434	8618	8802	8987	9171	9356	9540	9725	184
5	3720094	0278	3720462	0647	3720831	1015	3721200	1384	3721569	4
6	1937	2122	2306	2490	2674	2859	3043	3227	3412	4
7	3780	3964	4149	4333	4517	4701	4885	5070	5254	4
8	5622	5806	5991	6175	6359	6543	6727	6911	7095	4
9	7464	7646	7832	8016	8200	8384	8568	8752	8936	4
2360	9304	9488	9672	9856	3730040	0224	3730408	0592	3730776	4
1	3731144	1328	3731512	1696	1879	2063	2247	2431	2615	4
2	2983	3167	3350	3534	3718	3902	4086	4270	4453	4
3	4821	5005	5189	5372	5556	5740	5924	6107	6291	4
4	6658	6842	7026	7210	7393	7577	7761	7944	8128	4
5	8495	8679	8862	9046	9230	9413	9597	9780	9964	4
6	3740331	0515	3740698	0882	3741065	1249	3741432	1616	3741799	4
7	2166	2350	2533	2716	2900	3083	3267	3450	3634	183
8	4000	4184	4367	4551	4734	4917	5101	5284	5467	3
9	5834	6017	6201	6384	6567	6750	6934	7117	7300	3
2370	7667	7850	8033	8216	8400	8583	8766	8949	9132	3
1	9499	9682	9865	/ 0048	3750231	0414	3750598	0781	3750964	3
2	3751330	1513	3751696	1879	2062	2245	2428	2611	2794	3
3	3160	3343	3526	3709	3892	4075	4258	4441	4624	3
4	4990	5173	5356	5539	5722	5905	6088	6270	6453	3
5	6819	7002	7185	7367	7550	7733	7916	8099	8282	3
6	8647	8630	9013	9195	9378	9561	9744	9926	3760109	3
7	3760475	0657	3760840	1023	3761205	1388	3761571	1753	1936	3
8	2301	2484	2666	2849	3032	3214	3397	3579	3762	3
9	4127	4310	4492	4675	4857	5040	5222	5405	5587	3
2380	5952	6135	6317	6499	6682	6864	7047	7229	7412	182
1	7776	7959	8141	8323	8506	8688	8871	9053	9235	2
2	9600	9782	9965	0147	3770329	0511	3770694	0876	3771058	2
3	3771423	1605	3771787	/ 1969	2152	2334	2516	2698	2880	2
4	3245	3427	3609	3791	3973	4155	4338	4520	4702	2
5	5066	5248	5430	5612	5794	5976	6158	6340	6522	2
6	6886	7068	7250	7432	7614	7796	7978	8160	8342	2
7	8706	8888	9070	9252	9434	9616	9798	9979	3780161	2
8	3780525	0707	3780889	1071	3781252	1434	3781616	1798	1960	2
9	2343	2525	2707	2889	3070	3252	3434	3616	3797	2
2390	4161	4342	4524	4706	4887	5069	5251	5432	5614	2
1	5977	6159	6341	6522	6704	6885	7067	7249	7430	2
2	7793	7975	8156	8338	8519	8701	8882	9064	9245	2
3	9608	9790	9971	/ 0153	3790334	0516	3790697	0879	3791060	181
4	3791423	1604	3791786	1967	2148	2330	2511	2692	2874	1
5	3237	3418	3599	3780	3962	4143	4324	4506	4687	1
6	5049	5231	5412	5593	5774	5956	6137	6318	6499	1
7	6862	7043	7224	7405	7586	7767	7948	8130	8311	1
8	8673	8854	9035	9216	9397	9578	9759	9940	3800121	1
9	3800484	0665	3800846	1027	3801208	1389	3801570	1750	1931	1
	1	2	3	4	5	6	7	8	9	

Between 24000 = log.$^{-1}$ 4·3802112, and 24600 = log.$^{-1}$ 4·3909351.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
2400	3802293	2474	3802655	2836	3803017	3198	3803379	3560	3803741	181
1	4102	4283	4464	4645	4826	5007	5188	5368	5549	1
2	5911	6092	6272	6453	6634	6815	6995	7176	7357	1
3	7718	7899	8080	8261	8441	8622	8803	8983	9164	1
4	9525	9706	9887	0067	3810248	0428	3810609	0790	3810970	1
5	3811331	1512	3811693	1873	2054	2234	2415	2595	2776	1
6	3137	3317	3498	3678	3859	4039	4220	4400	4580	0
7	4941	5122	5302	5483	5663	5843	6024	6204	6384	0
8	6745	6926	7106	7286	7467	7647	7827	8007	8188	0
9	8548	8729	8909	9089	9269	9450	9630	9810	9990	0
2410	3820351	0531	3820711	0891	3821071	1252	3821432	1612	3821792	0
1	2152	2332	2512	2693	2873	3053	3233	3413	3593	0
2	3953	4133	4313	4493	4673	4853	5033	5213	5393	0
3	5753	5933	6113	6293	6473	6653	6833	7013	7193	0
4	7553	7732	7912	8092	8272	8452	8632	8812	8992	0
5	9351	9531	9711	9891	3830070	0250	3830430	0610	3830790	0
6	3831149	1329	3831509	1688	1868	2048	2227	2407	2587	0
7	2946	3126	3306	3485	3665	3844	4024	4204	4383	0
8	4743	4922	5102	5281	5461	5640	5820	6000	6179	0
9	6538	6718	6897	7077	7256	7436	7615	7795	7974	179
2420	8333	8513	8692	8871	9051	9230	9410	9589	9769	9
1	3840127	0307	3840486	0665	3840845	1024	3841203	1383	3841562	9
2	1921	2100	2279	2459	2638	2817	2996	3176	3355	9
3	3713	3893	4072	4251	4430	4609	4789	4968	5147	9
4	5505	5684	5864	6043	6222	6401	6580	6759	6938	9
5	7297	7476	7655	7834	8013	8192	8371	8550	8729	9
6	9087	9266	9445	9624	9803	9982	3850161	0340	3850519	9
7	3850877	1056	3851235	1413	3851592	1771	1950	2129	2308	9
8	2666	2845	3023	3202	3381	3560	3739	3918	4097	9
9	4454	4633	4812	4990	5169	5348	5527	5705	5884	9
2430	6241	6420	6599	6778	6956	7135	7314	7492	7671	9
1	8028	8207	8386	8564	8743	8921	9100	9279	9457	9
2	9814	9993	3860171	0350	3860528	0707	3860886	1064	3861243	9
3	3861600	1778	1957	2135	2314	2492	2670	2849	3027	178
4	3384	3563	3741	3919	4098	4276	4455	4633	4811	8
5	5168	5346	5525	5703	5881	6060	6238	6416	6595	8
6	6951	7129	7308	7486	7664	7842	8021	8199	8377	8
7	8733	8912	9090	9268	9446	9624	9803	9981	3870159	8
8	3870515	0693	3870871	1049	3871228	1406	3871584	1762	1940	8
9	2296	2474	2652	2830	3008	3186	3364	3542	3720	8
2440	4076	4254	4432	4610	4788	4966	5144	5322	5500	8
1	5856	6034	6212	6389	6567	6745	6923	7101	7279	8
2	7934	7812	7990	8168	8346	8524	8701	8879	9057	8
3	9412	9590	9768	9946	3880123	0301	3880479	0657	3880834	8
4	3881190	1367	3881545	1723	1900	2078	2256	2433	2611	8
5	2966	3144	3321	3499	3677	3854	4032	4209	4387	8
6	4742	4920	5097	5275	5452	5630	5807	5985	6162	8
7	6517	6695	6872	7050	7227	7404	7582	7759	7937	177
8	8292	8469	8646	8824	9001	9178	9356	9533	9711	7
9	3890065	0243	3890420	0597	3890774	0952	3891129	1306	3891484	7
2450	1838	2015	2193	2370	2547	2724	2902	3079	3256	7
1	3610	3787	3965	4142	4319	4496	4673	4850	5028	7
2	5382	5559	5736	5913	6090	6267	6444	6621	6798	7
3	7153	7330	7507	7684	7861	8038	8215	8392	8569	7
4	8923	9100	9276	9453	9630	9807	9984	0161	3900338	7
5	3900692	0869	3901046	1223	3901399	1576	3901753	1930	2107	7
6	2460	2637	2814	2991	3168	3344	3521	3698	3875	7
7	4228	4405	4582	4759	4935	5112	5289	5465	5642	7
8	5995	6172	6349	6525	6702	6879	7055	7232	7409	7
9	7762	7939	8115	8292	8468	8645	8821	8998	9175	7
	1	2	3	4	5	6	7	8	9	

Between 24600 = log.⁻¹ 4·3909351, and 25200 = log.⁻¹ 4·4014005.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
2460	3909528	9704	3909881	/0057	3910234	0410	3910587	0763	3910940	177
1	3911293	1469	3911646	1822	1998	2175	2351	2528	2704	6
2	3057	3233	3410	3586	3762	3939	4115	4291	4468	6
3	4820	4997	5173	5349	5526	5702	5878	6055	6231	6
4	6583	6760	6936	7112	7288	7464	7641	7817	7993	6
5	8345	8522	8698	8874	9050	9226	9402	9578	9755	6
6	3920107	0283	3920459	0635	3920811	0987	3921163	1339	3921515	6
7	1868	2044	2220	2396	2572	2748	2924	3100	3276	6
8	3628	3803	3979	4155	4331	4507	4683	4859	5035	6
9	5387	5563	5739	5914	6090	6266	6442	6618	6794	6
2470	7145	7321	7497	7673	7849	8024	8200	8376	8552	6
1	8903	9079	9255	9430	9606	9782	9958	/0133	3930309	6
2	3930660	0836	3931012	1187	3931363	1539	3931714	1890	2066	6
3	2417	2592	2768	2944	3119	3295	3470	3646	3821	6
4	4172	4348	4524	4699	4875	5050	5226	5401	5577	6
5	5928	6103	6278	6454	6629	6805	6980	7156	7331	175
6	7682	7857	8033	8208	8383	8559	8734	8909	9085	5
7	9435	9611	9786	9961	3940137	0312	3940487	0662	3940838	5
8	3941188	1364	3941539	1714	1889	2064	2240	2415	2590	5
9	2940	3116	3291	3466	3641	3816	3991	4167	4342	5
2480	4692	4867	5042	5217	5392	5567	5742	5918	6093	5
1	6443	6618	6793	6968	7143	7318	7493	7668	7843	5
2	8193	8368	8543	8718	8893	9068	9242	9417	9592	5
3	9942	/0117	3950292	0467	3950642	0817	3950991	1166	3951341	5
4	3951691	1866	2040	2215	2390	2565	2740	2914	3089	5
5	3439	3613	3788	3963	4138	4312	4487	4662	4837	5
6	5186	5361	5535	5710	5885	6059	6234	6409	6583	5
7	6932	7107	7282	7456	7631	7805	7980	8155	8329	5
8	8678	8853	9027	9202	9376	9551	9725	9900	3960074	5
9	3960423	0598	3960772	0947	3961121	1296	3961470	1645	1819	174
2490	2168	2342	2517	2691	2865	3040	3214	3389	3563	4
1	3912	4086	4260	4435	4609	4783	4958	5132	5306	4
2	5655	5829	6003	6177	6352	6526	6700	6874	7049	4
3	7397	7571	7745	7920	8094	8268	8442	8616	6790	4
4	9139	9313	9487	9661	9835	/0009	3970183	0357	3970531	4
5	3970880	1054	3971228	1402	3971576	1750	1924	2098	2272	4
6	2620	2794	2968	3142	3316	3490	3664	3838	4011	4
7	4359	4533	4707	4881	5055	5229	5403	5577	5750	4
8	6098	6272	6446	6620	6794	6967	7141	7315	7489	4
9	7836	8010	8184	8358	8531	8705	8879	9053	9226	4
2500	9574	9748	9921	/0095	3980269	0442	3980616	0790	3980963	4
1	3981311	1484	3981658	1831	2005	2179	2352	2526	2699	4
2	3047	3220	3394	3567	3741	3914	4088	4261	4435	4
3	4782	4956	5129	5302	5476	5649	5823	5996	6170	173
4	6517	6690	6864	7037	7210	7384	7557	7731	7904	3
5	8251	8424	8597	8771	8944	9117	9291	9464	9637	3
6	9984	/0157	3990331	0504	3990677	0850	3991024	1197	3991370	3
7	3991717	1890	2063	2236	2409	2583	2756	2929	3102	3
8	3448	3622	3795	3968	4141	4314	4487	4660	4834	3
9	5180	5353	5526	5699	5872	6045	6218	6391	6564	3
2510	6910	7083	7256	7429	7602	7775	7948	8121	8294	3
1	8640	8813	8986	9159	9332	9505	9678	9851	4000023	3
2	4000369	0542	4000715	0888	4001061	1234	4001406	1579	1752	3
3	2098	2271	2443	2616	2789	2962	3134	3307	3480	3
4	3825	3998	4171	4344	4516	4689	4862	5035	5207	3
5	5553	5725	5898	6071	6243	6416	6588	6761	6934	3
6	7279	7452	7624	7797	7969	8142	8314	8487	8660	3
7	9005	9177	9350	9522	9695	9867	4010040	0212	4010385	3
8	4010730	0902	4011075	1247	4011420	1592	1764	1937	2109	172
9	2454	2626	2799	2971	3144	3316	3488	3661	3833	2
	1	2	3	4	5	6	7	8	9	

Between 25200 = log.$^{-1}$ 4·4014005, and 25800 = log.$^{-1}$ 4·4116197.

tens.	1	2	3	4	5	6	7	8	9	dif.
2520	4014178	4350	4014522	4695	4014867	5039	4015212	5384	4015556	172
1	5901	6073	6245	6417	6590	6762	6934	7106	7279	2
2	7623	7795	7967	8140	8312	8484	8656	8828	9000	2
3	9345	9517	9689	9861	4020033	0205	4020377	0549	4020721	2
4	4021066	1238	4021410	1582	1754	1926	2098	2270	2442	2
5	2786	2958	3130	3302	3474	3646	3818	3990	4162	2
6	4505	4677	4849	5021	5193	5365	5537	5709	5881	2
7	6224	6396	6568	6740	6912	7083	7255	7427	7599	2
8	7942	8114	8286	8458	8630	8801	8973	9145	9317	2
9	9660	9832	4030003	0175	4030347	0519	4030690	0862	4031034	2
2530	4031377	1549	1720	1892	2063	2235	2407	2578	2750	2
1	3093	3265	3436	3608	3779	3951	4122	4294	4465	2
2	4809	4980	5152	5323	5495	5666	5838	6009	6180	171
3	6523	6695	6866	7038	7209	7381	7552	7723	7895	1
4	8237	8409	8580	8752	8923	9094	9266	9437	9608	1
5	9951	0122	4040294	0465	4040636	0807	4040979	1150	4041321	1
6	4041664 /	1835	2006	2177	2349	2520	2691	2862	3033	1
7	3376	3547	3718	3889	4061	4232	4403	4574	4745	1
8	5087	5258	5429	5601	5772	5943	6114	6285	6456	1
9	6798	6969	7140	7311	7482	7653	7824	7995	8166	1
2540	8508	8679	8850	9021	9192	9363	9534	9705	9876	1
1	4050218	0388	4050559	0730	4050901	1072	4051243	1414	4051585	1
2	1926	2097	2268	2439	2610	2780	2951	3122	3293	1
3	2634	3805	3976	4147	4317	4488	4659	4830	5000	1
4	5342	5512	5683	5854	6025	6195	6366	6537	6707	1
5	7049	7219	7390	7560	7731	7902	8072	8243	8413	1
6	8755	8925	9096	9266	9437	9607	9778	9948	4060119	1
7	4060460	0630	4060801	0971	4061142	1312	4061483	1653	1824	170
8	2165	2335	2506	2676	2846	3017	3187	3358	3528	0
9	3869	4039	4209	4380	4550	4721	4891	5061	5231	0
2550	5572	5742	5913	6083	6253	6424	6594	6764	6934	0
1	7275	7445	7615	7786	7956	8126	8296	8466	8637	0
2	8977	9147	9317	9487	9658	9828	9998 /	0168	4070338	0
3	1070678	0848	4071018	1189	4071359	1529	4071699	1869	2039	0
4	2379	2549	2719	2889	3059	3229	3399	3569	3739	0
5	4079	4249	4419	4589	4759	4929	5099	5269	5439	0
6	5778	5948	6118	6288	6458	6628	6798	6968	7137	0
7	7477	7647	7817	7987	8156	8326	8496	8666	8836	0
8	9175	9345	9515	9684	9854 /	0024	4080194	0363	4080533	0
9	4080873	1042	4081212	1382	4081551	1721	1891	2060	2230	0
2560	2569	2739	2909	3078	3248	3417	3587	3757	3926	0
1	4265	4435	4604	4774	4944	5113	5283	5452	5622	169
2	5961	6130	6300	6469	6639	6808	6978	7147	7317	9
3	7656	7825	7994	8164	8333	8503	8672	8841	9011	9
4	9350	9519	9688	9858	4090027	0196	4090366	0535	4090704	9
5	4091043	1212	4091382	1551	1720	1889	2059	2228	2397	9
6	2736	2905	3074	3243	3413	3582	3751	3920	4089	9
7	4428	4597	4766	4935	5105	5274	5443	5612	5781	9
8	6119	6288	6458	6627	6796	6965	7134	7303	7472	9
9	7810	7979	8148	8317	8486	8655	8824	8993	9162	9
2570	9500	9669	9838 /	0007	4100176	0345	4100514	0683	4100852	9
1	4101190	1359	4101527	1696	1865	2034	2203	2372	2541	9
2	2878	3047	3216	3385	3554	3723	3891	4060	4229	9
3	4567	4735	4904	5073	5242	5410	5579	5748	5917	9
4	6254	6423	6592	6760	6929	7098	7266	7435	7604	9
5	7941	8110	8278	8447	8616	8784	8953	9121	9290	9
6	9627	9796	9964 /	0133	4110301	0470	4110639	0807	4110976	9
7	4111313	1481	4111650	1818	1987	2155	2324	2492	2661	168
8	2998	3166	3334	3503	3671	3840	4008	4177	4345	8
9	4682	4850	5019	5187	5355	5524	5692	5860	6029	8
	1	2	3	4	5	6	7	8	9	

Between 25800 = log.$^{-1}$ 4·4116197, and 26400 = log.$^{-1}$ 4·4216039.

tens.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	*dif.*
2580	4116365	6534	4116702	6870	4117039	7207	4117375	7544	4117712	168
1	8048	8217	8385	8553	8721	8890	9058	9226	9394	8
2	9731	9899	4120067	0235	4120403	0571	4120740	0908	4121076	8
3	4121412	1580	1748	1917	2085	2253	2421	2589	2757	8
4	3093	3261	3429	3597	3765	3933	4101	4269	4437	8
5	4773	4941	5109	5277	5445	5613	5781	5949	6117	8
6	6453	6621	6789	6957	7125	7293	7461	7629	7796	8
7	8132	8300	8468	8636	8804	8971	9139	9307	9475	8
8	9811	9978	4130146	0314	4130482	0649	4130817	0985	4131153	8
9	4131488	1656	1824	1991	2159	2327	2495	2662	2830	8
2590	3165	3333	3501	3668	3836	4004	4171	4339	4507	8
1	4842	5009	5177	5345	5512	5680	5847	6015	6182	8
2	6518	6685	6853	7020	7188	7355	7523	7690	7858	8
3	8193	8360	8528	8695	8863	9030	9197	9365	9532	167
4	9867	/0035	4140202	0369	4140537	0704	4140872	1039	4141206	7
5	4141541	1708	1876	2043	2210	2378	2545	2712	2880	7
6	3214	3381	3549	3716	3883	4051	4218	4385	4552	7
7	4887	5054	5221	5388	5556	5723	5890	6057	6224	7
8	6559	6726	6893	7060	7227	7394	7561	7729	7896	7
9	8230	8397	8564	8731	8898	9065	9232	9399	9566	7
2600	9901	/0068	4150235	0402	4150569	0736	4150903	1070	4151237	7
1	4151570	1737	1904	2071	2238	2405	2572	2739	2906	7
2	3240	3407	3574	3741	3907	4074	4241	4408	4575	7
3	4909	5075	5242	5409	5576	5743	5909	6076	6243	7
4	6577	6743	6910	7077	7244	7410	7577	7744	7911	7
5	8244	8411	8577	8744	8911	9077	9244	9411	9577	7
6	9911	/0077	4160244	0411	4160577	0744	4160911	1077	4161244	7
7	4161577	1743	1910	2077	2243	2410	2576	2743	2909	7
8	3242	3409	3575	3742	3908	4075	4241	4408	4574	166
9	4907	5074	5240	5407	5573	5739	5906	6072	6239	6
2610	6571	6738	6904	7071	7237	7403	7570	7736	7902	6
1	8235	8401	8568	8734	8900	9067	9233	9399	9565	6
2	9898	/0064	4170231	0397	4170563	0729	4170895	1062	4171228	6
3	4171560	1726	1893	2059	2225	2391	2557	2724	2890	6
4	3222	3388	3554	3720	3886	4053	4219	4385	4551	6
5	4883	5049	5215	5318	5547	5713	5879	6045	6211	6
6	6543	6709	6875	7041	7207	7373	7539	7705	7871	6
7	8203	8369	8535	8701	8867	9033	9199	9365	9531	6
8	9862	/0028	4180194	0360	4180526	0692	4180857	1023	4181189	6
9	4181521	1687	1852	2018	2184	2350	2516	2681	2847	6
2620	3179	3344	3510	3676	3842	4007	4173	4339	4505	6
1	4836	5002	5167	5333	5499	5664	5830	5996	6161	6
2	6493	6658	6824	6989	7155	7321	7486	7652	7817	6
3	8148	8314	8480	8645	8811	8976	9142	9307	9473	6
4	9804	9969	4190135	0300	4190466	0631	4190797	0962	4191128	165
5	4191459	1624	1789	1955	2120	2286	2451	2616	2782	5
6	3113	3278	3443	3609	3774	3939	4105	4270	4435	5
7	4766	4931	5097	5262	5427	5593	5758	5923	6088	5
8	6419	6584	6749	6915	7080	7245	7410	7575	7741	5
9	8071	8236	8401	8567	8732	8897	9062	9227	9392	5
2630	9723	9888	4200053	0218	4200383	0548	4200713	0878	4201043	5
1	4201374	1539	1704	1869	2034	2199	2364	2529	2694	5
2	3024	3189	3354	3519	3684	3849	4014	4179	4344	5
3	4674	4838	5003	5168	5333	5498	5663	5828	5993	5
4	6323	6487	6652	6817	6982	7147	7312	7477	7641	5
5	7971	8136	8301	8465	8630	8795	8960	9125	9289	5
6	9619	9784	9948	/0113	4210278	0442	4210607	0772	4210937	5
7	4211266	1431	4211595	1760	1925	2089	2254	2419	2583	5
8	2913	3077	3242	3406	3571	3736	3900	4065	4229	5
9	4558	4723	4888	5052	5217	5381	5546	5710	5875	5
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	

Between 26400 = $\log.^{-1}$ 4·4216039, and 27000 = $\log.^{-1}$ 4·4313638.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
2640	4216204	6368	4216533	6697	4216862	7026	4217191	7355	4217520	164
1	7848	8013	8177	8342	8506	8671	8835	8999	9164	4
2	9493.	9657	9821	9986	4220150	0314	4220479	0643	4220807	4
3	4221136	1300	4221465	1629	1793	1957	2122	2286	2450	4
4	2779	2943	3107	3271	3436	3600	3764	3928	4093	4
5	4421	4585	4749	4913	5078	5242	5406	5570	5734	4
6	6063	6227	6391	6555	6719	6883	7047	7211	7375	4
7	7703	7868	8032	8196	8360	8524	8688	8852	9016	4
8	9344	9508	9672	9836	4230000	0164	4230328	0492	4230656	4
9	4230984	1147	4231311	1475	1639	1803	1967	2131	2295	4
2650	2623	2786	2950	3114	3278	3442	3606	3770	3933	4
1	4261	4425	4589	4753	4916	5080	5244	5408	5571	4
2	5899	6063	6226	6390	6554	6718	6881	7045	7209	4
3	7536	7700	7864	8027	8191	8355	8518	8682	8846	4
4	9173	9336	9500	9664	9827	9991	4240154	0318	4240482	4
5	4240809	0972	4241136	1300	4241463	1627	1790	1954	2117	4
6	2444	2608	2771	2935	3098	3262	3425	3589	3752	163
7	4079	4242	4406	4569	4733	4896	5060	5223	5386	3
8	5713	5877	6040	6203	6367	6530	6693	6857	7020	3
9	7347	7510	7673	7837	8000	8163	8327	8490	8653	3
2660	8980	9143	9306	9469	9633	9796	9959	/0122	4250286	3
1	4250612	0775	4250938	1102	4251265	1428	4251591	1754	1917	3
2	2244	2407	2570	2733	2896	3059	3222	3385	3549	3
3	3875	4038	4201	4364	4527	4690	4853	5016	5179	3
4	5505	5668	5831	5994	6157	6320	6483	6646	6809	3
5	7135	7298	7461	7624	7787	7950	8113	8276	8439	3
6	8764	8927	9090	9253	9416	9579	9742	9904	4260067	3
7	4260393	0556	4260719	0881	4261044	1207	4261370	1533	1695	3
8	2021	2184	2347	2509	2672	2835	2998	3160	3323	3
9	3648	3811	3974	4137	4299	4462	4625	4787	4950	3
2670	5275	5438	5601	5763	5926	6088	6251	6414	6576	3
1	6901	7064	7227	7389	7552	7714	7877	8039	8202	3
2	8527	8690	8852	9015	9177	9340	9502	9665	9827	3
3	4270152	0315	4270477	0639	4270802	0964	4271127	1289	4271452	162
4	1776	1939	2101	2264	2426	2588	2751	2913	3076	2
5	3400	3563	3725	3887	4050	4212	4374	4536	4699	2
6	5023	5186	5348	5510	5672	5835	5997	6159	6321	2
7	6646	6808	6970	7133	7295	7457	7619	7781	7944	2
8	8268	8430	8592	8754	8917	9079	9241	9403	9565	2
9	9889	/0051	4280213	0376	4280538	0700	4280862	1024	4281186	2
2680	4281510	1672	1834	1996	2158	2320	2482	2644	2806	2
1	3130	3292	3454	3616	3778	3940	4102	4264	4426	2
2	4750	4912	5073	5235	5397	5559	5721	5883	6045	2
3	6369	6530	6692	6854	7016	7178	7340	7501	7663	2
4	7987	8149	8311	8472	8634	8796	8958	9119	9281	2
5	9605	9766	9928	/0090	4290252	0413	4290575	0737	4290898	2
6	4291222	1383	4291545	1707	1868	2030	2192	2353	2515	2
7	2838	3000	3162	3323	3485	3646	3808	3969	4131	2
8	4454	4616	4777	4939	5100	5262	5423	5585	5747	2
9	6070	6231	6393	6554	6715	6877	7038	7200	7361	161
2690	7684	7846	8007	8169	8330	8491	8653	8814	8976	1
1	9298	9460	9621	9782	9944	0105	4300267	0428	4300589	1
2	4300912	1073	4301235	1396	4301557	1718	1880	2041	2202	1
3	2525	2686	2847	3009	3170	3331	3492	3653	3815	1
4	4137	4298	4460	4621	4782	4943	5104	5265	5427	1
5	5749	5910	6071	6232	6393	6554	6716	6877	7038	1
6	7360	7521	7682	7843	8004	8165	8326	8487	8648	1
7	8970	9132	9293	9454	9615	9776	9937	/0098	4310258	1
8	4310580	0741	4310902	1063	4311224	1385	4311546	1707	1868	1
9	2190	2351	2512	2672	2833	2994	3155	3316	3477	1
	1	2	3	4	5	6	7	8	9	

Between 27000 = log.$^{-1}$ 4·4313638, and 27600 = log.$^{-1}$ 4·4409091.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
2700	4313798	3959	4314120	4281	4314442	4603	4314763	4924	4315085	161
1	5407	5567	5728	5889	6050	6210	6371	6532	6693	1
2	7014	7175	7336	7496	7657	7818	7978	8139	8300	1
3	8621	8782	8942	9103	9264	9424	9585	9746	9906	1
4	4320227	0388	4320549	0709	4320870	1030	4321191	1352	4321512	1
5	1833	1994	2154	2315	2475	2636	2796	2957	3117	1
6	3438	3599	3759	3920	4080	4241	4401	4562	4722	160
7	5043	5203	5364	5524	5685	5845	6005	6166	6326	0
8	6647	6807	6968	7128	7288	7449	7609	7769	7930	0
9	8250	8411	8571	8731	8892	9052	9212	9372	9533	0
2710	9853	/0013	4330174	0334	4330494	0654	4330815	0975	4331135	0
1	4341455	1616	1776	1936	2096	2256	2416	2577	2737	0
2	3057	3217	3377	3537	3697	3858	4018	4178	4338	0
3	4658	4818	4978	5138	5298	5458	5618	5778	5938	0
4	6258	6418	6578	6738	6898	7058	7218	7378	7538	0
5	7858	8018	8178	8338	8498	8658	8818	8978	9138	0
6	9458	9617	9777	9937	4340097	0257	4340417	0577	4340737	0
7	4341056	1216	4341376	1536	1696	1855	2015	2175	2335	0
8	2654	2814	2974	3134	3293	3453	3613	3773	3932	0
9	4252	4412	4571	4731	4891	5050	5210	5370	5529	0
2720	5849	6008	6168	6328	6487	6647	6807	6966	7126	0
1	7445	7605	7764	7924	8083	8243	8403	8562	8722	0
2	9041	9200	9360	9519	9679	9838	9998	/0157	4350317	0
3	4350636	0795	4350955	1114	4351274	1433	4351593	1752	1912	0
4	2230	2390	2549	2709	2868	3028	3187	3346	3506	159
5	3824	3984	4143	4303	4462	4621	4781	4940	5099	9
6	5418	5577	5736	5896	6055	6214	6374	6533	6692	9
7	7011	7170	7329	7488	7648	7807	7966	8125	8284	9
8	8603	8762	8921	9080	9240	9399	9558	9717	9876	9
9	4360194	0354	4360513	0672	4360831	0990	4361149	1308	4361467	9
2730	1786	1945	2104	2263	2422	2581	2740	2899	3058	9
1	3376	3535	3694	3853	4012	4171	4330	4489	4648	9
2	4966	5125	5284	5443	5602	5761	5920	6078	6237	9
3	6555	6714	6873	7032	7191	7350	7509	7667	7826	9
4	8144	8303	8462	8620	8779	8938	9097	9256	9415	9
5	9732	9891	4370050	0208	4370367	0526	4370685	0843	4371002	9
6	4371320	1478	1637	1796	1955	2113	2272	2431	2589	9
7	2907	3065	3224	3383	3541	3700	3859	4017	4176	9
8	4493	4652	4810	4969	5127	5286	5445	5603	5762	9
9	6079	6237	6396	6555	6713	6872	7030	7189	7347	9
2740	7664	7823	7981	8140	8298	8457	8615	8773	8932	158
1	9249	9407	9566	9724	9883	/0041	4380199	0358	4380516	8
2	4380833	0991	4381150	1308	4381466	1625	1783	1941	2100	8
3	2416	2575	2733	2891	3050	3208	3366	3525	3683	8
4	3999	4158	4316	4474	4632	4791	4949	5107	5265	8
5	5582	5740	5898	6056	6214	6373	6531	6689	6847	8
6	7163	7322	7480	7638	7796	7954	8112	8270	8428	8
7	8745	8903	9061	9219	9377	9535	9693	9851	4390009	8
8	4390325	0483	4390641	0799	4390957	1115	4391273	1431	1589	8
9	1905	2063	2221	2379	2537	2695	2853	3011	3169	8
2750	3485	3643	3801	3959	4116	4274	4432	4590	4748	8
1	5064	5222	5379	5537	5695	5853	6011	6169	6326	8
2	6642	6800	6958	7115	7273	7431	7589	7747	7904	8
3	8220	8378	8535	8693	8851	9009	9166	9324	9482	8
4	9797	9955	4400112	0270	4400428	0585	4400743	0901	4401058	8
5	4401374	1531	1689	1347	2004	2162	2319	2477	2635	8
6	2950	3107	3265	3422	3580	3738	3895	4053	4210	8
7	4525	4683	4840	4998	5155	5313	5470	5628	5785	157
8	6100	6258	6415	6572	6730	6887	7045	7202	7360	7
9	7674	7832	7989	8147	8304	8461	8619	8776	8933	7
	1	2	3	4	5	6	7	8	9	

Between 27600 = log.⁻¹ 4·4409091, and 28200 = log.⁻¹ 4·4502491.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
2760	4409248	9406	4409563	9720	4409878	/0035	4410192	0349	4410507	157
1	4410821	0979	4411136	1293	4411450	1608	1765	1922	2080	7
2	2394	2551	2708	2866	3023	3180	3337	3494	3652	7
3	3966	4123	4280	4438	4595	4752	4909	5066	5223	7
4	5538	5695	5852	6009	6166	6323	6480	6637	6794	7
5	7108	7265	7423	7580	7737	7894	8051	8208	8365	7
6	8679	8836	8993	9150	9307	9464	9621	9778	9935	7
7	4420249	0405	4420562	0719	4420876	1033	4421190	1347	4421504	7
8	1818	1975	2132	2288	2445	2602	2759	2916	3073	7
9	3386	3543	3700	3857	4014	4171	4327	4484	4641	7
2770	4954	5111	5268	5425	5582	5738	5895	6052	6209	7
1	6522	6679	6835	6992	7149	7306	7462	7619	7776	7
2	8089	8246	8402	8559	8716	8872	9029	9185	9342	7
3	9655	9812	9969	/0125	4430282	0438	4430595	0751	4430908	7
4	4431221	1378	4431534	1691	1847	2004	2160	2317	2473	7
5	2786	2943	3099	3256	3412	3569	3725	3882	4038	156
6	4351	4507	4664	4820	4977	5133	5290	5446	5602	6
7	5915	6072	6228	6384	6541	6697	6853	7010	7166	6
8	7479	7635	7791	7948	8104	8260	8417	8573	8729	6
9	9042	9198	9354	9511	9667	9823	9979	/0136	4440292	6
2780	4440604	0760	4440917	1073	4441229	1385	4441541	1698	1854	6
1	2166	2322	2478	2635	2791	2947	3103	3259	3415	6
2	3727	3883	4040	4196	4352	4508	4664	4820	4976	6
3	5288	5444	5600	5756	5912	6068	6224	6380	6536	6
4	6848	7004	7160	7316	7472	7628	7784	7940	8096	6
5	8408	8564	8720	8876	9032	9188	9343	9499	9655	6
6	9967	/0123	4450279	0435	4450590	0746	4450902	1058	4451214	6
7	4451526	1681	1837	1993	2149	2305	2460	2616	2772	6
8	3083	3239	3395	3551	3706	3862	4018	4174	4329	6
9	4641	4797	4952	5108	5264	5419	5575	5731	5886	6
2790	6198	6353	6509	6665	6820	6976	7132	7287	7443	6
1	7754	7910	8065	8221	8376	8532	8687	8843	8999	6
2	9310	9465	9621	9776	9932	/0087	4460243	0398	4460554	6
3	4460865	1020	4461176	1331	4461487	1642	1798	1953	2109	155
4	2419	2575	2730	2886	3041	3197	3352	3507	3663	5
5	3974	4129	4284	4440	4595	4750	4906	5061	5216	5
6	5527	5682	5838	5993	6148	6304	6459	6614	6769	5
7	7080	7235	7390	7546	7701	7856	8011	8167	8322	5
8	8632	8788	8943	9098	9253	9408	9563	9719	9874	5
9	4470184	0339	4470494	0650	4470805	0960	4471115	1270	4471425	5
2800	1735	1891	2046	2201	2356	2511	2666	2821	2976	5
1	3286	3441	3596	3751	3906	4061	4216	4371	4526	5
2	4836	4991	5146	5301	5456	5611	5766	5921	6076	5
3	6386	6541	6696	6851	7006	7161	7315	7470	7625	5
4	7935	8090	8245	8400	8554	8709	8864	9019	9174	5
5	9483	9638	9793	9948	4480103	0258	4480412	0567	4480722	5
6	4481031	1186	4481341	1496	1650	1805	1960	2115	2269	5
7	2579	2734	2888	3043	3198	3352	3507	3662	3816	5
8	4126	4280	4435	4590	4744	4899	5054	5208	5363	5
9	5672	5827	5981	6136	6290	6445	6600	6754	6909	5
2810	7218	7372	7527	7681	7836	7990	8145	8299	8454	5
1	8763	8917	9072	9226	9381	9535	9690	9844	9999	154
2	4490308	0462	4490616	0771	4490925	1080	4491234	1389	4491543	4
3	1852	2006	2160	2315	2469	2624	2778	2932	3087	4
4	3395	3550	3704	3858	4013	4167	4321	4475	4630	4
5	4938	5093	5247	5401	5555	5710	5864	6018	6172	4
6	6481	6635	6789	6943	7098	7252	7406	7560	7714	4
7	8023	8177	8331	8485	8639	8793	8948	9102	9256	4
8	9564	9718	9872	/0026	4500180	0334	4500489	0643	4500797	4
9	4501105	1259	4501413	1567	1721	1875	2029	2183	2337	4
	1	2	3	4	5	6	7	8	9	

Between 28200 = log.$^{-1}$ 4·4502491, and 28800 = log.$^{-1}$ 4·4593925.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
2820	4502645	2799	4502953	3107	4503261	3415	4503569	3723	4503877	154
1	4185	4339	4493	4647	4801	4954	5108	5262	5416	4
2	5724	5878	6032	6186	6340	6493	6647	6801	6955	4
3	7263	7416	7570	7724	7878	8032	8186	8339	8493	4
4	8801	8954	9108	9262	9416	9570	9723	9877	4510031	4
5	4510338	0492	4510646	0799	4510953	1107	4511261	1414	1568	4
6	1875	2029	2183	2336	2490	2644	2797	2951	3104	4
7	3412	3565	3719	3873	4026	4180	4333	4487	4640	4
8	4948	5101	5255	5408	5562	5715	5869	6022	6176	153
9	6483	6636	6790	6943	7097	7250	7404	7557	7711	3
2830	8018	8171	8325	8478	8632	8785	8938	9092	9245	3
1	9552	9705	9859	/0012	4520166	0319	4520472	0626	4520779	3
2	4521086	1239	4521393	1546	1699	1853	2006	2159	2312	3
3	2619	2772	2926	3079	3232	3385	3539	3692	3845	3
4	4152	4305	4458	4611	4765	4918	5071	5224	5377	3
5	5684	5837	5990	6143	6297	6450	6603	6756	6909	3
6	7215	7369	7522	7675	7828	7981	8134	8287	8440	3
7	8746	8900	9053	9206	9359	9512	9665	9818	9971	3
8	4530277	0430	4530583	0736	4530889	1042	4531195	1348	4531501	3
9	1807	1960	2113	2266	2419	2572	2725	2878	3030	3
2840	3336	3489	3642	3795	3948	4101	4254	4407	4559	3
1	4865	5018	5171	5324	5477	5629	5782	5935	6088	3
2	6394	6546	6699	6852	7005	7158	7310	7463	7616	3
3	7921	8074	8227	8380	8532	8685	8838	8990	9143	3
4	9449	9601	9754	9907	4540059	0212	4540365	0517	4540670	3
5	4540975	1128	4541281	1433	1586	1739	1891	2044	2196	3
6	2502	2654	2807	2959	3112	3264	3417	3570	3722	3
7	4027	4180	4332	4485	4637	4790	4942	5095	5247	3
8	5552	5705	5857	6010	6162	6315	6467	6620	6772	152
9	7077	7229	7382	7534	7687	7839	7991	8144	8296	2
2850	8601	8753	8906	9058	9210	9363	9515	9668	9820	2
1	4550125	0277	4550429	0581	4550734	0886	4551038	1191	4551343	2
2	1647	1800	1952	2104	2257	2409	2561	2713	2865	2
3	3170	3322	3474	3627	3779	3931	4083	4235	4388	2
4	4692	4844	4996	5148	5300	5453	5605	5757	5909	2
5	6213	6365	6517	6670	6822	6974	7126	7278	7430	2
6	7734	7886	8038	8190	8342	8494	8646	8798	8950	2
7	9254	9406	9558	9710	9862	0014	4560166	0318	4560470	2
8	4560774	0926	4561078	1230	4561382	/1534	1686	1838	1990	2
9	2293	2445	2597	2749	2901	3053	3205	3357	3508	2
2860	3812	3964	4116	4268	4420	4571	4723	4875	5027	2
1	5330	5482	5634	5786	5938	6089	6241	6393	6545	2
2	6848	7000	7152	7303	7455	7607	7758	7910	8062	2
3	8365	8517	8669	8820	8972	9124	9275	9427	9578	2
4	9882	/0033	4570185	0337	4570488	0640	4570791	0943	4571095	2
5	4571398	1549	1701	1853	2004	2156	2307	2459	2610	2
6	2913	3065	3216	3368	3519	3671	3822	3974	4125	2
7	4428	4580	4731	4883	5034	5186	5337	5489	5640	151
8	5943	6094	6246	6397	6549	6700	6851	7003	7154	1
9	7457	7608	7760	7911	8062	8214	8365	8516	8668	1
2870	8970	9122	9273	9424	9576	9727	9878	0029	4580181	1
1	4580483	0634	4580786	0937	4581088	1239	4581391	/1542	1693	1
2	1996	2147	2298	2449	2600	2752	2903	3054	3205	1
3	3507	3659	3810	3961	4112	4263	4414	4565	4717	1
4	5019	5170	5321	5472	5623	5774	5925	6076	6227	1
5	6530	6681	6832	6983	7134	7285	7436	7587	7738	1
6	8040	8191	8342	8493	8644	8795	8946	9097	9248	1
7	9550	9701	9851	/0002	4590153	0304	4590455	0606	4590757	1
8	4591059	1210	4591361	1511	1662	1813	1964	2115	2266	1
9	2567	2718	2869	3020	3171	3322	3472	3623	3774	1
	1	2	3	4	5	6	7	8	9	

Between 28800 = log.$^{-1}$ 4·4593925, and 29400 = log.$^{-1}$ 4·4683473.

tens.	1	2	3	4	5	6	7	8	9	dif.
2880	4594076	4226	4594377	4528	4594679	4830	4594980	5131	4595282	151
1	5583	5734	5885	6036	6186	6337	6488	6638	6789	1
2	7090	7241	7392	7542	7693	7844	7994	8145	8296	1
3	8597	8748	8898	9049	9200	9350	9501	9651	9802	1
4	4600103	0254	4600404	0555	4600705	0856	4601007	1157	4601308	1
5	1609	1759	1910	2060	2211	2361	2512	2662	2813	150
6	3114	3264	3415	3565	3716	3866	4017	4167	4317	0
7	4618	4769	4919	5070	5220	5370	5521	5671	5822	0
8	6122	6273	6423	6573	6724	6874	7024	7175	7325	0
9	7626	7776	7926	8077	8227	8377	8528	8678	8828	0
2890	9129	9279	9429	9579	9730	9880	4610030	0180	4610331	0
1	4610631	0781	4610932	1082	4611232	1382	1532	1683	1833	0
2	2133	2283	2433	2584	2734	2884	3034	3184	3334	0
3	3634	3785	3935	4085	4235	4385	4535	4685	4835	0
4	5135	5285	5435	5585	5736	5886	6036	6186	6336	0
5	6636	6786	6936	7086	7236	7386	7536	7686	7836	0
6	8136	8285	8435	8585	8735	8885	9035	9185	9335	0
7	9635	9785	9935	/ 0085	4620234	0384	4620534	0684	4620834	0
8	4621134	1284	4621433	1583	1733	1883	2033	2183	2332	0
9	2632	2782	2932	3081	3231	3381	3531	3680	3830	0
2900	4130	4279	4429	4579	4729	4878	5028	5178	5328	0
1	5627	5777	5926	6076	6226	6375	6525	6675	6824	0
2	7124	7273	7423	7573	7722	7872	8022	8171	8321	0
3	8620	8770	8919	9069	9218	9368	9517	9667	9817	0
4	4630116	0265	4630415	0564	4630714	0863	4631013	1162	4631312	149
5	1611	1760	1910	2059	2209	2358	2508	2657	2807	9
6	3106	3255	3404	3554	3703	3853	4002	4152	4301	9
7	4600	4749	4898	5048	5197	5347	5496	5645	5795	9
8	6093	6243	6392	6541	6691	6840	6989	7139	7288	9
9	7587	7736	7885	8034	8184	8333	8482	8631	8781	9
2910	9079	9228	9378	9527	9676	9825	9974	/ 0124	4640273	9
1	4640571	0720	4640870	1019	4641168	1317	4641466	1615	1765	9
2	2063	2212	2361	2510	2659	2808	2958	3107	3256	9
3	3554	3703	3852	4001	4150	4299	4448	4597	4746	9
4	5045	5194	5343	5492	5641	5790	5939	6088	6237	9
5	6535	6684	6833	6981	7130	7279	7428	7577	7726	9
6	8024	8173	8322	8471	8620	8769	8918	9067	9215	9
7	9513	9662	9811	9960	4650109	0258	4650406	0555	4650704	9
8	4651002	1151	4651299	1448	1597	1746	1895	2043	2192	9
9	2490	2639	2787	2936	3085	3234	3382	3531	3680	9
2920	3977	4126	4275	4423	4572	4721	4870	5018	5167	9
1	5464	5613	5762	5910	6059	6208	6356	6505	6653	9
2	6951	7099	7248	7397	7545	7694	7842	7991	8140	9
3	8437	8585	8734	8882	9031	9180	9328	9477	9625	9
4	9922	/ 0071	4660219	0368	4660516	0665	4660813	0962	4661110	9
5	4661407	1556	1704	1853	2001	2149	2298	2446	2595	148
6	2892	3040	3188	3337	3485	3634	3782	3930	4079	8
7	4376	4524	4672	4821	4969	5117	5266	5414	5562	8
8	5859	6007	6156	6304	6452	6601	6749	6897	7045	8
9	7342	7490	7639	7787	7935	8083	8232	8380	8528	8
2930	8824	8973	9121	9269	9417	9565	9714	9862	4670010	8
1	4670306	0455	4670603	0751	4670899	1047	4671195	1343	1492	8
2	1788	1936	2084	2232	2380	2528	2676	2824	2973	8
3	3269	3417	3565	3713	3861	4009	4157	4305	4453	8
4	4749	4897	5045	5193	5341	5489	5637	5785	5933	8
5	6229	6377	6525	6673	6821	6969	7117	7265	7413	8
6	7708	7856	8004	8152	8300	8448	8596	8744	8892	8
7	9187	9335	9483	9631	9779	9927	4680074	0222	4680370	8
8	4680666	0814	4680961	1109	4681257	1405	1553	1700	1848	8
9	2144	2291	2439	2587	2735	2882	3030	3178	3326	8
	1	2	3	4	5	6	7	8	9	

Between 29400 = $\log.^{-1}$ 4·4683473, and 30000 = $\log.^{-1}$ 4·4771213.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
2940	4683621	3769	4683916	4064	4684212	4360	4684507	4655	4684803	148
1	5098	5246	5393	5541	5689	5836	5984	6131	6279	8
2	6574	6722	6870	7017	7165	7312	7460	7607	7755	8
3	8050	8198	8345	8493	8640	8788	8935	9083	9231	8
4	9526	9673	9821	9968	4690116	0263	4690411	0558	4690706	147
5	4691000	1148	4691295	1443	1590	1738	1885	2033	2180	7
6	2475	2622	2770	2917	3064	3212	3359	3507	3654	7
7	3949	4096	4243	4391	4538	4685	4833	4980	5127	7
8	5422	5569	5717	5864	6011	6159	6306	6453	6600	7
9	6895	7042	7190	7337	7484	7631	7778	7926	8073	7
2950	8367	8515	8662	8809	8956	9103	9251	9398	9545	7
1	9839	9986	4700134	0281	4700428	0575	4700722	0869	4701016	7
2	4701311	1458	1605	1752	1899	2046	2193	2340	2487	7
3	2782	2929	3076	3223	3370	3517	3664	3811	3958	7
4	4252	4399	4546	4693	4840	4987	5134	5281	5428	7
5	5722	5869	6016	6163	6310	6457	6604	6750	6897	7
6	7191	7338	7485	7632	7779	7926	8073	8219	8366	7
7	8660	8807	8954	9101	9248	9394	9541	9688	9835	7
8	4710129	0275	4710422	0569	4710716	0863	4711009	1156	4711303	7
9	1596	1743	1890	2037	2183	2330	2477	2624	2770	7
2960	3064	3211	3357	3504	3651	3797	3944	4091	4237	7
1	4531	4677	4824	4971	5117	5264	5411	5557	5704	7
2	5997	6144	6290	6437	6584	6730	6877	7023	7170	7
3	7463	7610	7756	7903	8049	8196	8342	8489	8635	7
4	8929	9075	9222	9368	9515	9661	9808	9954	4720101	146
5	4720393	0540	4720686	0833	4720979	1126	4721272	1419	1565	6
6	1858	2004	2151	2297	2444	2590	2736	2883	3029	6
7	3322	3468	3615	3761	3907	4054	4200	4346	4493	6
8	4785	4932	5078	5224	5371	5517	5663	5809	5956	6
9	6248	6395	6541	6687	6833	6980	7126	7272	7418	6
2970	7711	7857	8003	8149	8296	8442	8588	8734	8880	6
1	9173	9319	9465	9611	9757	9903	4730050	0196	4730342	6
2	4730634	0780	4730926	1073	4731219	1365	1511	1657	1803	6
3	2095	2241	2387	2533	2679	2825	2972	3118	3264	6
4	3556	3702	3848	3994	4140	4286	4432	4578	4724	6
5	5016	5162	5308	5454	5600	5746	5891	6037	6183	6
6	6475	6621	6767	6913	7059	7205	7351	7497	7642	6
7	7934	8080	8226	8372	8518	8664	8809	8955	9101	6
8	9393	9539	9684	9830	9976	/ 0122	4740268	0413	4740559	6
9	4740851	0997	4741142	1288	4741434	1580	1725	1871	2017	6
2980	2308	2454	2600	2746	2891	3037	3183	3328	3474	6
1	3765	3911	4057	4202	4348	4494	4639	4785	4931	6
2	5222	5368	5513	5659	5805	5950	6096	6241	6387	6
3	6678	6824	6969	7115	7260	7406	7552	7697	7843	6
4	8134	8279	8425	8570	8716	8861	9007	9152	9298	145
5	9589	9734	9880	/ 0025	4750171	0316	4750462	0607	4750753	5
6	4751043	1189	4751334	1480	1625	1771	1916	2061	2207	5
7	2498	2643	2788	2934	3079	3225	3370	3515	3661	5
8	3951	4097	4242	4387	4533	4678	4823	4969	5114	5
9	5404	5550	5695	5840	5986	6131	6276	6421	6567	5
2990	6857	7002	7148	7293	7438	7583	7729	7874	8019	5
1	8309	8455	8600	8745	8890	9035	9180	9326	9471	5
2	9761	9906	4760051	0196	4760342	0487	4760632	0777	4760922	5
3	4761212	1357	1502	1648	1793	1938	2083	2228	2373	5
4	2663	2808	2953	3098	3243	3388	3533	3678	3823	5
5	4113	4258	4403	4548	4693	4838	4983	5128	5273	5
6	5563	5708	5853	5998	6143	6288	6433	6578	6723	5
7	7012	7157	7302	7447	7592	7737	7882	8027	8171	5
8	8461	8606	8751	8896	9041	9185	9330	9475	9620	5
9	9909	/ 0054	4770199	0344	4770489	0633	4770778	0923	4771068	5
	1	2	3	4	5	6	7	8	9	

Between $30000 = \log.^{-1} 4{\cdot}4771213$, and $30600 = \log.^{-1} 4{\cdot}4857214$.

tens.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	*dif.*
3000	4771357	1502	4771647	1792	4771936	2081	4772226	2371	4772515	145
1	2805	2949	3094	3239	3383	3528	3673	3818	3962	5
2	4252	4396	4541	4686	4830	4975	5119	5264	5409	5
3	5698	5843	5987	6132	6276	6421	6566	6710	6855	5
4	7144	7288	7433	7578	7722	7867	8011	8156	8300	5
5	8589	8734	8878	9023	9167	9312	9456	9601	9745	144
6	4780034	0179	4780323	0468	4780612	0757	4780901	1045	4781190	4
7	1479	1623	1768	1912	2056	2201	2345	2490	2634	4
8	2923	3067	3211	3356	3500	3645	3789	3933	4078	4
9	4366	4511	4655	4799	4943	5088	5232	5376	5521	4
3010	5809	5954	6098	6242	6386	6531	6675	6819	6963	4
1	7252	7396	7540	7684	7829	7973	8117	8261	8405	4
2	8694	8838	8982	9126	9271	9415	9559	9703	9847	4
3	4790135	0280	4790424	0568	4790712	0856	4791000	1144	4791288	4
4	1577	1721	1865	2009	2153	2297	2441	2585	2729	4
5	3017	3161	3305	3449	3593	3737	3881	4025	4169	4
6	4457	4601	4745	4889	5033	5177	5321	5465	5609	4
7	5897	6041	6185	6329	6473	6617	6761	6905	7048	4
8	7336	7480	7624	7768	7912	8056	8200	8343	8487	4
9	8775	8919	9063	9207	9350	9494	9638	9782	9926	4
3020	4800213	0357	4800501	0645	4800788	0932	4801076	1220	4801363	4
1	1651	1795	1939	2082	2226	2370	2513	2657	2801	4
2	3088	3232	3376	3519	3663	3807	3950	4094	4238	4
3	4525	4669	4812	4956	5100	5243	5387	5531	5674	4
4	5961	6105	6249	6392	6536	6679	6823	6967	7110	4
5	7397	7541	7684	7828	7972	8115	8259	8402	8546	4
6	8833	8976	9120	9263	9407	9550	9694	9837	9981	143
7	4810268	0411	4810555	0698	4810842	0985	4811128	1272	4811415	3
8	1702	1846	1989	2132	2276	2419	2563	2706	2849	3
9	3136	3279	3423	3566	3710	3853	3996	4140	4283	3
3030	4570	4713	4856	5000	5143	5286	5429	5573	5716	3
1	6003	6146	6289	6432	6576	6719	6862	7005	7149	3
2	7435	7578	7722	7865	8008	8151	8295	8438	8581	3
3	8867	9010	9154	9297	9440	9583	9726	9869	4820013	3
4	4820299	0442	4820585	0728	4820871	1015	4821158	1301	1444	3
5	1730	1873	2016	2159	2302	2445	2589	2732	2875	3
6	3161	3304	3447	3590	3733	3876	4019	4162	4305	3
7	4591	4734	4877	5020	5163	5306	5449	5592	5735	3
8	6021	6164	6307	6449	6592	6735	6878	7021	7164	3
9	7450	7593	7736	7879	8021	8164	8307	8450	8593	3
3040	8879	9022	9164	9307	9450	9593	9736	9879	4830021	3
1	4830307	0450	4830593	0735	4830878	1021	4831164	1307	1449	3
2	1735	1878	2020	2163	2306	2449	2591	2734	2877	3
3	3162	3305	3448	3590	3733	3876	4018	4161	4304	3
4	4589	4732	4874	5017	5160	5302	5445	5588	5730	3
5	6016	6158	6301	6443	6586	6729	6871	7014	7156	3
6	7442	7584	7727	7869	8012	8154	8297	8439	8582	3
7	8867	9010	9152	9295	9437	9580	9722	9865	4840007	3
8	4840292	0435	4840577	0720	4840862	1004	4841147	1289	1432	142
9	1717	1859	2002	2144	2286	2429	2571	2714	2856	2
3050	3141	3283	3426	3568	3710	3853	3995	4137	4280	2
1	4564	4707	4849	4991	5134	5276	5418	5561	5703	2
2	5988	6130	6272	6414	6557	6699	6841	6984	7126	2
3	7410	7553	7695	7837	7979	8121	8264	8406	8548	2
4	8833	8975	9117	9259	9401	9543	9686	9828	9970	2
5	4850254	0396	4850539	0681	4850823	0965	4851107	1249	4851391	2
6	1676	1818	1960	2102	2244	2386	2528	2670	2812	2
7	3096	3239	3381	3523	3665	3807	3949	4091	4233	2
8	4517	4659	4801	4943	5085	5227	5369	5511	5653	2
9	5937	6079	6221	6363	6505	6647	6788	6930	7072	2
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	

Between 30600 = log.⁻¹ 4·4857214, and 31200 = log.⁻¹ 4·4941546.

	1	2	3	4	5	6	7	8	9	*dif.*
0	4857356	7498	4857640	7782	4857924	8066	4858208	8350	4858491	142
1	8775	8917	9059	9201	9343	9484	9626	9768	9910	2
2	4860194	0336	4860477	0619	4860761	0903	4861045	1186	4861328	2
3	1612	1754	1895	2037	2179	2321	2462	2604	2746	2
4	3029	3171	3313	3455	3596	3738	3880	4021	4163	2
5	4446	4588	4730	4872	5013	5155	5297	5438	5580	2
6	5863	6005	6146	6288	6430	6571	6713	6855	6996	2
7	7279	7421	7563	7704	7846	7987	8129	8270	8412	2
8	8695	8837	8978	9120	9261	9403	9544	9686	9827	2
9	4870110	0252	4870393	0535	4870676	0818	4870959	1101	4871242	2
70	1525	1667	1808	1950	2091	2232	2374	2515	2657	2
1	2940	3081	3222	3364	3505	3647	3788	3929	4071	2
2	4353	4495	4636	4778	4919	5060	5202	5343	5484	2
3	5767	5908	6050	6191	6332	6473	6615	6756	6897	2
4	7180	7321	7462	7604	7745	7886	8027	8169	8310	2
5	8592	8734	8875	9016	9157	9299	9440	9581	9722	2
6	4880004	0146	4880287	0428	4880569	0710	4880852	0993	4881134	2
7	1416	1557	1698	1839	1981	2122	2263	2404	2545	2
8	2827	2968	3109	3251	3392	3533	3674	3815	3956	2
9	4238	4379	4520	4661	4802	4943	5084	5225	5366	141
80	5648	5789	5930	6071	6212	6353	6494	6635	6776	1
1	7058	7199	7340	7481	7622	7763	7904	8045	8185	1
2	8467	8608	8749	8890	9031	9172	9313	9454	9594	1
3	9876	/0017	4890158	0299	4890440	0580	4890721	0862	4861003	1
4	4891285	1425	1566	1707	1848	1989	2129	2270	2411	1
5	2692	2833	2974	3115	3256	3396	3537	3678	3818	1
6	4100	4241	4381	4522	4663	4804	4944	5085	5226	1
7	5507	5648	5788	5929	6070	6210	6351	6492	6632	1
8	6914	7054	7195	7335	7476	7617	7757	7898	8038	1
9	8320	8460	8601	8741	8882	9023	9163	9304	9444	1
090	9725	9866	4900006	0147	4900287	0428	4900569	0709	4900850	1
1	4901131	1271	1412	1552	1693	1833	1973	2114	2254	1
2	2535	2676	2816	2957	3097	3238	3378	3518	3659	1
3	3940	4080	4220	4361	4501	4642	4782	4922	5063	1
4	5343	5484	5624	5765	5905	6045	6186	6326	6466	140
5	6747	6887	7027	7168	7308	7448	7589	7729	7869	0
6	8150	8290	8430	8571	8711	8851	8991	9132	9272	0
7	9552	9693	9833	9973	4910113	0253	4910394	0534	4910674	0
8	4910954	1094	4911235	1375	1515	1655	1795	1935	2076	0
9	2356	2496	2636	2776	2916	3057	3197	3337	3477	0
3100	3757	3897	4037	4177	4317	4457	4597	4738	4878	0
1	5158	5298	5438	5578	5718	5858	5998	6138	6278	0
2	6558	6698	6838	6978	7118	7258	7398	7538	7678	0
3	7958	8098	8238	8378	8517	8657	8797	8937	9077	0
4	9357	9497	9637	9777	9917	/0057	4920196	0336	4920476	0
5	4920756	0896	4921036	1175	4921315	1455	1595	1735	1875	0
6	2154	2294	2434	2574	2714	2853	2993	3133	3273	0
7	3552	3692	3832	3972	4111	4251	4391	4531	4670	0
8	4950	5090	5229	5369	5509	5648	5788	5928	6068	0
9	6347	6487	6626	6766	6906	7045	7185	7325	7464	0
3110	7744	7883	8023	8162	8302	8442	8581	8721	8861	0
1	9140	9279	9419	9558	9698	9838	9977	/0117	4930256	0
2	4930535	0675	4930815	0954	4931094	1233	4931373	1512	1652	0
3	1931	2070	2210	2349	2489	2628	2768	2907	3047	0
4	3326	3465	3604	3744	3883	4023	4162	4302	4441	0
5	4720	4859	4999	5138	5278	5417	5556	5696	5835	0
6	6114	6253	6393	6532	6671	6811	6950	7089	7229	0
7	7507	7647	7786	7925	8065	8204	8343	8483	8622	0
8	8900	9040	9179	9318	9457	9597	9736	9875	4940015	0
9	4940293	0432	4940571	0711	4940850	0989	4941128	1268	1407	0
	1	2	3	4	5	6	7	8	9	

Between 31200 = log.⁻¹ 4·4941546, and 31800 = log.⁻¹ 4·5024271.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
3120	4941685	1824	4941964	2103	4942242	2381	4942520	2659	4942799	140
1	3077	3216	3355	3494	3633	3773	3912	4051	4190	139
2	4468	4607	4746	4885	5024	5164	5303	5442	5581	9
3	5859	5998	6137	6276	6415	6554	6693	6832	6971	9
4	7249	7388	7527	7666	7805	7944	8083	8222	8361	9
5	8639	8778	8917	9056	9195	9334	9473	9612	9751	9
6	4950029	0168	4950307	0445	4950584	0723	4950862	1001	4951140	9
7	1418	1557	1695	1834	1973	2112	2251	2390	2529	9
8	2806	2945	3084	3223	3362	3500	3639	3778	3917	9
9	4194	4333	4472	4611	4750	4888	5027	5166	5305	9
3130	5582	5721	5860	5998	6137	6276	6415	6553	6692	9
1	6969	7108	7247	7385	7524	7663	7802	7940	8079	9
2	8356	8495	8634	8772	8911	9049	9188	9327	9465	9
3	9743	9881	4960020	0158	4960297	0436	4960574	0713	4960851	9
4	4961128	1267	1406	1544	1683	1821	1960	2098	2237	9
5	2514	2653	2791	2930	3068	3207	3345	3484	3622	9
6	3899	4038	4176	4314	4453	4591	4730	4868	5007	9
7	5284	5422	5560	5699	5837	5976	6114	6253	6391	9
8	6668	6806	6945	7083	7221	7360	7498	7636	7775	9
9	8052	8190	8328	8467	8605	8743	8882	9020	9158	9
3140	9435	9573	9711	9850	9988	/0126	4970265	0403	4970541	138
1	4970819	0956	4971094	1232	4971371	1509	1647	1785	1924	8
2	2200	2338	2476	2615	2753	2891	3029	3167	3306	8
3	3582	3720	3858	3996	4135	4273	4411	4549	4687	8
4	4964	5102	5240	5378	5516	5654	5792	5930	6068	8
5	6345	6483	6621	6759	6897	7035	7173	7311	7449	8
6	7725	7863	8001	8139	8277	8415	8553	8691	8829	8
7	9105	9243	9381	9519	9657	9795	9933	/0071	4980209	8
8	4980485	0623	4980761	0899	4981037	1175	4981313	1451	1589	8
9	1865	2002	2140	2278	2416	2554	2692	2830	2968	8
3150	3243	3381	3519	3657	3795	3933	4071	4208	4346	8
1	4622	4760	4897	5035	5173	5311	5449	5587	5724	8
2	6000	6138	6275	6413	6551	6689	6826	6964	7102	8
3	7377	7515	7653	7791	7928	8066	8204	8341	8479	8
4	8755	8892	9030	9168	9305	9443	9581	9718	9856	8
5	4990131	0269	4990407	0544	4990682	0819	4990957	1095	4991232	8
6	1508	1645	1783	1920	2058	2196	2333	2471	2608	8
7	2883	3021	3158	3296	3434	3571	3709	3846	3984	8
8	4259	4396	4534	4671	4809	4946	5084	5221	5359	8
9	5634	5771	5909	6046	6184	6321	6459	6596	6733	8
3160	7008	7146	7283	7421	7558	7695	7833	7970	8108	8
1	8382	8520	8657	8794	8932	9069	9207	9344	9481	8
2	9756	9893	5000031	0168	5000305	0443	5000580	0717	5000855	8
3	5001129	1267	1404	1541	1678	1816	1953	2090	2227	8
4	2502	2639	2777	2914	3051	3188	3325	3463	3600	8
5	3874	4012	4149	4286	4423	4560	4698	4835	4972	8
6	5246	5383	5521	5658	5795	5932	6069	6206	6344	8
7	6618	6755	6892	7029	7166	7303	7440	7578	7715	8
8	7989	8126	8263	8400	8537	8674	8811	8948	9085	8
9	9359	9496	9634	9771	9908	/0045	5010182	0319	5010456	137
3170	5010730	0867	5011004	1141	5011278	1415	1552	1688	1825	7
1	2099	2236	2373	2510	2647	2784	2921	3058	3195	7
2	3469	3606	3743	3879	4016	4153	4290	4427	4564	7
3	4838	4974	5111	5248	5385	5522	5659	5796	5932	7
4	6206	6343	6480	6617	6753	6890	7027	7164	7301	7
5	7574	7711	7848	7984	8121	8258	8395	8531	8668	7
6	8942	9078	9215	9352	9489	9625	9762	9899	5020035	7
7	5020309	0446	5020582	0719	5020856	0992	5021129	1266	1402	7
8	1676	1812	1949	2086	2222	2359	2495	2632	2769	7
9	3042	3178	3315	3452	3588	3725	3861	3998	4135	7
	1	2	3	4	5	6	7	8	9	

Between 31800 = $\log.^{-1}$ 4·5024271, and 32400 = $\log.^{-1}$ 4·5105450.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
3180	5024408	4544	5024681	4817	5024954	5091	5025227	5364	5025500	137
1	5773	5910	6046	6183	6319	6456	6592	6729	6865	7
2	7138	7275	7411	7548	7684	7821	7957	8093	8230	7
3	8503	8639	8776	8912	9049	9185	9321	9458	9594	7
4	9867	0003	5030140	0276	5030413	0549	5030685	0822	5030958	7
5	5031231	1367	1503	1640	1776	1912	2049	2185	2321	7
6	2594	2730	2867	3003	3139	3276	3412	3548	3684	7
7	3957	4093	4229	4366	4502	4638	4774	4911	5047	7
8	5319	5456	5592	5728	5864	6000	6137	6273	6409	7
9	6681	6818	6954	7090	7226	7362	7498	7635	7771	7
3190	8043	8179	8315	8451	8587	8724	8860	8996	9132	7
1	9404	9540	9676	9812	9948	0085	5040221	0357	5040493	7
2	5040765	0901	5041037	1173	5041309	1445	1581	1717	1853	7
3	2125	2261	2397	2533	2669	2805	2941	3077	3213	7
4	3485	3621	3757	3893	4029	4165	4301	4437	4573	136
5	4845	4980	5116	5252	5388	5524	5660	5796	5932	6
6	6204	6339	6475	6611	6747	6883	7019	7155	7291	6
7	7562	7698	7834	7970	8106	8241	8377	8513	8649	6
8	8920	9056	9192	9328	9464	9599	9735	9871	5050007	6
9	5050278	0414	5050550	0685	5050821	0957	5051093	1228	1364	6
3200	1635	1771	1907	2043	2178	2314	2450	2585	2721	6
1	2992	3128	3264	3399	3535	3671	3806	3942	4078	6
2	4349	4485	4620	4756	4891	5027	5163	5298	5434	6
3	5705	5841	5976	6112	6247	6383	6518	6654	6790	6
4	7061	7196	7332	7467	7603	7738	7874	8009	8145	6
5	8416	8551	8687	8822	8958	9093	9229	9364	9500	6
6	9771	9906	5060042	0177	5060312	0448	5060583	0719	5060854	6
7	5061125	1260	1396	1531	1667	1802	1937	2073	2208	6
8	2479	2614	2750	2885	3020	3156	3291	3426	3562	6
9	3833	3968	4103	4238	4374	4509	4644	4780	4915	6
3210	5186	5321	5456	5591	5727	5862	5997	6133	6268	6
1	6538	6674	6809	6944	7079	7214	7350	7485	7620	6
2	7891	8026	8161	8296	8431	8567	8702	8837	8972	6
3	9242	9378	9513	9648	9783	9918	5070053	0188	5070324	6
4	5070594	0729	5070864	0999	5071134	1269	1405	1540	1675	6
5	1945	2080	2215	2350	2485	2620	2755	2890	3025	6
6	3295	3430	3566	3701	3836	3971	4106	4241	4376	6
7	4646	4781	4916	5051	5186	5321	5456	5590	5725	6
8	5995	6130	6265	6400	6535	6670	6805	6940	7075	6
9	7345	7480	7614	7749	7884	8019	8154	8289	8424	6
3220	8694	8828	8963	9098	9233	9368	9503	9638	9772	135
1	5080042	0177	5080312	0447	5080581	0716	5080851	0986	5081121	5
2	1390	1525	1660	1794	1929	2064	2199	2334	2468	5
3	2738	2873	3007	3142	3277	3411	3546	3681	3816	5
4	4085	4220	4354	4489	4624	4758	4893	5028	5163	5
5	5432	5567	5701	5836	5970	6105	6240	6374	6509	5
6	6778	6913	7047	7182	7317	7451	7586	7720	7855	5
7	8124	8259	8393	8528	8663	8797	8932	9066	9201	5
8	9470	9604	9739	9873	5090008	0142	5090277	0411	5090546	5
9	5090815	0949	5091084	1218	1353	1487	1622	1756	1891	5
3230	2160	2294	2429	2563	2697	2832	2966	3101	3235	5
1	3504	3638	3773	3907	4042	4176	4310	4445	4579	5
2	4848	4982	5117	5251	5385	5520	5654	5788	5923	5
3	6191	6326	6460	6594	6729	6863	6997	7132	7266	5
4	7534	7669	7803	7937	8072	8206	8340	8474	8609	5
5	8877	9011	9146	9280	9414	9548	9682	9817	9951	5
6	5100219	0354	5100488	0622	5100756	0890	5101024	1159	5101293	5
7	1561	1695	1829	1964	2098	2232	2366	2500	2634	5
8	2903	3037	3171	3305	3439	3573	3707	3841	3975	5
9	4244	4378	4512	4646	4780	4914	5048	5182	5316	134
	1	2	3	4	5	6	7	8	9	

Between 32400 = log.$^{-1}$ 4·5105450, and 33000 = log.$^{-1}$ 4·5185139.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
3240	5105584	5718	5105852	5986	5106120	6254	5106388	6522	5106656	134
1	6924	7058	7192	7326	7460	7594	7728	7862	7996	4
2	8264	8398	8532	8666	8800	8934	9068	9202	9336	4
3	9603	9737	9871	/0005	5110139	0273	5110407	0541	5110675	4
4	5110942	1076	5111210	1344	1478	1612	1745	1879	2013	4
5	2281	2415	2548	2682	2816	2950	3084	3218	3351	4
6	3619	3753	3887	4020	4154	4288	4422	4555	4689	4
7	4957	5090	5224	5358	5492	5625	5759	5893	6026	4
8	6294	6428	6561	6695	6829	6962	7096	7230	7363	4
9	7631	7764	7898	8032	8165	8299	8433	8566	8700	4
3250	8967	9101	9234	9368	9502	9635	9769	9903	5120036	4
1	5120303	0437	5120570	0704	5120838	0971	5121105	1236	1372	4
2	1639	1772	1906	2040	2173	2307	2440	2574	2707	4
3	2974	3108	3241	3375	3508	3642	3775	3909	4042	4
4	4309	4443	4576	4709	4843	4976	5110	5243	5377	4
5	5643	5777	5910	6044	6177	6310	6444	6577	6711	4
6	6977	7111	7244	7377	7511	7644	7778	7911	8044	4
7	8311	8444	8578	8711	8844	8978	9111	9244	9377	4
8	9644	9777	9911	/0044	5130177	0311	5130444	0577	5130710	4
9	5130977	1110	5131243	1377	1510	1643	1776	1910	2043	4
3260	2309	2442	2576	2709	2842	2975	3108	3242	3375	4
1	3641	3774	3908	4041	4174	4307	4440	4573	4706	4
2	4973	5106	5239	5372	5505	5638	5771	5905	6038	133
3	6304	6437	6570	6703	6836	6969	7102	7235	7368	3
4	7635	7768	7901	8034	8167	8300	8433	8566	8699	3
5	8965	9098	9231	9364	9497	9630	9763	9896	5140029	3
6	5140295	0428	5140561	0694	5140827	0960	5141093	1225	1358	3
7	1624	1757	1890	2023	2156	2289	2422	2555	2688	3
8	2953	3086	3219	3352	3485	3618	3751	3883	4016	3
9	4282	4415	4548	4681	4813	4946	5079	5212	5345	3
3270	5610	5743	5876	6009	6142	6274	6407	6540	6673	3
1	6938	7071	7204	7336	7469	7602	7735	7867	8000	3
2	8266	8398	8531	8664	8797	8929	9062	9195	9327	3
3	9593	9725	9858	9991	5150123	0256	5150389	0521	5150654	3
4	5150919	1052	5151185	1317	1450	1583	1715	1848	1980	3
5	2246	2378	2511	2643	2776	2909	3041	3174	3306	3
6	3571	3704	3837	3969	4102	4234	4367	4499	4632	3
7	4897	5029	5162	5294	5427	5560	5692	5825	5957	3
8	6222	6354	6487	6619	6752	6884	7017	7149	7282	3
9	7547	7679	7811	7944	8076	8209	8341	8474	8606	3
3280	8871	9003	9136	9268	9400	9533	9665	9798	9930	3
1	5160195	0327	5160459	0592	5160724	0856	5160989	1121	5161253	3
2	1518	1650	1783	1915	2047	2180	2312	2444	2577	3
3	2841	2973	3106	3238	3370	3502	3635	3767	3899	3
4	4164	4296	4428	4560	4693	4825	4957	5089	5222	3
5	5486	5618	5750	5883	6015	6147	6279	6411	6543	3
6	6808	6940	7072	7204	7336	7469	7601	7733	7865	132
7	8129	8261	8393	8526	8658	8790	8922	9054	9186	2
8	9450	9582	9714	9846	9978	/0111	5170243	0375	5170507	2
9	5170771	0903	5171035	1167	5171299	1431	1563	1695	1827	2
3290	2091	2223	2355	2487	2619	2751	2883	3015	3147	2
1	3411	3543	3675	3807	3939	4071	4202	4334	4466	2
2	4730	4862	4994	5126	5258	5390	5522	5654	5785	2
3	6049	6181	6313	6445	6577	6709	6840	6972	7104	2
4	7368	7500	7631	7763	7895	8027	8159	8291	8422	2
5	8686	8818	8950	9081	9213	9345	9477	9608	9740	2
6	5180004	0136	5180267	0399	5180531	0663	5180794	0926	5181058	2
7	1321	1453	1585	1716	1848	1980	2111	2243	2375	2
8	2638	2770	2902	3033	3165	3297	3428	3560	3692	2
9	3955	4086	4218	4350	4481	4613	4745	4876	5008	2
	1	2	3	4	5	6	7	8	9	

Between 33000 = $\log.^{-1}$ 4·5185139, and 33600 = $\log.^{-1}$ 4·5263393.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
3300	5185271	5403	5185534	5666	5185797	5929	5186061	6192	5186324	132
1	6587	6718	6850	6981	7113	7245	7376	7508	7639	2
2	7902	8034	8165	8297	8428	8560	8691	8823	8954	2
3	9217	9349	9480	9612	9743	9875	5190006	0137	5190269	2
4	5190532	0663	5190795	0926	5191058	1189	1320	1452	1583	2
5	1846	1977	2109	2240	2372	2503	2634	2766	2697	2
6	3160	3291	342[illegible]	3554	3685	3817	3948	4079	4211	2
7	4473	4605	4736	4867	4999	5130	5261	5392	5524	2
8	5786	5918	6049	6180	6311	6443	6574	6705	6836	2
9	7099	7230	7361	7493	7624	7755	7886	8018	8149	2
3310	8411	8542	8674	8805	8936	9067	9198	9329	9461	2
1	9723	9854	9985	/0116	5200248	0379	5200510	0641	5200772	131
2	5201034	1166	5201297	1428	1559	1690	1821	1952	2083	1
3	2345	2477	2608	2739	2870	3001	3132	3263	3394	1
4	3656	3787	3918	4049	4180	4311	4442	4573	4704	1
5	4966	5097	5228	5359	5490	5621	5752	5883	6014	1
6	6276	6407	6538	6669	6800	6931	7062	7193	7324	1
7	7586	7717	7847	7978	8109	8240	8371	8502	8633	1
8	8895	9026	9156	9287	9418	9549	9680	9811	9942	1
9	5210203	0334	5210465	0596	5210727	0858	5210988	1119	5211250	1
3320	1512	1642	1773	1904	2035	2166	2296	2427	2558	1
1	2820	2950	3081	3212	3343	3473	3604	3735	3866	1
2	4127	4258	4388	4519	4650	4781	4911	5042	5173	1
3	5434	5565	5695	5826	5957	6088	6218	6349	6479	1
4	6741	6871	7002	7133	7263	7394	7525	7655	7786	1
5	8047	8178	8308	8439	8570	8700	8831	8961	9092	1
6	9353	9484	9614	9745	9875	/0006	5220136	0267	5220397	1
7	5220659	0789	5220920	1050	5221181	1311	1442	1572	1703	1
8	1964	2094	2225	2355	2486	2616	2747	2877	3007	1
9	3268	3399	3529	3660	3790	3921	4051	4181	4312	130
3330	4573	4703	4834	4964	5094	5225	5355	5486	5616	0
1	5877	6007	6137	6268	6398	6529	6659	6789	6920	0
2	7180	7311	7441	7571	7702	7832	7962	8093	8223	0
3	8483	8614	8744	8874	9005	9135	9265	9395	9526	0
4	9786	9916	5230047	0177	5230307	0437	5230568	0698	5230828	0
5	5231089	1219	1349	1479	1609	1740	1870	2000	2130	0
6	2391	2521	2651	2781	2911	3041	3172	3302	3432	0
7	3692	3822	3952	4083	4213	4343	4473	4603	4733	0
8	4993	5124	5254	5384	5514	5644	5774	5904	6034	0
9	6294	6424	6554	6684	6814	6945	7075	7205	7335	0
3340	7595	7725	7855	7985	8115	8245	8375	8505	8635	0
1	8895	9025	9155	9285	9415	9545	9675	9805	9935	0
2	5240194	0324	5240454	0584	5240714	0844	5240974	1104	5241234	0
3	1494	1624	1753	1883	2013	2143	2273	2403	2533	0
4	2793	2922	3052	3182	3312	3442	3572	3702	3831	0
5	4091	4221	4351	4481	4610	4740	4870	5000	5130	0
6	5389	5519	5649	5779	5908	6038	6168	6298	6427	0
7	6687	6817	6946	7076	7206	7336	7465	7595	7725	0
8	7984	8114	8244	8373	8503	8633	8762	8892	9022	0
9	9281	9411	9540	9670	9800	9929	5250059	0189	5250318	0
3350	5250578	0707	5250837	0967	5251096	1226	1355	1485	1615	0
1	1874	2003	2133	2263	2392	2522	2651	2781	2911	0
2	3170	3299	3429	3558	3688	3817	3947	4076	4206	0
3	4465	4595	4724	4854	4983	5113	5242	5372	5501	0
4	5760	5890	6019	6148	6278	6407	6537	6666	6796	0
5	7055	7184	7314	7443	7572	7702	7831	7961	8090	0
6	8349	8478	8608	8737	8867	8996	9125	9255	9384	0
7	9643	9772	9902	/0031	5260160	0290	5260419	0548	5260678	0
8	5260936	1066	5261195	1324	1454	1583	1712	1841	1971	0
9	2229	2359	2488	2617	2746	2876	3005	3134	3264	0
	1	2	3	4	5	6	7	8	9	

Between 33600 = log.$^{-1}$ 4·5263393, and 34200 = log.$^{-1}$ 4·5340261.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
3360	5263522	3651	5263781	3910	5264039	4168	5264297	4427	5264556	130
1	4814	4944	5073	5202	5331	5460	5590	5719	5848	0
2	6106	6235	6365	6494	6623	6752	6881	7010	7140	0
3	7398	7527	7656	7785	7914	8043	8173	8302	8431	0
4	8689	8818	8947	9076	9205	9334	9463	9593	9722	129
5	9980	0109	5270238	0367	5270496	0625	5270754	0883	5271012	9
6	5271270	1399	1528	1657	1786	1915	2044	2173	2302	9
7	2560	2689	2818	2947	3076	3205	3334	3463	3592	9
8	3850	3979	4108	4237	4366	4494	4623	4752	4881	9
9	5139	5268	5397	5526	5655	5783	5912	6041	6170	9
3370	6428	6557	6686	6814	6943	7072	7201	7330	7459	9
1	7716	7845	7974	8103	8232	8360	8489	8618	8747	9
2	9004	9133	9262	9391	9520	9648	9777	9906	5280035	9
3	5280292	0421	5280550	0678	5280807	0936	5281065	1193	1322	9
4	1579	1708	1837	1966	2094	2223	2352	2480	2609	9
5	2866	2995	3124	3252	3381	3510	3638	3767	3896	9
6	4153	4282	4410	4539	4668	4796	4925	5053	5182	9
7	5439	5568	5696	5825	5954	6082	6211	6339	6468	9
8	6725	6854	6982	7111	7239	7368	7496	7625	7753	9
9	8010	8139	8267	8396	8525	8653	8782	8910	9039	9
3380	9295	9424	9552	9681	9809	9938	5290066	0195	5290323	9
1	5290580	0709	5290837	0965	5291094	1222	1351	1479	1608	9
2	1864	1993	2121	2250	2378	2506	2635	2763	2892	9
3	3148	3277	3405	3533	3662	3790	3919	4047	4175	9
4	4432	4560	4688	4817	4945	5074	5202	5330	5458	9
5	5715	5843	5972	6100	6228	6356	6485	6613	6741	9
6	6998	7126	7254	7383	7511	7639	7767	7896	8024	9
7	8280	8408	8537	8665	8793	8921	9049	9178	9306	9
8	9562	9690	9819	9947	5300075	0203	5300331	0459	5300588	9
9	5300844	0972	5301100	1228	1356	1485	1613	1741	1869	9
3390	2125	2253	2381	2509	2637	2766	2894	3022	3150	9
1	3406	3534	3662	3790	3918	4046	4174	4302	4430	9
2	4686	4814	4943	5071	5199	5327	5455	5583	5711	128
3	5967	6095	6223	6351	6479	6607	6734	6862	6990	8
4	7246	7374	7502	7630	7758	7886	8014	8142	8270	8
5	8526	8654	8782	8909	9037	9165	9293	9421	9549	8
6	9805	9933	5310060	0188	5310316	0444	5310572	0700	5310828	8
7	5311083	1211	1339	1467	1595	1722	1850	1978	2106	8
8	2362	2489	2617	2745	2873	3001	3128	3256	3384	8
9	3639	3767	3895	4023	4150	4278	4406	4534	4661	8
3400	4917	5045	5172	5300	5428	5556	5683	5811	5939	8
1	6194	6322	6449	6577	6705	6832	6960	7088	7215	8
2	7471	7598	7726	7854	7981	8109	8237	8364	8492	8
3	8747	8875	9002	9130	9258	9385	9513	9640	9768	8
4	5320023	0151	5320278	0406	5320533	0661	5320789	0916	5321044	8
5	1299	1426	1554	1681	1809	1936	2064	2191	2319	8
6	2574	2701	2829	2956	3084	3211	3339	3466	3594	8
7	3849	3976	4104	4231	4359	4486	4614	4741	4868	8
8	5123	5251	5378	5506	5633	5760	5888	6015	6143	8
9	6397	6525	6652	6780	6907	7034	7162	7289	7416	8
3410	7671	7799	7926	8053	8181	8308	8435	8563	8690	8
1	8945	9072	9199	9326	9454	9581	9708	9836	9963	8
2	5330218	0345	5330472	0599	5330727	0854	5330981	1108	5331236	8
3	1490	1617	1745	1872	1999	2126	2254	2381	2508	8
4	2762	2890	3017	3144	3271	3398	3526	3653	3780	8
5	4034	4161	4289	4416	4543	4670	4797	4924	5051	8
6	5306	5433	5560	5687	5814	5941	6068	6196	6323	8
7	6577	6704	6831	6958	7085	7212	7339	7466	7594	8
8	7848	7975	8102	8229	8356	8483	8610	8737	8864	8
9	9118	9245	9372	9499	9626	9753	9880	0007	5340134	127
	1	2	3	4	5	6	7	8	9	

Between 34200 = $\log.^{-1}$ 4·5340261, and 34800 = $\log.^{-1}$ 4·5415792.

ens.	1	2	3	4	5	6	7	8	9	*dif.*
3420	5340388	0515	5340642	0769	5340896	1023	5341150	1277	5341404	127
1	1658	1785	1912	2039	2165	2292	2419	2546	2673	7
2	2927	3054	3181	3308	3435	3561	3688	3815	3942	7
3	4196	4323	4450	4576	4703	4830	4957	5084	5211	7
4	5464	5591	5718	5845	5972	6099	6225	6352	6479	7
5	6733	6859	6986	7113	7240	7366	7493	7620	7747	7
6	8000	8127	8254	8381	8507	8634	8761	8888	9014	7
7	9268	9394	9521	9648	9775	9901	5350028	0155	5350281	7
8	5350535	0662	5350788	0915	5351042	1168	1295	1422	1548	7
9	1802	1928	2055	2181	2308	2435	2561	2688	2815	7
3430	3068	3194	3321	3448	3574	3701	3827	3954	4081	7
1	4334	4460	4587	4713	4840	4967	5093	5220	5346	7
2	5599	5726	5852	5979	6105	6232	6359	6485	6612	7
3	6865	6991	7118	7244	7371	7497	7623	7750	7876	7
4	8129	8256	8382	8509	8635	8762	8888	9015	9141	7
5	9394	9520	9647	9773	9900	/0026	5360152	0279	5360405	7
6	5360658	0784	5360911	1037	5361163	1290	1416	1543	1669	7
7	1922	2048	2174	2301	2427	2553	2680	2806	2932	7
8	3185	3311	3438	3564	3690	3817	3943	4069	4195	7
9	4448	4574	4701	4827	4953	5079	5206	5332	5458	7
3440	5711	5837	5963	6089	6216	6342	6468	6594	6721	7
1	6973	7099	7225	7352	7478	7604	7730	7856	7982	7
2	8235	8361	8487	8613	8739	8866	8992	9118	9244	7
3	9496	9622	9749	9875	5370001	0127	5370253	0379	5370505	7
4	5370758	0884	5371010	1136	1262	1388	1514	1640	1766	7
5	2018	2144	2270	2396	2523	2649	2775	2901	3027	7
6	3279	3405	3531	3657	3783	3909	4035	4161	4287	7
7	4539	4665	4791	4917	5043	5169	5295	5421	5547	126
8	5799	5924	6050	6176	6302	6428	6554	6680	6806	6
9	7058	7184	7310	7436	7561	7687	7813	7939	8065	6
3450	8317	8443	8569	8694	8820	8946	9072	9198	9324	6
1	9575	6701	9827	9953	5380079	0205	5380330	0456	5380582	6
2	5380834	0959	5381085	1211	1337	1463	1588	1714	1840	6
3	2092	2217	2343	2469	2595	2720	2846	2972	3098	6
4	3349	3475	3601	3726	3852	3978	4103	4229	4355	6
5	4606	4732	4858	4983	5109	5235	5360	5486	5612	6
6	5863	5989	6114	6240	6366	6491	6617	6743	6868	6
7	7119	7245	7371	7496	7622	7747	7873	7999	8124	6
8	8375	8501	8627	8752	8878	9003	9129	9255	9380	6
9	9631	9757	9882	/0008	5390133	0259	5390384	0510	5390635	6
3460	5390887	1012	5391138	1263	1389	1514	1640	1765	1891	6
1	2141	2267	2392	2518	2643	2769	2894	3020	3145	6
2	3396	3522	3647	3772	3898	4023	4149	4274	4400	6
3	4650	4776	4901	5027	5152	5277	5403	5528	5653	6
4	5904	6030	6155	6280	6406	6531	6656	6782	6907	6
5	7158	7283	7408	7534	7659	7784	7910	8035	8160	6
6	8411	8536	8661	8787	8912	9037	9163	9288	9413	125
7	9664	9789	9914	/0039	5400165	0290	5400415	0540	5400666	5
8	5400916	1041	5401167	1292	1417	1542	1667	1793	1918	5
9	2168	2293	2419	2544	2669	2794	2919	3044	3170	5
3470	3420	3545	3670	3795	3920	4046	4171	4296	4421	5
1	4671	4796	4921	5047	5172	5297	5422	5547	5672	5
2	5922	6047	6172	6297	6423	6548	6673	6798	6923	5
3	7173	7298	7423	7548	7673	7798	7923	8048	8173	5
4	8423	8548	8673	8798	8923	9048	9173	9298	9423	5
5	9673	9798	9923	/0048	5410173	0298	5410423	0548	5410673	5
6	5410923	1048	5411172	1297	1422	1547	1672	1797	1922	5
7	2172	2297	2422	2546	2671	2796	2921	3046	3171	5
8	3421	3546	3670	3795	3920	4045	4170	4295	4419	5
9	4669	4794	4919	5044	5168	5293	5418	5543	5668	5
	1	2	3	4	5	6	7	8	9	

Between 34800 = log.$^{-1}$ 4·5415792, and 35400 = log.$^{-1}$ 4·5490033.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
3480	5415917	6042	5416167	6292	5416416	6541	5416666	6791	5416915	125
1	7165	7290	7415	7539	7664	7789	7913	8038	8163	5
2	8412	8537	8662	8787	8911	9036	9161	9285	9410	5
3	9659	9784	9909	0033	5420158	0283	5420407	0532	5420657	5
4	5420906	1031	5421155	1280	1404	1529	1654	1779	1903	5
5	2152	2277	2402	2526	2651	2775	2900	3025	3149	5
6	3398	3523	3648	3772	3897	4021	4146	4270	4395	5
7	4644	4769	4893	5018	5142	5267	5391	5516	5640	5
8	5889	6014	6138	6263	6387	6512	6636	6761	6885	5
9	7134	7259	7383	7508	7632	7756	7881	8005	8130	5
3490	8379	8503	8628	8752	8876	9001	9125	9250	9374	5
1	9623	9747	9872	9996	5430120	0245	5430369	0494	5430618	5
2	5430867	0991	5431115	1240	1364	1488	1613	1737	1862	5
3	2110	2235	2359	2483	2607	2732	2856	2980	3105	5
4	3353	3478	3602	3726	3850	3975	4099	4223	4348	5
5	4596	4720	4845	4969	5093	5217	5342	5466	5590	5
6	5838	5963	6087	6211	6335	6460	6584	6708	6832	5
7	7081	7205	7329	7453	7577	7701	7826	7950	8074	5
8	8322	8446	8571	8695	8819	8943	9067	9191	9315	5
9	9564	9688	9812	9936	5440060	0184	5440308	0432	5440556	124
3500	5440805	0929	5441053	1117	1301	1425	1549	1673	1797	4
1	2045	2169	2293	2417	2541	2665	2789	2913	3037	4
2	3285	3409	3533	3657	3781	3905	4029	4153	4277	4
3	4525	4649	4773	4897	5021	5145	5269	5393	5517	4
4	5765	5889	6013	6137	6261	6385	6508	6632	6756	4
5	7004	7128	7252	7376	7500	7624	7747	7871	7995	4
6	8243	8367	8491	8615	8738	8862	8986	9110	9234	4
7	9481	9605	9729	9853	9977	0101	5450224	0348	5450472	4
8	5450720	0843	5450967	1091	5451215	1339	1462	1586	1710	4
9	1957	2081	2205	2329	2452	2576	2700	2824	2947	4
3510	3195	3319	3442	3566	3690	3813	3937	4061	4185	4
1	4432	4556	4679	4803	4927	5050	5174	5298	5421	4
2	5669	5792	5916	6040	6163	6287	6411	6534	6658	4
3	6905	7029	7152	7276	7400	7523	7647	7770	7894	4
4	8141	8265	8388	8512	8635	8759	8883	9006	9130	4
5	9377	9500	9624	9747	9871	9995	5460118	0242	5460365	4
6	5460612	0736	5460859	0983	5461106	1230	1353	1477	1600	4
7	1847	1971	2094	2218	2341	2465	2588	2711	2835	4
8	3082	3205	3329	3452	3576	3699	3822	3946	4069	4
9	4316	4439	4563	4686	4810	4933	5056	5180	5303	4
3520	5550	5673	5797	5920	6043	6167	6290	6414	6537	4
1	6784	6907	7030	7154	7277	7400	7524	7647	7770	4
2	8017	8140	8263	8387	8510	8633	8757	8880	9003	4
3	9250	9373	9496	9620	9743	9866	9989	0113	5470236	4
4	5470482	0605	5470729	0852	5470975	1098	5471222	1345	1468	4
5	1714	1838	1961	2084	2207	2330	2454	2577	2700	4
6	2946	3069	3193	3316	3439	3562	3685	3808	3931	123
7	4178	4301	4424	4547	4670	4793	4916	5040	5163	3
8	5409	5532	5655	5778	5901	6024	6147	6270	6394	3
9	6640	6763	6886	7009	7132	7255	7378	7501	7624	3
3530	7870	7993	8116	8239	8362	8485	8608	8731	8854	3
1	9100	9223	9346	9469	9592	9715	9838	9961	5480084	3
2	5480330	0453	5480576	0699	5480822	0945	5481068	1191	1313	3
3	1559	1682	1805	1928	2051	2174	2297	2420	2543	3
4	2788	2911	3034	3157	3280	3403	3526	3648	3771	3
5	4017	4140	4263	4386	4508	4631	4754	4877	5000	3
6	5245	5368	5491	5614	5737	5859	5982	6105	6228	3
7	6473	6596	6719	6842	6964	7087	7210	7333	7456	3
8	7701	7824	7947	8069	8192	8315	8437	8560	8683	3
9	8928	9051	9174	9296	9419	9542	9665	9787	9910	3
	1	2	3	4	5	6	7	8	9	

Between 35400 = log.$^{-1}$ 4·5490033, and 36000 = log.$^{-1}$ 4·5563025.

tens.	1	2	3	4	5	6	7	8	9	*dif.*
3540	5490155	0278	5490401	0523	5490646	0769	5490891	1014	5491137	123
1	1382	1505	1627	1750	1872	1995	2118	2240	2363	3
2	2608	2731	2853	2976	3099	3221	3344	3466	3589	3
3	3834	3957	4079	4202	4324	4447	4569	4692	4815	3
4	5060	5182	5305	5427	5550	5672	5795	5917	6040	3
5	6285	6407	6530	6652	6775	6897	7020	7142	7265	3
6	7510	7632	7755	7877	8000	8122	8245	8367	8489	3
7	8734	8857	8979	9102	9224	9346	9469	9591	9714	3
8	9959	/0081	5500203	0326	5500448	0570	5500693	0815	5500938	3
9	5501182	1305	1427	1549	1672	1794	1917	2039	2161	3
3550	2406	2528	2651	2773	2895	3017	3140	3262	3384	3
1	3629	3751	3874	3996	4118	4240	4363	4485	4607	3
2	4852	4974	5096	5219	5341	5463	5585	5708	5830	3
3	6074	6197	6319	6441	6563	6685	6808	6930	7052	3
4	7296	7419	7541	7663	7785	7907	8030	8152	8274	3
5	8518	8640	8763	8885	9007	9129	9251	9373	9495	3
6	9740	9862	9984	/0106	5510228	0350	5510472	0594	5510717	3
7	5510961	1083	5511205	1327	1449	1571	1693	1815	1937	3
8	2181	2304	2426	2548	2670	2792	2914	3036	3158	3
9	3402	3524	3646	3768	3890	4012	4134	4256	4378	3
3560	4622	4744	4866	4988	5110	5232	5354	5476	5598	122
1	5842	5964	6086	6208	6329	6451	6873	6695	6817	2
2	7061	7183	7305	7427	7549	7671	7793	7914	8036	2
3	8280	8402	8524	8646	8768	8890	9011	9133	9255	2
4	9499	9621	9743	9864	9986	/0108	5520230	0352	5520474	2
5	5520717	0839	5520961	1083	5521204	1326	1448	1570	1692	2
6	1935	2057	2179	2301	2422	2544	2666	2788	2909	2
7	3153	3275	3396	3518	3640	3762	3883	4005	4127	2
8	4370	4492	4614	4735	4857	4979	5100	5222	5344	2
9	5587	5709	5831	5952	6074	6196	6317	6439	6561	2
3570	6804	6925	7047	7169	7290	7412	7534	7655	7777	2
1	8020	8142	8263	8385	8507	8628	8750	8871	8993	2
2	9236	9358	9479	9601	9722	9844	9965	/0087	5530209	2
3	5530452	0573	5530695	0816	5530938	1059	5531181	1302	1424	2
4	1667	1789	1910	2032	2153	2275	2396	2517	2639	2
5	2882	3003	3125	3246	3368	3489	3611	3732	3854	2
6	4097	4218	4339	4461	4582	4704	4825	4947	5068	2
7	5311	5432	5554	5675	5796	5918	6039	6161	6282	2
8	6525	6646	6767	6889	7010	7132	7253	7374	7496	2
9	7738	7860	7981	8102	8224	8345	8466	8588	8709	2
3580	8952	9073	9194	9315	9437	9558	9679	9801	9922	2
1	5540164	0286	5540407	0528	5540650	0771	5540892	1013	5541135	2
2	1377	1498	1620	1741	1862	1983	2104	2226	2347	2
3	2589	2710	2832	2953	3074	3195	3316	3438	3559	2
4	3801	3922	4044	4165	4286	4407	4528	4649	4770	2
5	5013	5134	5255	5376	5497	5618	5740	5861	5982	2
6	6224	6345	6466	6587	6708	6829	6951	7072	7193	121
7	7435	7556	7677	7798	7919	8040	8161	8282	8403	1
8	8645	8766	8887	9008	9130	9251	9372	9493	9614	1
9	9856	9977	5550098	0219	5550340	0461	5550582	0703	5550824	1
3590	5551065	1186	1307	1428	1549	1670	1791	1912	2033	1
1	2275	2396	2517	2638	2759	2880	3001	3121	3242	1
2	3484	3605	3726	3847	3968	4089	4210	4330	4451	1
3	4693	4814	4935	5056	5176	5297	5418	5539	5660	1
4	5902	6022	6143	6264	6385	6506	6627	6747	6868	1
5	7110	7231	7351	7472	7593	7714	7835	7955	8076	1
6	8318	8438	8559	8680	8801	8921	9042	9163	9284	1
7	9525	9646	9767	9887	5560008	0129	5560249	0370	5560491	1
8	5560732	0853	5560974	1094	1215	1336	1456	1577	1698	1
9	1939	2060	2180	2301	2422	2542	2663	2784	2904	1
	1	2	3	4	5	6	7	8	9	

LOGARITHMS

OF

SINES, COSINES, TANGENTS, AND COTANGENTS.

LOG. SINE 0°.

″	0′	1′	2′	3′	4′	5′	6′	7′	″
0	— ∞	6·4637261	6·7647561	6 9408473	7·0657860	7·1626960	7·2418771	7·3088239	60
1	4·6855749	709047	683602	432534	675918	41412	30818	98567	59
2	9866049	779665	719347	456462	693901	55817	42832	7·3108870	58
3	5·1626961	849154	754800	480259	711810	70173	54813	19149	57
4	2876349	917548	789965	503926	729646	84483	66760	29404	56
5	3845449	984882	824849	527465	747408	98745	78675	39635	55
6	4637261	6·5051188	859454	550878	765099	7·1712961	90557	49842	54
7	5306729	116497	893786	574164	782717	27131	7·2502407	60024	53
8	5886649	180838	927848	597327	800264	41254	14225	70183	52
9	6398174	244239	961645	620366	817741	55332	26010	80318	51
10	6855749	306729	995182	643284	835148	69364	37764	90430	50
11	7269676	368332	6·8028461	666082	852485	83351	49485	7·3200518	49
12	7647561	429074	061488	688760	869753	97293	61176	10583	48
13	7995182	488977	094265	711321	886953	7·1811190	72835	20624	47
14	8317029	548066	126796	733765	904085	25043	84462	30643	46
15	8616661	606361	159086	756094	921149	38853	96059	40638	45
16	8896948	663884	191137	778309	938147	52618	7·2607625	50610	44
17	9160238	720656	222954	800410	955079	66340	19160	60560	43
18	9408474	776695	254539	822400	971945	80018	30664	70487	42
19	9643285	832019	285896	844279	988745	93654	42138	80391	41
20	9866049	886648	317029	866048	7·1005481	7·1907247	53582	90272	40
21	6·0077942	940599	347939	887709	22153	20797	64996	7·3300131	39
22	0279975	993887	378632	909262	38760	34306	76380	09968	38
23	0473027	6 6046529	409109	930708	55305	47772	87734	19783	37
24	0657861	098541	439373	952050	71787	61197	99058	29575	36
25	0835149	149938	469428	973287	88206	74580	7·2710353	39345	35
26	1005482	200733	499277	994420	7 1104564	87923	21619	49094	34
27	1169386	250941	528922	7·0015451	20860	7·2001224	32856	58821	33
28	1327329	300575	558365	036381	37095	14485	44063	68525	32
29	1479729	349649	587611	057211	53270	27706	55242	78209	31
30	1626961	398174	616661	077941	69385	40886	66392	87870	30
31	1769366	446162	645518	098572	85440	54027	77514	97511	29
32	1907248	493627	674184	119107	7·1201436	67128	88607	7·3407130	28
33	2040888	540578	702663	139544	17374	80189	99672	16727	27
34	2170538	587027	730955	159886	33253	93211	7·2810708	26304	26
35	2296429	632985	759065	180132	49074	7·2106195	21717	35859	25
36	2418774	678461	786994	200285	64838	19140	32698	45394	24
37	2537766	723466	814745	220345	80545	32046	43651	54907	23
38	2653585	768009	842319	240313	96195	44914	54577	64400	22
39	2766395	812100	869719	260189	7·1311789	57744	65475	73872	21
40	2876349	855748	896948	279975	27328	70536	76346	83323	20
41	2983587	898962	924007	299671	42811	83290	87190	92754	19
42	3088242	941750	950898	319278	58238	96008	98006	7·3502165	18
43	3190433	984121	977624	338796	73612	7·2208688	7·2908796	11555	17
44	3290275	6·7026082	6·9004187	358228	88931	21331	19560	20925	16
45	3387874	067641	030588	377573	7 1404196	33938	30296	30275	15
46	3483327	108807	056829	396832	19408	46508	41006	39604	14
47	3576727	149585	082913	416006	34566	59041	51690	48914	13
48	3668161	189986	108841	435096	49672	71539	62347	58203	12
49	3757709	230013	134615	454103	64726	84001	72979	67473	11
50	3845449	269675	160237	473026	79727	96427	83584	76723	10
51	3931450	308978	185709	491868	94677	7·2308818	94164	85954	9
52	4015782	347929	211033	510628	7·1509576	21173	7·3004718	95165	8
53	4098507	386533	236209	529307	24423	33494	15246	7·3604356	7
54	4179686	424797	261241	547906	39221	45779	25749	13528	6
55	4259376	462727	286129	566426	53967	58030	36227	22681	5
56	4337629	500328	310875	584868	68664	70246	46679	31814	4
57	4414497	537607	335481	603231	83312	82429	57106	40929	3
58	4490029	574569	359948	621517	97910	94577	67509	50024	2
59	4564269	611218	384278	639727	7·1612459	7·2406691	77886	59100	1
60	4637261	647561	408473	657860	26960	18771	88239	68157	0
″	59′	58′	57′	56′	55′	54′	53′	52′	″

LOG. COSINE 89°.

LOG. TAN. 0°.

″	0′	1′	2′	3′	4′	5′	6′	7′	″
0	— ∞	6·4637261	6·7647562	6·9408475	7·0657863	7·1626964	7·2418778	7·3088248	60
1	4·6855749	709047	683603	432536	75921	41417	30825	98576	59
2	9866049	779666	719347	456464	93904	55821	42839	7·3108879	58
3	5·1626961	849154	754800	480261	7·0711813	70178	54819	19158	57
4	2876349	917549	789966	503928	29649	84488	66767	29413	56
5	3845449	984882	824849	527467	47412	98750	78682	39644	55
6	4637261	6·5051188	859455	550879	65102	7·1712966	90564	49851	54
7	5306729	116497	893786	574166	82720	27136	7·2502414	60034	53
8	5886649	180838	927849	597328	7·0800268	41259	14231	70193	52
9	6398174	244240	961646	620368	17744	55337	26017	80328	51
10	6855749	306729	995183	643286	35151	69369	37771	90440	50
11	7269676	368332	6·8028462	666084	52488	83356	49492	7·3200528	49
12	7647561	429074	061489	688762	69756	97298	61183	10592	48
13	7995182	488977	094266	711323	86956	7·1811195	72842	20634	47
14	8317029	548066	126797	733767	7·0904088	25049	84469	30652	46
15	8616661	606361	159087	756096	21153	38858	96066	40648	45
16	8896948	663885	191138	778311	38151	52623	7·2607632	50620	44
17	9160238	720656	222955	800412	55082	66345	19167	60570	43
18	9408474	776695	254540	822402	71948	80023	30672	70496	42
19	9643285	832020	285897	844281	88749	93659	42146	80400	41
20	9866049	886649	317030	866050	7·1005484	7·1907252	53590	90282	40
21	6·0077942	940599	347940	887711	22156	20802	65003	7·3300141	39
22	0279975	993887	378633	909264	38764	34311	76387	09978	38
23	0473027	6·6046530	409110	930710	55309	47777	87741	19793	37
24	0657861	098542	439374	952052	71790	61202	99066	29585	36
25	0835149	149938	469429	973289	88210	74586	7·2710361	39356	35
26	1005482	200733	499278	994422	7·1104567	87928	21627	49104	34
27	1169386	250941	528923	7·0015454	20864	7·2001230	32863	58831	33
28	1327329	300576	558367	036383	37099	14491	44071	68536	32
29	1479729	349649	587612	057213	53274	27711	55250	78219	31
30	1626961	398174	616662	077943	69389	40892	66400	87881	30
31	1769366	446163	645519	098575	85444	54032	77521	97521	29
32	1907248	493627	674185	119109	7·1201440	67133	88615	7·3407140	28
33	2040888	540578	702664	139546	17378	80195	99679	16738	27
34	2170538	587027	730957	159888	33257	93217	7·2810716	26314	26
35	2296429	632985	759066	180135	49078	7·2106201	21725	35870	25
36	2418774	678461	786995	200288	64842	19145	32706	45404	24
37	2537766	723466	814746	220348	80549	32052	43659	54918	23
38	2653585	768010	842320	240315	96199	44920	54585	64411	22
39	2766395	812101	869721	260191	7·1311793	57750	65483	73883	21
40	2876349	855749	896949	279977	27332	70542	76354	83334	20
41	2983587	898963	924008	299673	42815	83296	87198	92765	19
42	3088242	941751	950900	319280	58242	96014	98015	7·3502176	18
43	3190433	984121	977626	338799	73616	7·2208694	7·2908805	11566	17
44	3290275	6·7026082	6·9004188	358231	88935	21337	19568	20936	16
45	3387874	067642	030589	377576	7·1404200	33944	30304	30286	15
46	3483327	108808	056830	396835	19412	46514	41015	39615	14
47	3576727	149587	082914	416009	34570	59048	51698	48925	13
48	3668161	189987	108842	435099	49676	71545	62356	58215	12
49	3757709	230014	134617	454105	64730	84007	72987	67485	11
50	3845449	269676	160239	473029	79732	96433	83593	76735	10
51	3931450	308979	185711	491870	94681	7·2308824	94173	85965	9
52	4015782	347929	211034	510630	7·1509580	21180	7·3004727	95176	8
53	4098507	386534	236211	529310	24428	33500	15255	7·3604368	7
54	4179686	424798	261242	547909	39225	45786	25758	13540	6
55	4259376	462728	286130	566429	53972	58036	36235	22692	5
56	4337629	500329	310876	584871	68669	70253	46688	31826	4
57	4414497	537608	335482	603234	83316	82435	57115	40940	3
58	4490029	574570	359950	621520	97914	94583	67517	50035	2
59	4564269	611219	384280	639730	7·1612464	7·2406698	77895	59112	1
60	4637261	647562	408475	657863	26964	18778	88248	68169	0
″	59′	58′	57′	56′	55′	54′	53′	52′	″

LOG. COTAN. 89°.

LOG. SINE 0°.

″	8′	9′	10′	11′	12′	13′	14′	15′	″
0	7·3668157	7·4179681	7·4637255	7·5051181	7·5429065	7·5776684	7·6 98530	7·6398160	60
1	77195	87716	44487	57756	35092	82249	7·6103697	7·6402983	59
2	86215	95737	51707	64321	41112	87806	08858	07800	58
3	95216	7·4203742	58916	70876	47123	93356	14012	12612	57
4	7·3704198	11733	66112	77422	53125	98899	19161	17419	56
5	13162	19709	73296	83958	59120	7·5804435	24304	22221	55
6	22107	27670	80469	90483	65106	09964	29440	27017	54
7	31034	35617	87629	96999	71084	15485	34571	31808	53
8	39943	43549	94778	7·5103506	77053	21000	39695	36593	52
9	48832	51467	7·4701915	10002	83015	26508	44813	41373	51
10	57705	59370	09041	16489	88968	32009	49926	46149	50
11	66559	67259	16154	22966	94913	37503	55032	50918	49
12	75396	75134	23257	29434	7·5500850	42990	60132	55683	48
13	84214	82995	30347	35892	06779	48470	65227	60442	47
14	93014	90841	37426	42340	12700	53943	70315	65196	46
15	7·3801796	98673	44493	48779	18613	59409	75397	69945	45
16	10561	7·4306491	51549	55208	24518	64869	80474	74689	44
17	19308	14295	58594	61628	30414	70321	85544	79428	43
18	28038	22085	65627	68038	36303	75767	90609	84161	42
19	36750	29861	72649	74439	42184	81206	95668	88889	41
20	45444	37624	79659	80830	48057	86638	7·6200721	93613	40
21	54122	45372	86658	87212	53921	92063	05768	98331	39
22	62782	53106	93646	93585	59778	97481	10809	7·6⁻03043	38
23	71424	60827	7·4800623	99948	65627	7·5902893	15844	07751	37
24	80050	68534	07588	7·5206302	71469	08298	20873	12454	36
25	88658	76228	14542	12646	77302	13696	25897	17151	35
26	97249	83908	21485	18982	83127	19088	30915	21844	34
27	7·3905824	91574	28417	25308	88945	24473	35927	26531	33
28	14381	99227	35338	31625	94755	29851	40933	31214	32
29	22922	7·4406866	42248	37933	7·5600557	35223	45934	35891	31
30	31446	14492	49147	44231	06352	40588	50928	40563	30
31	39953	22104	56035	50521	12138	45946	55917	45231	29
32	48444	29703	62913	56801	17917	51298	60901	49893	28
33	56918	37289	69779	63073	23689	56643	65878	54550	27
34	65375	44862	76634	69335	29452	61981	70850	59203	26
35	73816	52421	83479	75588	35208	67313	75816	63850	25
36	82241	59968	90313	81833	40957	72639	80777	68492	24
37	90650	67501	97136	88068	46698	77958	85732	73130	23
38	99042	75021	7·4903949	94295	52431	83270	90681	77762	22
39	7·4007418	82529	10750	7·5300512	58157	88576	95624	82390	21
40	15778	90023	17541	06721	63875	93876	7·6300562	87012	20
41	24121	97504	24322	12920	69585	99169	05495	91630	19
42	32449	7·4504973	31092	19111	75289	7·6004455	10421	96243	18
43	40761	12428	37851	25294	80984	09735	15342	7·6600850	17
44	49057	19871	44600	31467	86672	15009	20258	05453	16
45	57337	27302	51339	37631	92353	20277	25168	10052	15
46	65601	34719	58067	43787	98026	25538	30073	14645	14
47	73850	42124	64784	49934	7·5703692	30792	34971	19233	13
48	82083	49516	71492	56073	09351	36040	39865	23817	12
49	90301	56896	78188	62202	15002	41282	44753	28395	11
50	98503	64263	84875	68324	20646	46518	49635	32969	10
51	7·4106689	71618	91551	74436	26282	51747	54512	37538	9
52	14860	78960	98217	80540	31912	56970	59384	42103	8
53	23016	86290	7·5004873	86635	37533	62187	64250	46662	7
54	31156	93607	11519	92722	43148	67397	69110	51217	6
55	39282	7·4600912	18154	98800	48755	72602	73965	55767	5
56	47392	08205	24780	7·5404870	54356	77800	78815	60312	4
57	55487	15486	31395	10931	59949	82991	83659	64852	3
58	63567	22754	38000	16984	65534	88177	88498	69388	2
59	71631	30011	44595	23029	71113	93356	93332	73919	1
60	79681	37255	51181	29065	76684	98530	98160	78445	0
″	51′	50′	49′	48′	47′	46′	45′	44′	″

LOG. COSINE 89°.

LOG. TAN. 0°.

″	8′	9′	10′	11′	12′	13′	14′	15′	″
0	7·3668169	7·4179696	7·4637273	7·5051203	7·5429091	7·5776715	7·6098566	7·6398201	60
1	77207	87731	44506	57778	35119	82280	7·6103733	7·6403024	59
2	86227	95752	51726	64343	41138	87837	08894	07842	58
3	95228	7·4203757	58934	70899	47149	93387	14049	12654	57
4	7·3704210	11748	66130	77444	53152	98930	19197	17461	56
5	13174	19724	73315	83980	59147	7·5804466	24340	22262	55
6	22119	27685	80487	90506	65133	09995	29477	27059	54
7	31046	35632	87648	97022	71111	15517	34607	31850	53
8	39955	43564	94797	7·5103528	77080	21032	39732	36635	52
9	48845	51482	7·4701934	10025	83042	26540	44850	41416	51
10	57718	59386	09060	16512	88995	32041	49963	46191	50
11	66572	67275	16173	22989	94941	37535	55069	50961	49
12	75408	75150	23276	29457	7·5500878	43022	60169	55725	48
13	84226	83010	30366	35915	06807	48502	65264	60485	47
14	93026	90857	37445	42363	12728	53975	70352	65239	46
15	7·3801809	98689	44513	48802	18640	59441	75435	69988	45
16	10574	7·4306507	51569	55231	24545	64901	80511	74732	44
17	19321	14311	58613	61651	30442	70353	85582	79471	43
18	28051	22101	65646	68061	36331	75799	90647	84204	42
19	36763	29877	72668	74462	42212	81238	95705	88933	41
20	45457	37640	79679	80854	48084	86670	7·6200758	93656	40
21	54134	45388	86678	87236	53949	92096	05805	98374	39
22	62794	53123	93666	93608	59806	97514	10847	7·6503087	38
23	71437	60843	7·4800642	99972	65656	7·5902926	15882	07795	37
24	80063	68551	07608	7·5206326	71497	08331	20911	12497	36
25	88671	76244	14562	12670	77330	13730	25935	17195	35
26	97263	83924	21505	19006	83156	19121	30953	21888	34
27	7·3905837	91590	28437	25332	88974	24506	35965	26575	33
28	14395	99243	35359	31649	94784	29884	40972	31258	32
29	22935	7·4406882	42269	37957	7·5600586	35256	45972	35935	31
30	31459	14508	49168	44256	06380	40621	50967	40608	30
31	39967	22121	56056	50545	12167	45980	55956	45275	29
32	48457	29720	62933	56826	17946	51331	60939	49937	28
33	56931	37306	69799	63097	23718	56677	65917	54595	27
34	65389	44879	76655	69360	29481	62015	70889	59247	26
35	73830	52438	83500	75613	35238	67347	75855	63895	25
36	82255	59985	90334	81858	40986	72673	80816	68537	24
37	90663	67518	97157	88093	46727	77992	85771	73174	23
38	99055	75038	7·4903969	94319	52460	83304	90720	77807	22
39	7·4007431	82546	10771	7·5300537	58186	88611	95664	82435	21
40	15791	90040	17562	06746	63904	93910	7·6300602	87057	20
41	24135	97521	24343	12946	69615	99203	05534	91675	19
42	32463	7·4504990	31113	19137	75318	7·6004490	10461	96288	18
43	40775	12446	37872	25319	81014	09770	15382	7·6600896	17
44	49071	19889	44621	31492	86702	15044	20298	05499	16
45	57351	27319	51360	37657	92383	20311	25208	10097	15
46	65616	34737	58088	43813	98056	25572	30113	14690	14
47	73864	42141	64806	49960	7·5703722	30827	35012	19279	13
48	82097	49534	71513	56098	09381	36075	39905	23863	12
49	90315	56913	78210	62228	15032	41317	44793	28441	11
50	98517	64281	84897	68349	20676	46553	49676	33015	10
51	7·4106703	71635	91573	74462	26313	51782	54553	37585	9
52	14875	78978	98239	80566	31942	57005	59424	42149	8
53	23030	86308	7·5004895	86661	37564	62222	64290	46709	7
54	31171	93625	11541	92748	43179	67433	69151	51263	6
55	39296	7·4600930	18176	98826	48786	72637	74006	55813	5
56	47406	08223	24802	7·5404896	54386	77835	78856	60359	4
57	55501	15504	31417	10958	59979	83027	83700	64899	3
58	63581	22773	38022	17011	65565	88213	88539	69435	2
59	71646	30030	44618	23055	71144	93392	93373	73966	1
60	79696	37273	51203	29091	76715	98566	98201	78492	0
″	51′	50′	49′	48′	47′	46′	45′	44′	″

LOG. COTAN. 89°.

LOG. SINE 0°.

″	16′	17′	18′	19′	20′	21′	22′	23′	″
0	7·6678445	7·6941733	7·7189966	7·7424775	7·7647537	7·7859427	7·8061458	7·8254507	60
1	82967	45988	93986	28583	51154	62872	64747	57653	59
2	87484	50240	98001	32388	54769	66315	68033	60797	58
3	91996	54487	7·7202013	36189	58380	69755	71317	63938	57
4	96503	58730	06021	39987	61989	73192	74599	67077	56
5	7·6701006	62969	10026	43781	65594	76627	77878	70214	55
6	05504	67204	14027	47573	69197	80058	81154	73348	54
7	09998	71435	18024	51360	72797	83488	84428	76481	53
8	14486	75662	22017	55145	76393	86914	87699	79611	52
9	18970	79884	26007	58926	79987	90337	90966	82738	51
10	23450	84103	29993	62705	83577	93758	94235	85864	50
11	27925	88317	33976	66479	87165	97177	97499	88987	49
12	32395	92528	37955	70251	90750	7·7900592	7·8100761	92108	48
13	36861	96734	41930	74019	94332	04005	04020	95227	47
14	41322	7·7000936	45902	77784	97910	07415	07227	98343	46
15	45779	05134	49869	81546	7·7701486	10823	10531	7·8301458	45
16	50231	09328	53834	85304	05059	14228	13783	04570	44
17	54678	13518	57794	89059	08629	17630	17032	07680	43
18	59121	17704	61752	92811	12196	21029	20279	10787	42
19	63559	21886	65705	96560	15760	24426	23524	13893	41
20	67993	26064	69655	7·7500306	19322	27820	26766	16996	40
21	72422	30238	73601	04048	22880	31212	30006	20097	39
22	76847	34407	77544	07787	26435	34601	33243	23195	38
23	81267	38573	81483	11523	29988	37987	36478	26292	37
24	85683	42735	85419	15255	33537	41371	39711	29386	36
25	90094	46893	89351	18985	37084	44752	42941	32478	35
26	94501	51047	93279	22711	40628	48130	46168	35568	34
27	98904	55197	97204	26434	44169	51506	49394	38656	33
28	7·6803302	59343	7·7301125	30154	47707	54879	52617	41741	32
29	07695	63485	05043	33871	51242	58250	55837	44825	31
30	12084	67623	08957	37584	54774	61617	59055	47906	30
31	16469	71757	12868	41294	58303	64983	62271	50985	29
32	20849	75887	16776	45001	61830	68345	65484	54062	28
33	25224	80014	20679	48705	65354	71705	68695	57136	27
34	29596	84136	24579	52406	68874	75063	71904	60209	26
35	33963	88254	28476	56104	72392	78418	75110	63279	25
36	38325	92369	32369	59798	75907	81770	78314	66347	24
37	42683	96480	36259	63490	79420	85120	81516	69413	23
38	47037	7·7100586	40145	67178	82929	88467	84715	72477	22
39	51387	04689	44028	70863	86436	91811	87912	75538	21
40	55732	08788	47908	74545	89939	95153	91106	78598	20
41	60072	12883	51783	78224	93440	98493	94298	81655	19
42	64409	16975	55656	81900	96938	7·8001830	97488	84710	18
43	68741	21062	59525	85572	7·7800434	05164	7·8200676	87763	17
44	73069	25146	63390	89242	03926	08496	03861	90814	16
45	77392	29225	67252	92908	07416	11825	07043	93863	15
46	81711	33301	71111	96572	10903	15151	10224	96909	14
47	86026	37373	74966	7·7600232	14387	18475	13402	99954	13
48	90337	41442	78818	03889	17868	21797	16578	7·8402996	12
49	94643	45506	82666	07543	21347	25116	19751	06036	11
50	98945	49567	86511	11194	24822	28432	22922	09074	10
51	7·6903243	53624	90353	14842	28295	31746	26091	12110	9
52	07536	57677	94191	18487	31765	35058	29258	15144	8
53	11826	61726	98026	22129	35233	38367	32422	18176	7
54	16111	65772	7·7401857	25768	38697	41673	35584	21205	6
55	20392	69814	05685	29403	42159	44977	38743	24233	5
56	24668	73852	09510	33036	45618	48278	41901	27258	4
57	28941	77886	13331	36666	49075	51577	45056	30281	3
58	33209	81917	17149	40292	52528	54873	48209	33302	2
59	37473	85943	20964	43916	55979	58167	51359	36321	1
60	41733	89966	24775	47537	59427	61458	54507	39338	0
″	43′	42′	41′	40′	39′	38′	37′	36′	″

LOG. COSINE 89°.

″	16′	17′	18′	19′	20′	21′	22′	23′	″
0	7·6678492	7·6941786	7·7190026	7·7424841	7·7647610	7·7859508	7·8061547	7·8254604	60
1	83014	46042	94045	28649	51228	62954	64836	57750	59
2	87531	50293	98061	32454	54843	66396	68123	60894	58
3	92043	54541	7·7202073	36255	58454	69836	71407	64036	57
4	96551	58784	06081	40053	62063	73274	74688	67175	56
5	7·6701053	63023	10086	43848	65669	76708	77967	70312	55
6	05552	67258	14087	47640	69271	80140	81244	73446	54
7	10045	71489	18084	51428	72871	83569	84518	76579	53
8	14534	75716	22078	55212	76468	86996	87789	79709	52
9	19018	79938	26068	58994	80061	90420	91059	82837	51
10	23498	84157	30054	62772	83652	93841	94325	85962	50
11	27973	88371	34037	66547	87240	97259	97590	89086	49
12	32443	92582	38016	70319	90825	7·7900675	7·8100851	92207	48
13	36909	96788	41991	74087	94407	04088	04111	95326	47
14	41371	7·7000990	45963	77852	97986	07498	07368	98443	46
15	45827	05189	49931	81614	7·7701562	10906	10622	7·8301557	45
16	50279	09383	53895	85372	05135	14311	13874	04669	44
17	54727	13573	57856	89126	08705	17713	17124	07779	43
18	59170	17759	61813	92880	12272	21113	20371	10887	42
19	63608	21941	65767	96629	15836	24510	23615	13992	41
20	68042	26119	69717	7·7500374	19398	27904	26858	17096	40
21	72471	30293	73663	04117	22956	31296	30098	20197	39
22	76896	34463	77606	07856	26512	34685	33335	23296	38
23	81317	38629	81545	11592	30064	38071	36570	26392	37
24	85733	42791	85481	15325	33614	41455	39803	29487	36
25	90144	46949	89413	19054	37161	44836	43033	32579	35
26	94551	51103	93342	22780	40705	48215	46261	35669	34
27	98953	55253	97267	26504	44246	51590	49486	38757	33
28	7·6803351	59399	7·7301188	30224	47784	54964	52709	41843	32
29	07745	63541	05106	33940	51319	58334	55930	44926	31
30	12134	67679	09020	37654	54851	61702	59148	48007	30
31	16519	71813	12931	41364	58381	65068	62364	51087	29
32	20899	75944	16839	45072	61907	68431	65578	54163	28
33	25275	80070	20742	48776	65431	71791	68789	57238	27
34	29646	84193	24643	52477	68952	75148	71998	60311	26
35	34013	88311	28540	56174	72470	78503	75204	63381	25
36	38376	92426	32433	59869	75985	81856	78408	66449	24
37	42734	96537	36323	63560	79498	85206	81610	69515	23
38	47088	7·7100643	40209	67249	83007	88553	84809	72580	22
39	51438	04746	44092	70934	86514	91898	88006	75641	21
40	55783	08846	47972	74616	90018	95240	91201	78701	20
41	60124	12941	51848	78295	93519	98579	94393	81758	19
42	64460	17032	55720	81971	97017	7·8001916	97583	84813	18
43	68792	21120	59589	85644	7·7800513	05251	7·8200770	87867	17
44	73120	25203	63455	89313	04005	08582	03956	90918	16
45	77444	29283	67317	92980	07495	11912	07139	93966	15
46	81763	33359	71176	96643	10982	15238	10319	97013	14
47	86078	37432	75031	7·7600304	14466	18563	13497	7·8400058	13
48	90389	41500	78883	03961	17948	21884	16673	03100	12
49	94695	45565	82731	07615	21426	25203	19847	06140	11
50	98997	49625	86577	11266	24902	28520	23018	09179	10
51	7·6903295	53682	90418	14915	28375	31834	26187	12215	9
52	07589	57736	94257	18560	31845	35146	29354	15249	8
53	11878	61785	98091	22202	35313	38455	32518	18280	7
54	16163	65831	7·7401923	25840	38778	41761	35680	21310	6
55	20444	69873	05751	29476	42240	45065	38840	24338	5
56	24721	73911	09576	33109	45699	48366	41997	27363	4
57	28993	77945	13397	36739	49155	51665	45153	30387	3
58	33262	81976	17215	40366	52609	54962	48305	33408	2
59	37526	86003	21030	43989	56060	58256	51456	36427	1
60	41786	90026	24841	47610	59508	61547	54604	39444	0
″	43′	42′	41′	40′	39′	38′	37′	36′	″

LOG. COTAN. 89°.

LOG. SINE 0°.

″	24′	25′	26′	27′	28′	29′	30′	31′	″
0	7·8439338	7·8616623	7·8786953	7·8950854	7·9108793	7·9261190	7·9408419	7·9550819	60
1	42353	19517	89736	53534	11378	63685	10831	53153	59
2	45366	22410	92517	56212	13960	66179	13241	55486	58
3	48377	25300	95297	58889	16542	68671	15651	57818	57
4	51385	28189	98075	61564	19121	71162	18059	60149	56
5	54392	31075	7·8800850	64237	21699	73651	20465	62478	55
6	57396	33960	03625	66909	24276	76139	22871	64806	54
7	60398	36843	06397	69579	26851	78626	25275	67133	53
8	63399	39723	09167	72248	29425	81111	27677	69458	52
9	66397	42602	11936	74914	31997	83595	30079	71782	51
10	69393	45479	14703	77580	34567	86077	32479	74105	50
11	72387	48354	17469	80243	37136	88558	34877	76427	49
12	75379	51228	20232	82905	39704	91037	37275	78747	48
13	78369	54099	22994	85565	42269	93516	39671	81067	47
14	81357	56968	25754	88224	44834	95992	42066	83385	46
15	84343	59836	28512	90881	47397	98467	44459	85702	45
16	87326	62702	31269	93536	49958	7·9300941	46851	88017	44
17	90308	65565	34023	96190	52518	03414	49242	90331	43
18	93288	68427	36776	98842	55076	05885	51631	92645	42
19	96265	71287	39528	7·9001493	57633	08354	54019	94956	41
20	99241	74145	42277	04141	60189	10823	56406	97267	40
21	7·8502215	77001	45025	06789	62743	13289	58792	99576	39
22	05186	79856	47771	09434	65295	15755	61176	7·9601885	38
23	08156	82708	50515	12078	67846	18219	63559	04192	37
24	11123	85559	53258	14721	70395	20682	65940	06497	36
25	14088	88408	55999	17362	72943	23143	68321	08802	35
26	17052	91254	58738	20001	75489	25603	70700	11105	34
27	20013	94099	61475	22639	78034	28061	73077	13407	33
28	22973	96942	64211	25275	80578	30518	75454	15708	32
29	25930	99784	66945	27909	83120	32974	77829	18008	31
30	28885	7·8702623	69677	30542	85660	35428	80203	20306	30
31	31839	05461	72407	33173	88199	37881	82575	22603	29
32	34790	08296	75136	35803	90736	40332	84946	24899	28
33	37739	11130	77863	38431	93272	42783	87316	27194	27
34	40687	13962	80589	41057	95807	45231	89685	29487	26
35	43632	16792	83312	43682	98340	47679	92052	31780	25
36	46575	19621	86034	46305	7·9200871	50125	94418	34071	24
37	49517	22447	88754	48927	03401	52569	96783	36361	23
38	52456	25272	91473	51547	05930	55012	99146	38649	22
39	55393	28095	94190	54166	08457	57454	7·9501508	40937	21
40	58329	30916	96905	56783	10983	59895	03869	43223	20
41	61262	33735	99618	59398	13507	62334	06229	45508	19
42	64193	36552	7·8902330	62012	16030	64772	08587	47792	18
43	67123	39367	05040	64624	18551	67208	10944	50075	17
44	70050	42181	07749	67235	21071	69643	13300	52356	16
45	72976	44993	10455	69844	23589	72077	15654	54637	15
46	75899	47803	13160	72451	26106	74509	18008	56916	14
47	78821	50611	15864	75057	28621	76940	20360	59194	13
48	81740	53417	18565	77662	31135	79369	22710	61470	12
49	84658	56222	21265	80265	33648	81798	25060	63746	11
50	87574	59025	23963	82866	36159	84224	27408	66020	10
51	90487	61826	26660	85466	38668	86650	29755	68293	9
52	93399	64625	29355	88064	41177	89074	32100	70565	8
53	96309	67422	32048	90660	43683	91497	34444	72836	7
54	99217	70218	34740	93256	46188	93918	36787	75106	6
55	7·8602123	73011	37430	95849	48692	96338	39129	77374	5
56	05027	75803	40118	98441	51195	98757	41470	79641	4
57	07929	78594	42804	7·9101031	53696	7·9401175	43809	81907	3
58	10829	81382	45489	03620	56195	03591	46147	84172	2
59	13727	84168	48173	06208	58693	06005	48484	86436	1
60	16623	86953	50854	08793	61190	08419	50819	88698	0
″	35′	34′	33′	32′	31′	30′	39′	28′	″

LOG. COSINE 89°.

″	24′	25′	26′	27′	28′	29′	30′	31′	″
0	7·8439444	7·8616738	7·8787077	7.8950988	7·9108938	7·9261344	7·9408584	7·9550996	60
1	42459	19632	89861	53668	7·9111522	3840	7·94 0996	3330	59
2	45472	22525	92642	56347	4105	6333	3407	5663	58
3	48483	25415	95422	59023	6686	8826	5817	7995	57
4	51492	28304	98199	61699	9266	7·9271317	8225	7·9560326	56
5	54498	31191	7·8800975	64372	7·9121844	3807	7·9420632	2655	55
6	57503	34076	03750	67044	4421	6295	3037	4984	54
7	60505	36958	06522	69714	6996	8782	5441	7310	53
8	63506	39839	09293	72383	9570	7·9281267	7844	9636	52
9	66504	42719	12062	75050	7·9132142	3751	7·9430246	7·9571961	51
10	69500	45596	14829	77715	4713	6233	2646	4284	50
11	72494	48471	17594	80379	7282	8714	5045	6606	49
12	75487	51344	20358	83041	9850	7·9291194	7442	8926	48
13	78477	54216	23120	85701	7·9142416	3672	9839	7·9581246	47
14	81465	57085	25880	88360	4980	6149	7·9442233	3564	46
15	84451	59953	28639	91017	7543	8625	4627	5881	45
16	87435	62819	31395	93673	7·9150105	7·9301099	7019	8197	44
17	90416	65683	34150	96327	2665	3571	9410	7·9590511	43
18	93396	68545	36903	98979	5224	6043	7·9451800	2825	42
19	96374	71405	39655	7·9001630	7781	8512	4188	5137	41
20	99350	74263	42404	04279	7·9160336	7·9310981	6575	7447	40
21	7·8502323	77120	45152	06926	2890	3448	8961	9757	39
22	05295	79974	47899	09572	5443	5913	7·9461345	7·9602065	38
23	08265	82827	50643	12216	7994	8378	3728	4373	37
24	11232	85677	53386	14859	7·9170543	7·9320840	6110	6678	36
25	14198	88526	56127	17500	3091	3302	8491	8983	35
26	17161	91373	58866	20139	5638	5762	7·9470870	7·9611287	34
27	20123	94218	61604	22777	8183	8220	3248	3589	33
28	23083	97062	64339	25413	7·9180727	7·9330678	5624	5890	32
29	26040	99903	67074	28048	3269	3133	8000	8190	31
30	28996	7·8702743	69806	30681	5809	5588	7·9480374	7·9620488	30
31	31949	05580	72537	33312	8348	8041	2746	2786	29
32	34900	08416	75266	35942	7·9190886	7·9340493	5118	5082	28
33	37850	11250	77993	38570	3422	2943	7488	7377	27
34	40797	14082	80718	41197	5957	5392	9856	9670	26
35	43743	16913	83442	43822	8490	7839	7·9492224	7·9631963	25
36	46686	19741	86164	46445	7·9201022	7·9350286	4590	4254	24
37	49628	22568	88885	49067	3552	2730	6955	6544	23
38	52567	25393	91603	51687	6081	5174	9319	8833	22
39	55505	28215	94320	54306	8608	7616	7·9501681	7·9641121	21
40	58440	31037	97036	56923	7·9211134	7·9360057	4042	3408	20
41	61374	33856	99749	59539	3658	2496	6402	5693	19
42	64305	36673	7·8902461	62153	6181	4934	8760	7977	18
43	67235	39489	05171	64765	8702	7370	7·9511118	7·9650260	17
44	70163	42303	07880	67376	7·9221222	9805	3474	2541	16
45	73088	45115	10587	69985	3741	7·9372239	5828	4822	15
46	76012	47925	13292	72593	6258	4672	8182	7101	14
47	78934	50733	15995	75199	8774	7103	7·9520534	9379	13
48	81853	53540	18697	77804	7·9231288	9533	2885	7·9661656	12
49	84771	56344	21397	80407	3800	7·9381961	5234	3932	11
50	87687	59147	24096	83008	6312	4388	7582	6206	10
51	90601	61949	26792	85608	8821	6814	9929	8480	9
52	93513	64748	29487	88207	7·9241330	9238	7·0532275	7·9670752	8
53	96423	67545	32181	90803	3836	7·9391661	4620	3023	7
54	99331	70341	34873	93399	6342	4083	6963	5293	6
55	7·8602237	73135	37563	95992	8846	6503	9305	7561	5
56	05141	75927	40251	98584	7·9251348	8922	7·9541646	9829	4
57	08043	78717	42938	7 9101175	3850	7·9401339	3985	7·9682095	3
58	10943	81506	45623	03764	6349	3756	6323	4360	2
59	13841	84293	48306	06352	8847	6170	8660	6624	1
60	16738	87077	50988	08938	7·9261344	8584	7·9550996	8886	0
′	35′	34′	33′	32′	31′	30′	29′	28′	′

LOG. COTAN. 89°.

LOG. SINE.

″	32′	33′	34′	35′	36′	37′	38′	39′	″
0	7·9688698	7·9822334	7·9951980	8·0077867	8·0200207	8·0319195	8·0435009	8·0547814	60
1	7·9690960	4527	4108	9934	2217	8·0321150	6913	9670	59
2	3220	6718	6235	8·0082001	4226	3105	8816	8·0551524	58
3	5479	8909	8361	4066	6234	5059	8·0440719	3378	57
4	7736	7·9831098	7·9960487	6131	8242	7012	2621	5231	56
5	9993	3287	2611	8194	8·0210248	8965	4522	7084	55
6	7·9702248	5474	4734	8·0090257	2253	8·0330916	6422	8935	54
7	4503	7660	6856	2318	4258	2866	8321	8·0560786	53
8	6756	9845	8977	4379	6261	4816	80·450220	2636	52
9	9008	7·9842029	7·9971097	6439	8264	6765	2117	4485	51
10	7·9711258	4212	3216	8497	8·0220266	8713	4014	6333	60
11	3508	6394	5334	8·0100555	2267	8·0340660	5910	8181	49
12	5756	8574	7451	2612	4267	2606	7805	8·0570028	48
13	8004	7·9850754	9566	4668	6266	4551	9700	1874	47
14	7·9720250	2933	7·9981681	6722	8264	6495	8·0461593	3719	46
15	2495	5110	3795	8776	8·0230261	8439	3486	5563	45
16	4738	7286	5908	8·0110829	2257	8·0350382	5378	7407	44
17	6981	9461	8020	2881	4252	2323	7269	9250	43
18	9222	7·9861636	7·9990130	4932	6247	4264	9159	8·0581092	42
19	7·9731463	3809	2240	6982	8240	6204	8·0471048	2933	41
20	3702	5981	4349	9031	8·0240233	8143	2937	4774	40
21	5940	8151	6456	8·0121079	2224	8·0360082	4825	6614	39
22	8177	7·9870321	8563	3126	4215	2019	6712	8453	38
23	7·9740412	2490	8·0000669	5172	6205	3956	8598	8·0590291	37
24	2647	4658	2773	7217	8194	5892	8·0480483	2129	36
25	4880	6824	4877	9261	8·0250182	7826	2368	3965	35
26	7113	8989	6979	8·0131304	2169	9760	4251	5801	34
27	9344	7·9881154	9081	3347	4155	8·0371693	6134	7636	33
28	7·9751574	3317	8·0011181	5388	6140	3626	8016	9470	32
29	3802	5479	3281	7428	8125	5557	9897	8·0601304	31
30	6030	7641	5379	9468	8·0260108	7488	8·0491778	3137	30
31	8257	9801	7477	8·0141506	2091	9417	3657	4969	29
32	7·9760482	7·9891960	9573	3543	4072	8·0381346	5536	6800	28
33	2706	4117	8·0021669	5580	6053	3274	7414	8630	27
34	4929	6274	3763	7615	8033	5201	9291	8·0610460	26
35	7151	8430	5856	9650	8·0270012	7128	8·0501167	2289	25
36	9372	7·9900585	7949	8·0151684	1990	9053	3043	4117	24
37	7·9771592	2738	8·0030040	3716	3967	8·0390978	4918	5944	23
38	3810	4891	2131	5748	5943	2901	6792	7771	22
39	6028	7043	4220	7779	7919	4824	8665	9597	21
40	8244	9193	6308	9808	9893	6746	8·0510537	8·0621422	20
41	7·9780459	7·9911342	8396	8·0161837	8·0281867	8667	2408	3246	19
42	2673	3491	8·0040482	3865	3839	8·0400588	4279	5070	18
43	4886	5638	2568	5892	5811	2507	6149	6892	17
44	7098	7784	4652	7918	7782	4426	8018	8714	16
45	9309	9929	6735	9943	9752	6343	9886	8·0630536	15
46	7·9791518	7·9922073	8818	8·0171967	8·0291721	8260	8·0521754	2356	14
47	3726	4216	8·0050899	3991	3689	8·0410176	3620	4176	13
48	5934	6358	2979	6013	5656	2092	5486	5995	12
49	8140	8499	5059	8034	7623	4006	7351	7813	11
50	7·9800345	7·9930639	7137	8·0180055	9588	5920	9216	9630	10
51	2549	2778	9215	2074	8·0301553	7832	8·0531079	8·0641447	9
52	4752	4915	8·0061291	4093	3517	9744	2942	3263	8
53	6953	7052	3366	6110	5479	8·0421655	4803	5078	7
54	9154	9188	5441	8127	7441	3565	6665	6893	6
55	7·9811353	7·9941322	7514	8·0190142	9403	5475	8525	8706	5
56	3552	3456	9587	2157	8·0311363	7383	8·0540384	8·0650519	4
57	5749	5588	8·0071658	4171	3322	9291	2243	2331	3
58	7945	7720	3729	6184	5280	8·0431198	4101	4143	2
59	7·9820140	9850	5798	8196	7238	3104	5958	5953	1
60	2334	7·9951980	7867	8·0200207	9195	5009	7814	7763	0
″	27′	26′	25′	24′	23′	22′	21′	20′	″

LOG. COSINE 89°.

″	32′	33′	34′	35′	36′	37′	38′	39′	″
0	7·9688886	7·9822534	7·9952192	8·0078092	8·0200445	8·0319446	8·0435274	8·0548094	60
1	7·9691148	4727	4320	8·0080159	2455	8·0321402	7179	9949	59
2	3408	6919	6448	2226	4465	3357	9062	8·0551804	58
3	5667	9110	8574	4292	6473	5311	8·0440985	3658	57
4	7925	7·9831299	7·9960700	6357	8481	7265	2887	5512	56
5	7·9700182	3488	2824	8420	8·0210487	9217	4788	7364	55
6	2438	5675	4947	8·0090483	2493	8·0331169	6689	9216	54
7	4692	7862	7070	2545	4498	3120	8588	8·0561067	53
8	6945	7·9840047	9191	4606	6501	5069	8·0450487	2917	52
9	9198	2231	7·9971311	6666	8504	7018	2385	4767	51
10	7.9711449	4414	3430	8725	8·0220506	8967	4282	6615	50
11	3698	6596	5548	8·0100783	2507	8·0340914	6178	8463	49
12	5947	8777	7666	2840	4507	2860	8074	8·0570310	48
13	8194	7·9850957	9782	4896	6507	4806	9968	2156	47
14	7.9720441	3135	7·9981897	6951	8505	6750	8·0461862	4002	46
15	2686	5313	4011	9005	8·0230502	8694	3755	5846	45
16	4930	7490	6124	8·0111058	2499	8·0350637	5647	7690	44
17	7173	9665	8236	3110	4494	2579	7538	9534	43
18	9414	7·9861839	7·9990346	5161	6489	4520	9429	8·0581376	42
19	7·9731655	4013	2456	7211	8483	6460	8·0471318	3217	41
20	3894	6185	4565	9260	8·0240475	8400	3207	5058	40
21	6132	8356	6673	8·0121308	2467	8·0360338	5095	6898	39
22	8369	7·9870526	8780	3356	4458	2276	6982	8737	38
23	7·9740605	2695	8·0000886	5402	6448	4213	8869	8·0590576	37
24	2840	4862	2991	7447	8437	6149	8·0480754	2414	36
25	5073	7029	5094	9492	8·0250426	8084	2639	4250	35
26	7306	9195	7197	8·0131535	2413	8·0370018	4523	6087	34
27	9537	7·9881359	9299	3578	4399	1951	6406	7922	33
28	7·9751767	3523	8·0011400	5619	6385	3884	8288	9756	32
29	3996	5685	3499	7660	8369	5815	8·0490169	8·0601590	31
30	6224	7847	5598	9699	8·0260353	7746	2050	3423	30
31	8451	7·9890007	7696	8·0141438	2336	9676	3930	5255	29
32	7·9760676	2166	9792	3775	4318	8·0381605	5809	7087	28
33	2901	4324	8·0021888	5812	6299	3533	7687	8918	27
34	5124	6481	3983	7848	8279	5461	9564	8·0610748	26
35	7346	8637	6076	9883	8·0270258	7387	8·0501441	2577	25
36	9567	7·9900792	8169	8·0151916	2236	9313	3317	4405	24
37	7·9771787	2946	8·0030260	3949	4213	8·0391238	5192	6233	23
38	4006	5099	2351	5981	6190	3162	7066	8060	22
39	6224	7251	4441	8012	8166	5085	8939	9886	21
40	8440	9401	6529	8·0160042	8·0280140	7007	8·0510812	8·0621711	20
41	7·9780655	7·9911551	8617	2071	2114	8928	2683	3536	19
42	2870	3699	8·0040703	4099	4087	8·0400849	4554	5359	18
43	5083	5847	2789	6127	6059	2768	6424	7182	17
44	7295	7993	4874	8153	8030	4687	8294	9005	16
45	9506	7·9920138	6957	8·0170178	8·0290000	6605	8·0520162	8·0630826	15
46	7·9791715	2283	9040	2203	1969	8522	2030	2647	14
47	3924	4426	8·0051121	4226	3938	8·0410439	3897	4467	13
48	6131	6568	3202	6248	5905	2354	5763	6286	12
49	8338	8709	5282	8270	7872	4269	7628	8104	11
50	7·9800543	7·9930849	7360	8·0180291	9838	6183	9493	9922	10
51	2747	2988	9438	2310	8·0301802	8096	8·0531356	8·0641739	9
52	4950	5126	8·0061514	4329	3766	8·0420008	3219	3555	8
53	7152	7263	3590	6347	5729	1919	5081	5371	7
54	9353	9399	5665	8364	7692	3829	6943	7185	6
55	7·9811552	7·9941534	7738	8·0190379	9653	5739	8803	8999	5
56	3751	3667	9811	2394	8·0311613	7648	8·0640663	8·0650812	4
57	5948	5800	8·0071883	4408	3573	9555	2522	2625	3
58	8145	7932	3953	6422	5531	8·0431462	4380	4436	2
59	7·9820340	7·9950062	6023	8434	7489	3369	6237	6247	1
60	2534	2192	8092	8·0200445	9446	5274	8094	8057	0
″	27′	26′	25′	24′	23′	22′	21′	20′	″

LOG. COTAN. 89°.

″	40′	41′	42′	43′	44′	45′	46′	47′	″
0	8·0657763	8·0764997	8·0869646	8·0971832	8·1071669	8·1169262	1·1264710	8·1358104	60
1	9572	6762	8·0871369	3515	3314	8·1170870	6283	9644	59
2	8·0661381	8526	3091	5198	4958	2478	7856	8·1361183	58
3	3188	8·0770290	4813	6879	6601	4085	9428	2722	57
4	4995	2052	6534	8560	8244	5691	8·1270999	4260	56
5	6801	3815	8254	8·0980240	9886	7297	2570	5797	55
6	8606	5576	9974	1920	8·1081528	8902	4140	7334	54
7	8·0670411	7337	8·0881692	3599	3169	8·1180507	5710	8871	53
8	2215	9097	3411	5277	4809	2111	7279	8·1370407	52
9	4018	8·0780856	5128	6955	6449	3714	8848	1942	51
10	5820	2614	6845	8632	8088	5317	8·1280416	3477	50
11	7622	4372	8561	8·0990309	9726	6919	1983	5011	49
12	9423	6129	8·0890277	1984	8·1091364	8520	3550	6545	48
13	8·0681223	7886	1991	3659	3001	8·1190121	5117	8078	47
14	3022	9641	3706	5334	4638	1722	6682	9610	46
15	4821	8·0791396	5419	7008	6274	3322	8248	8·1381143	45
16	6619	3151	7132	8681	7909	4921	9812	2674	44
17	8416	4904	8844	8·1000353	9544	6519	8·1291376	4205	43
18	8·0690212	6657	8·0900555	2025	8·1101178	8118	2940	5736	42
19	2008	8409	2266	3697	2812	9715	4503	7265	41
20	3803	8·0800161	3976	5367	4445	8·1201312	6065	8795	40
21	5597	1912	5685	7037	6077	2908	7627	8·1390324	39
22	7390	3662	7394	8706	7709	4504	9188	1852	38
23	9183	5411	9102	8·1010375	9340	6099	8·1300749	3380	37
24	8·0700975	7160	8·0910810	2043	8·1110970	7693	2309	4907	36
25	2766	8908	2516	3710	2600	9287	3869	6434	35
26	4557	8·0810655	4222	5377	4229	8·1210881	5428	7960	34
27	6346	2401	5928	7043	5858	2474	6986	9485	33
28	8135	4147	7632	8709	7486	4066	8544	8·1401011	32
29	9923	5892	9336	8·1020374	9113	5657	8·1310101	2535	31
30	8·0711711	7637	8·0921040	2038	8·1120740	7248	1658	4059	30
31	3498	9380	2743	3701	2366	8839	3215	5583	29
32	5284	8·0821123	4445	5364	3992	8·1220429	4770	7105	28
33	7069	2866	6146	7027	5617	2018	6325	8628	27
34	8854	4607	7847	8688	7241	3607	7880	8·1410150	26
35	8·0720637	6348	9547	8·1030349	8865	5195	9434	1671	25
36	2421	8088	8·0931246	2010	8·1130488	6782	8·1320987	3192	24
37	4203	9828	2945	3669	2110	8369	2540	4712	23
38	5985	8·0831567	4643	5328	3732	9956	4093	6232	22
39	7765	3305	6340	6987	5354	8·1231541	5644	7751	21
40	9546	5042	8037	8645	6974	3127	7196	9270	20
41	8·0731325	6779	9733	8·1040302	8595	4711	8746	8·1420788	19
42	3104	8515	8·0941428	1959	8·1140214	6295	8·1330296	2306	18
43	4882	8·0840251	3123	3615	1833	7879	1846	3823	17
44	6659	1985	4817	5270	3451	9462	3395	5339	16
45	8436	3719	6510	6925	5069	8·1241044	4943	6855	15
46	8·0740211	5452	8203	8579	6686	2626	6491	8371	14
47	1986	7185	9895	8·1050232	8302	4207	8039	9886	13
48	3761	8917	8·0951587	1885	9918	5787	9586	8·1431400	12
49	5534	8·0850648	3277	3537	8·1151534	7367	8·1341132	2914	11
50	7307	2379	4968	5188	3148	8947	2678	4427	10
51	9080	4109	6657	6839	4762	8·1250526	4223	5940	9
52	8·0750851	5838	8346	8490	6376	2104	5767	7453	8
53	2622	7566	8·0960034	8·1060139	7989	3682	7311	8964	7
54	4392	9294	1721	1788	9601	5259	8855	8·1440476	6
55	6161	8·0861021	3408	3437	8·1161213	6836	8·1350398	1987	5
56	7930	2747	5094	5085	2824	8412	1940	3497	4
57	9698	4473	6780	6732	4434	9987	3482	5006	3
58	8·0761465	6198	8465	8378	6044	8·1261562	5023	6516	2
59	3231	7922	8·0970149	8·1070024	7654	3136	6564	8024	1
60	4997	9646	1832	1669	9262	4710	8104	9532	0
″	19′	18′	17′	16′	15′	14′	13′	12′	″

LOG. COSINE 89°.

″	40′	41′	42′	43′	44′	45′	46′	47′	″
0	8·0658057	8·0765306	8·0869970	8·0972172	8·1072025	8·1169634	8·1265099	8·1358510	60
1	9866	7071	8·0871693	3855	3670	8·1171243	6672	8·1360050	59
2	8·0661675	8835	3416	5538	5314	2851	8245	1590	58
3	3483	8·0770599	5138	7220	6958	4458	9817	3129	57
4	5290	2362	6859	8901	8601	6064	8·1271389	4667	56
5	7096	4125	8579	8·0980582	8·1080243	7670	2960	6205	55
6	8902	5886	8·0880299	2261	1885	9276	4531	7742	54
7	8·0670707	7647	2018	3941	3526	8·1180881	6101	9279	53
8	2511	9407	3737	5619	5167	2485	7670	8·1370815	52
9	4314	8·0781167	5455	7297	6807	4088	9239	2350	51
10	6117	2926	7172	8975	8446	5691	8·1280807	3886	50
11	7919	4684	8888	8·0990651	8·1090085	7294	2375	5420	49
12	9720	6441	8·0890604	2327	1723	8896	3942	6954	48
13	8·0681520	8198	2319	4003	3361	8·1190497	5509	8488	47
14	3320	9954	4033	5677	4998	2098	7075	8·1380020	46
15	5118	8·0791709	5747	7351	6634	3698	8641	1553	45
16	6917	3464	7460	9025	8269	5297	8·1290206	3085	44
17	8714	5218	9172	8·1000698	9904	6896	1770	4616	43
18	8·0690511	6971	8·0900884	2374	8·1101539	8495	3334	6147	42
19	2306	8723	2595	4041	3173	8·1200092	4897	7677	41
20	4102	8·0800475	4305	5712	4806	1689	6460	9207	40
21	5896	2226	6015	7382	6436	3286	8022	8·1390736	39
22	7690	3976	7724	9052	8070	4882	9583	2264	38
23	9483	5726	9432	8·1010721	9702	6477	8·1301144	3792	37
24	8·0701275	7475	8·0911140	2389	8·1111332	8072	2705	5320	36
25	3066	9223	2847	4057	2962	9666	4265	6847	35
26	4857	8·0810970	4553	5724	4592	8·1211260	5824	8373	34
27	6647	2717	6259	7390	6221	2853	7383	9899	33
28	8436	4463	7964	9056	7849	4446	8941	8·1401425	32
29	8·0710225	6208	9668	8·1020721	9477	6037	8·1310498	2949	31
30	2012	7953	8·0921372	2386	8·1121104	7629	2056	4474	30
31	3799	9697	3075	4049	2730	9219	3612	5997	29
32	5586	8·0821440	4777	5713	4356	8·1220810	5168	7521	28
33	7371	3183	6479	7375	5981	2399	6723	9043	27
34	9156	4925	8180	9037	7606	3988	8278	8·1410566	26
35	8·0720940	6666	9880	8·1030698	9230	5577	9833	2087	25
36	2723	8406	8·0931579	2359	8·1130653	7164	8·1321386	3608	24
37	4506	8·0830146	3278	4019	2476	8752	2940	5129	23
38	6288	1885	4977	5678	4098	8·1230338	4492	6649	22
39	8069	3624	6674	7337	5720	1924	6044	8168	21
40	9850	5361	8371	8995	7341	3510	7596	9687	20
41	8·0731629	7098	8·0940068	8·1040653	8961	5095	9147	8·1421206	19
42	3408	8835	1763	2309	8·1140581	6679	8·1330697	2724	18
43	5186	8·0840570	3458	3966	2200	8263	2247	4241	17
44	6964	2305	5153	5621	3819	9846	3796	5758	16
45	8741	4039	6846	7276	5437	8·1241429	5345	7274	15
46	8·0740517	5773	8539	8931	7054	3011	6893	8790	14
47	2292	7506	8·0950232	8·1050584	8671	4592	8441	8·1430305	13
48	4067	9238	1923	2237	8·1150287	6173	9988	1820	12
49	5841	8·0850969	3614	3890	1903	7753	8·1341535	3334	11
50	7614	2700	5305	5542	3518	9333	3081	4848	10
51	9386	4430	6994	7193	5132	8·1250912	4626	6361	9
52	8·0751158	6160	8683	8843	6746	2491	6171	7874	8
53	2929	7888	8·0960372	8·1060493	8359	4069	7715	9386	7
54	4699	9616	2060	2142	9972	5646	9259	8·1440897	6
55	6469	8·0861344	3747	3791	8·1161584	7223	8·1350802	2408	5
56	8238	3070	5433	5439	3195	8799	2345	3919	4
57	9·0760006	4796	7119	7087	4806	8·1260375	3887	5429	3
58	1773	6522	8804	8733	6416	1950	5429	6938	2
59	3540	8246	8·0970488	8·1070380	8025	3525	6970	8447	1
60	5306	9970	2172	2025	9634	5099	8510	9956	0
″	19′	18′	17′	16′	15′	14′	13′	12′	″

LOG. COTAN. 89°.

″	48′	49′	50′	51′	52′	53′	54′	55′	″
0	8·1449532	8·1539075	8·1626808	8·1712804	8·1797129	8·1879848	8·1961020	8·2040703	60
1	8·1451040	8·1540552	8255	4223	8521	8·1881213	2360	2019	59
2	2547	2028	9702	5641	9912	2578	3700	3334	58
3	4054	3504	8·1631149	7059	8·1801303	3943	5039	4649	57
4	5560	4979	2594	8477	2693	5307	6378	5963	56
5	7065	6454	4040	9894	4083	6670	7717	7277	55
6	8570	7928	5485	8·1721310	5472	8034	9055	8591	54
7	8·1460075	9402	6929	2726	6861	9397	8·1970392	9905	53
8	1579	8·1550876	8373	4142	8250	8·1890759	1729	8·2051218	52
9	3082	2348	9817	5557	9638	2121	3066	2530	51
10	4585	3821	8·1641259	6972	8·1811025	3482	4403	3842	50
11	6087	5293	2702	8386	2413	4843	5739	5154	49
12	7589	6764	4144	9800	3799	6204	7074	6465	48
13	9091	8235	5586	8·1731214	5186	7564	8409	7776	47
14	8·1470591	9705	7027	2627	6571	8924	9744	9087	46
15	2092	8·1561175	8467	4039	7957	8·1900284	8·1981078	8·2060397	45
16	3592	2644	9907	5451	9342	1643	2412	1707	44
17	5091	4113	8·1651347	6863	8·1820726	3001	3746	3016	43
18	6590	5582	2786	8274	2111	4359	5079	4325	42
19	8088	7049	4225	9684	3494	5717	6412	5634	41
20	9586	8517	5663	8·1741094	4877	7074	7744	6942	40
21	8·1481083	9984	7101	2504	6260	8431	9076	8250	39
22	2579	8·1571450	8538	3913	7643	9788	9·1990407	9557	38
23	4076	2916	9975	5322	9024	8·1911144	1738	8·2070864	37
24	5571	4381	8·1661411	6731	8·1830406	2499	3069	2171	36
25	7066	5846	2847	8138	1787	3854	4399	3477	35
26	8561	7310	4282	9546	3167	5209	5729	4783	34
27	8·1490055	8774	5717	8·1750953	4548	6563	7058	6088	33
28	1549	8·1580238	7151	2359	5927	7917	8387	7393	32
29	3042	1701	8585	3765	7307	9271	9716	8698	31
30	4534	3163	8·1670019	5171	8685	8·1920624	8·2001044	6·2080002	30
31	6027	4625	1452	6576	8·1840064	1976	2372	1306	29
32	7518	6086	2884	7981	1442	3329	3699	2610	28
33	9009	7547	4316	9385	2819	4680	5026	3913	27
34	8·1500500	9008	5748	8·1760789	4196	6032	6353	5216	26
35	1990	8·1590468	7179	2192	5573	7383	7679	6518	25
36	3479	1927	8610	3595	6949	8733	9005	7820	24
37	4968	3386	8·1680040	4998	8325	8·1930083	8·2010330	9121	23
38	6457	4845	1469	6400	9700	1433	1655	8·2090422	22
39	7945	6303	2899	7801	8·1851075	2782	2980	1723	21
40	9432	7760	4327	9202	2450	4131	4304	3024	20
41	8·1510919	9217	5756	8·1770603	3824	5479	5628	4324	19
42	2406	8·1600674	7183	2003	5197	6827	6951	5623	18
43	3891	2130	8611	3403	6570	8175	8274	6922	17
44	5377	3585	8·1690038	4802	7943	9522	9597	8221	16
45	6862	5040	1464	6201	9315	8·1940869	8·2020919	9520	15
46	8346	6495	2890	7599	8·1860687	2215	2241	8·2100818	14
47	9830	7949	4315	8997	2059	3561	3562	2115	13
48	8·1521314	9403	5740	8·1780394	3430	4907	4883	3412	12
49	2796	8·1610856	7165	1791	4800	6252	6203	4709	11
50	4279	2308	8589	3188	6170	7596	7523	6006	10
51	5761	3761	8·1700012	4584	7540	8941	8843	7302	9
52	7242	5212	1435	5980	8909	8·1950284	8·2030163	8598	8
53	8723	6663	2858	7375	8·1870278	1628	1481	9893	7
54	8·1530203	8114	4280	8770	1646	2971	2800	8·2111188	6
55	1683	9564	5702	8·1790164	3014	4313	4118	2482	5
56	3163	8·1621014	7123	1558	4382	5656	5436	3777	4
57	4641	2463	8544	2951	5749	6997	6753	5070	3
58	6120	3912	9964	4344	7116	8339	8070	6364	2
59	7598	5360	8·1711384	5737	8482	9680	9387	7657	1
60	9075	6808	2804	7129	9848	8·1961020	8·2040703	8949	0
″	11′	10′	9′	8′	7′	6′	5′	4′	″

LOG. COSINE.

″	48′	49′	50′	51′	52′	53′	54′	55′	″
0	8·1449956	8·1539516	8·1627267	8·1713282	8·1797626	8·1880364	8·1961556	8·2041259	60
1	8·1451464	8·1540993	8715	4701	9018	1730	2896	2575	59
2	2971	2470	8·1630162	6120	8·1800409	3095	4236	3890	58
3	4478	3946	1609	7538	1800	4460	5576	5206	57
4	5984	5422	3055	8956	3191	5824	6915	6521	56
5	7490	6897	4501	8·1720373	4581	7188	8254	7835	55
6	8995	8371	5946	1790	5971	8552	9592	9149	54
7	8·1460500	9846	7391	3207	7360	9915	8·1970930	8·2950463	53
8	2004	8·1551319	8835	4623	8749	8·1891278	2268	1776	52
9	3508	2792	8·1640279	6038	8·1810137	2640	3605	3089	51
10	5011	4265	1722	7453	1525	4002	4942	4401	50
11	6514	5737	3165	8868	2913	5363	6278	5714	49
12	8016	7209	4607	8·1730282	4300	6724	7614	7025	48
13	9518	8680	6049	1696	5687	8085	8949	8337	47
14	8·1471019	8·1560151	7490	3109	7073	9445	8·1980284	9647	46
15	2520	1621	8931	4522	8459	8·1900805	1619	8·2060958	45
16	4020	3090	8·1650372	5934	9844	2164	2953	2268	44
17	5519	4559	1812	7346	8·1821229	3523	4287	3578	43
18	7018	6028	3251	8757	2613	4881	5621	4887	42
19	8517	7496	4690	8·1740168	3997	6239	6954	6196	41
20	8·1480015	8964	6128	1579	5381	7597	8286	7505	40
21	1512	8·1570431	7566	2989	6764	8954	9619	8813	39
22	3009	1898	9004	4398	8146	8·1910311	8·1990950	8·2070120	38
23	4506	3364	8·1660441	5807	9529	1667	2282	1428	37
24	6002	4830	1878	7216	8·1830910	3023	3613	2735	36
25	7497	6295	3314	8624	2292	4379	4943	4041	35
26	8992	7759	4749	8·1750032	3673	5734	6273	5348	34
27	8·1490487	9224	6185	1439	5053	7088	7603	6653	33
28	1980	8·1580687	7619	2846	6433	8442	8933	7959	32
29	3474	2151	9054	4252	7813	9796	8·2000262	9264	31
30	4967	3613	8·1670487	5658	9192	8·1921150	1590	8·2080568	30
31	6459	5076	1921	7064	8·1840571	2503	2918	1873	29
32	7951	6537	3353	8469	1949	3855	4246	3176	28
33	9442	7999	4786	9873	3327	5207	5573	4480	27
34	8·1500933	9459	6218	8·1761278	4704	6559	6900	5783	26
35	2423	8·1590920	7649	2681	6081	7910	8227	7086	25
36	3913	2379	9080	4084	7458	9261	9553	8388	24
37	5402	3839	8·1680510	5487	8834	8·1930611	8·2010879	9690	23
38	6891	5297	1940	6889	8·1850209	1961	2204	8·2090991	22
39	8380	6756	3370	8291	1585	3311	3529	2292	21
40	9867	8213	4799	9693	2959	4660	4853	3593	20
41	8·1511355	9671	6228	8·1771094	4334	6009	6177	4893	19
42	2841	8·1601128	7656	2494	5708	7357	7501	6193	18
43	4328	2584	9083	3894	7081	8705	8824	7493	17
44	5813	4040	8·1690510	5294	8454	8·1940053	8·2020147	8792	16
45	7299	5495	1937	6693	9827	1400	1470	8·2100091	15
46	8783	6950	3363	8091	8·1861199	2746	2792	1389	14
47	8·1520267	8404	4789	9490	2571	4093	4113	2687	13
48	1751	9858	6214	8·1780887	3942	5439	5435	3985	12
49	3234	8·1611312	7639	2285	5313	6784	6756	5282	11
50	4717	2765	9064	3682	6683	8129	8076	6579	10
51	6199	4217	8·1700487	5078	8053	9473	9396	7875	9
52	7681	5669	1911	6474	9423	8·1950818	8·2030716	9171	8
53	9162	7121	3334	7870	8·1870792	2161	2035	8·2110467	7
54	8·1530643	8572	4756	9265	2161	3505	3354	1762	6
55	2123	8·1620022	6178	8·1790659	3529	4848	4672	3057	5
56	3603	1472	7600	2054	4897	6190	5990	4351	4
57	5082	2922	9021	3447	6264	7532	7308	5646	3
58	6560	4371	8·1710442	4841	7631	8874	8625	6939	2
59	8038	5819	1862	6233	8998	8·1960215	9942	8233	1
60	9516	7267	3282	7626	8·1880364	1556	8·2041259	9526	0
″	11′	10′	9′	8′	7′	6′	5′	4′	″

LOG. COTAN. 89°.

″	56′	57′	58′	59′	0′	1′	2′	3′	″
0	8·2118949	8·2195811	8·2271335	8·2345568	8·2418553	8·2490332	8·2560943	8·2630424	60
1	8·2120242	7080	2583	6795	9759	1518	2120	1572	59
2	1533	8349	3830	8021	8·2420965	2704	3277	2721	58
3	2825	9618	5077	9247	2170	3890	4443	3869	57
4	4116	8·2200887	6324	8·2350472	3376	5075	5609	5016	56
5	5407	2155	7570	1697	4580	6260	6775	6164	55
6	6697	3423	8816	2922	5785	7445	7941	7311	54
7	7987	4690	8·2280061	4147	6989	8629	9106	8458	53
8	9277	5957	1306	5371	8192	9813	8·2570271	9604	52
9	8·2130566	7223	2551	6594	9396	8·2500997	1436	8·2640750	51
10	1854	8490	3796	7818	8·2430599	2180	2600	1896	50
11	3143	9756	5040	9041	1802	3363	3764	3042	49
12	4431	8·2211021	6284	8·2360264	3004	4546	4928	4187	48
13	5719	2286	7527	1486	4206	5728	6091	5332	47
14	7006	3551	8770	2708	5408	6911	7255	6477	46
15	8293	4815	8·2290013	3930	6609	8092	8417	7621	45
16	9579	6079	1255	5151	7810	9274	9580	8766	44
17	8·2140865	7343	2497	6372	9011	8·2510455	8·2580742	9909	43
18	2151	8606	3739	7593	8·2440212	1636	1904	8·2651053	42
19	3436	9869	4980	8813	1412	2816	3065	2196	41
20	4721	8·2221132	6221	8·2370033	2611	3996	4227	3339	40
21	6006	2394	7461	1253	3811	5176	5388	4482	39
22	7290	3636	8701	2472	5010	6356	6548	5624	38
23	8574	4917	9941	3691	6209	7535	7709	6766	37
24	9857	6178	8·2301181	4910	7407	8714	8869	7908	36
25	8·2151140	7439	2420	6128	8605	9893	8·2590028	9049	35
26	2423	8699	3659	7346	9803	8·2521071	1188	8·2660190	34
27	3705	9959	4897	8563	8·2451000	2249	2347	1331	33
28	4987	8·2231219	6135	9781	2198	3426	3505	2471	32
29	6269	2478	7373	8·2380997	3394	4604	4664	3612	31
30	7550	3737	8610	2214	4591	5781	5822	4751	30
31	8831	4996	9847	3430	5787	6957	6980	5891	29
32	8·2160111	6254	8·2311084	4646	6983	8134	8137	7030	28
33	1391	7512	2320	5862	8178	9310	9295	8169	27
34	2671	8769	3556	7077	9373	8·2530485	8·2600452	9308	26
35	3950	8·2240026	4792	8292	8·2460568	1661	1608	8·2670446	25
36	5229	1283	6027	9506	1762	2836	2764	1585	24
37	6508	2539	7262	8·2390720	2957	4011	3920	2722	23
38	7786	3795	8496	1934	4150	5185	5076	3860	22
39	9064	5051	9731	3148	5344	6359	6232	4997	21
40	8·2170341	6306	8·2320965	4361	6537	7533	7387	6134	20
41	1618	7561	2198	5574	7730	8706	8541	7271	19
42	2895	8815	3431	6786	8922	9880	9696	8407	18
43	4171	8·2250070	4664	7998	8·2470115	8·2541052	8·2610850	9543	17
44	5447	1323	5896	9210	1306	2225	2004	8·2680679	16
45	6723	2577	7128	8·2400422	2498	3397	3157	1814	15
46	7998	3830	8360	1633	3689	4569	4311	2949	14
47	9273	5083	9592	2844	4880	5741	5463	4084	13
48	8·2180547	6335	8·2330823	4054	6071	6912	6616	5219	12
49	1821	7587	2053	5264	7261	8083	7768	6353	11
50	3095	8839	3284	6474	8451	9254	8920	7487	10
51	4368	8·2260090	4514	7683	9640	8·2550424	8·2620072	8620	9
52	5641	1341	5743	8892	8·2480829	1594	1223	9754	8
53	6913	2591	6973	8·2410101	2018	2764	2375	8·2690887	7
54	8186	3841	8202	1310	3207	3933	3525	2020	6
55	9457	5091	9430	2518	4395	5102	4676	3152	5
56	8·2190729	6341	8·2340659	3725	5583	6271	5826	4284	4
57	2000	7590	1886	4933	6771	7439	6976	5416	3
58	3270	8839	3114	6140	7958	8607	8125	6548	2
59	4541	8·2270087	4341	7347	9145	9775	9275	7679	1
60	5811	1335	5568	8553	8·2490332	8·2560943	8·2630424	8810	0
″	3′	2′	1′	0′	59′	58′	57′	56′	″

LOG. COSINE 89°. LOG. COSINE 88°.

″	56′	57′	58′	59′	0′	1′	2′	3′	″
0	8·2119526	8·2196408	8·2271953	8·2346208	8·2419215	8·2491015	8·2561649	8·2631153	60
1	8·2120818	7678	3201	7435	8·2420421	2202	2817	2302	59
2	2110	8947	4449	8661	1627	3388	3984	3451	58
3	3402	8·2200216	5696	9887	2833	4574	5151	4599	57
4	4694	1485	6943	8·2351113	4038	5760	6317	5747	56
5	5985	2754	8190	2339	5244	6946	7484	6895	55
6	7275	4022	9436	3564	6448	8131	8650	8043	54
7	8566	5289	8·2280682	4789	7653	9315	9815	9190	53
8	9855	6557	1927	6013	8857	8·2500500	8·2570981	8·2640337	52
9	8·2131145	7824	3173	7237	8·2430061	1684	2146	1483	51
10	2434	9090	4417	8461	1264	2868	3310	2630	50
11	3723	8·2210356	5662	9684	2467	4051	4475	3776	49
12	5011	1622	6906	8·2360908	3670	5234	5639	4921	48
13	6299	2888	8150	2130	4872	6417	6803	6067	47
14	7587	4153	9393	3353	6075	7600	7966	7212	46
15	8874	5418	8·2290636	4575	7276	8782	9129	8357	45
16	8·2140161	6682	1879	5796	8478	9964	8·2580292	9501	44
17	1447	7946	3121	7018	9679	8·2511145	1455	8·2650645	43
18	2733	9210	4363	8239	8·2440880	2326	2617	1789	42
19	4019	8·2220473	5605	9460	2080	3507	3779	2933	41
20	5304	1736	6846	8·2370680	3280	4688	4941	4076	40
21	6589	2998	8087	1900	4480	5868	6102	5219	39
22	7874	4260	9327	3120	5680	7048	7263	6362	38
23	9158	5522	8·2300568	4339	6879	8227	8424	7504	37
24	8·2150442	6784	1807	5558	8077	9407	9584	8646	36
25	1725	8045	3047	6776	9276	8·2520586	8·2590744	9788	35
26	3008	9305	4286	7995	8·2450474	1764	1904	8·2660929	34
27	4291	8·2230566	5525	9213	1672	2943	3063	2071	33
28	5573	1826	6763	8·2380430	2869	4121	4223	3212	32
29	6855	3085	8001	1648	4066	5298	5381	4352	31
30	8137	4345	9239	2865	5263	6476	6540	5492	30
31	9418	5604	8·2310476	4081	6460	7653	7698	6632	29
32	8·2160699	6862	1713	5297	7656	8829	8856	7772	28
33	1979	8120	2950	6513	8852	8·2530006	8·2600014	8911	27
34	3259	9378	4186	7729	8·2460047	1182	1171	8·2670051	26
35	4539	8·2240635	5422	8944	1242	2358	2328	1189	25
36	5818	1892	6658	8·2390159	2437	3533	3485	2328	24
37	7097	3149	7893	1373	3632	4708	4641	3466	23
38	8375	4405	9128	2588	4826	5883	5797	4604	22
39	9653	5661	8·2320363	3802	6020	7058	6953	5742	21
40	8·2170931	6917	1597	5015	7213	8232	8108	6879	20
41	2209	8172	2831	6228	8407	9406	9263	8016	19
42	3486	9427	4064	7441	9599	8·2540579	8·2610418	9153	18
43	4762	8·2250682	5297	8654	8·2470792	1752	1573	8·2680289	17
44	6038	1936	6530	9866	1984	2925	2727	1425	16
45	7314	3190	7763	8·2401078	3176	4098	3881	2561	15
46	8590	4443	8995	2289	4368	5270	5034	3696	14
47	9865	5696	8·2330227	3500	5559	6442	6189	4832	13
48	8·2181140	6949	1458	4711	6750	7614	7341	5967	12
49	2414	8201	2689	5922	7940	8785	8493	7101	11
50	3688	9453	3920	7132	9131	9956	9646	8236	10
51	4962	8·2260705	5150	8342	8·2480321	8·2551127	8·2620798	9370	9
52	6235	1956	6380	9551	1510	2297	1950	8·2690503	8
53	7508	3207	7610	8·2410760	2699	3467	3101	1637	7
54	8780	4457	8839	1969	3888	4637	4252	2770	6
55	8·2190053	5708	8·2340068	3177	5077	5806	5403	3903	5
56	1324	6957	1297	4386	6265	6976	6554	5035	4
57	2596	8207	2525	5593	7453	8144	7704	6168	3
58	3867	9456	3753	6801	8641	9313	8854	7300	2
59	5137	8·2270705	4980	8008	9828	8·2560481	8·2630004	8431	1
60	6408	1953	6208	9215	8·2491015	1649	1153	9563	0
″	3′	2′	1′	0′	59′	58′	57′	56′	″

LOG. COTAN. 89°. LOG. COTAN. 88°.

″	4′	5′	6′	7′	8′	9′	10′	11′	″
0	8·2698810	8·2766136	8·2832434	8·2897734	8·2962067	8·3025460	8·3087941	8·3149536	60
1	9941	7249	3530	8814	3131	6509	8975	8·3150555	59
2	8·2701071	8362	4626	9894	4195	7558	8·3090009	1574	58
3	2201	9475	5722	8·2900974	5259	8606	1042	2593	57
4	3331	8·2770587	6818	2053	6322	9654	2075	3611	56
5	4461	1700	7913	3132	7385	8·3030702	3108	4630	55
6	5590	2811	9008	4211	8448	1749	4140	5648	54
7	6719	3923	8·2840103	5289	9511	2796	5173	6665	53
8	7847	5034	1197	6367	8·2970573	3843	6205	7683	52
9	8976	6145	2292	7445	1635	4890	7237	8700	51
10	8·2710104	7256	3386	8523	2697	5937	8268	9717	50
11	1232	8367	4479	9600	3759	6983	9299	8·3160734	49
12	2359	9477	5573	8·2910677	4820	8029	8·3100330	1751	48
13	3486	8·2780587	6666	1754	5881	9075	1361	2767	47
14	4613	1696	7759	2831	6942	8·3040120	2392	3783	46
15	5740	2806	8851	3907	8002	1165	3422	4799	45
16	6866	3915	9943	4983	9063	2210	4452	5815	44
17	7992	5023	8·2851035	6059	8·2980123	3255	5482	6830	43
18	9118	6132	2127	7134	1183	4299	6512	7845	42
19	8·2720243	7240	3219	8210	2242	5344	7541	8860	41
20	1368	8348	4310	9285	3301	6388	8570	9875	40
21	2493	9456	5401	8·2920359	4360	7431	9599	8·3170889	39
22	3618	8·2790563	6491	1434	5419	8475	8·3110628	1903	38
23	4742	1670	7582	2508	6477	9518	1656	2917	37
24	5866	2777	8672	3582	7536	8·3050561	2684	3931	36
25	6990	3883	9762	4656	8594	1604	3712	4945	35
26	8113	4989	8·2860851	5729	9651	2646	4740	5958	34
27	9236	6095	1941	6802	8·2990709	3688	5767	6971	33
28	8·2730359	7201	3030	7875	1766	4730	6794	7984	32
29	1481	8306	4118	8948	2823	5772	7821	8996	31
30	2604	9411	5207	8·2930020	3879	6813	8848	8·3180008	30
31	3725	8·2800516	6295	1092	4936	7855	9874	1021	29
32	4847	1621	7383	2164	5992	8896	8·3120901	2032	28
33	5968	2725	8471	3235	7048	9936	1927	3044	27
34	7089	3829	9558	4306	8104	8·3060977	2952	4055	26
35	8210	4933	8·2870645	5378	9159	2017	3978	5067	25
36	9331	6036	1732	6448	8·3000214	3057	5003	6077	24
37	8·2740451	7139	2818	7519	1269	4097	6028	7088	23
38	1571	8242	3905	8589	2324	5136	7053	8098	22
39	2690	9345	4991	9659	3378	6175	8077	9109	21
40	3810	8·2810447	6076	8·2940729	4432	7214	9101	8·3190119	20
41	4929	1549	7162	1798	5486	8253	8·3130125	1128	19
42	6048	2650	8247	2867	6539	9291	1149	2138	18
43	7166	3752	9332	3936	7593	8·3070330	2173	3147	17
44	8284	4853	8·2880417	5005	8646	1368	3196	4156	16
45	9402	5954	1501	6073	9699	2405	4219	5165	15
46	·2750520	7055	2585	7141	8·3010751	3443	5242	6173	14
47	1637	8155	3669	8209	1804	4480	6264	7182	13
48	2754	9255	4752	9277	2856	5517	7287	8190	12
49	3871	8·2820355	5836	8·2950344	3907	6554	8309	9198	11
50	4987	1454	6919	1411	4959	7590	9331	8·3200205	10
51	6103	2553	8002	2478	6010	8626	8·3140352	1213	9
52	7219	3652	9084	3544	7061	9662	1374	2220	8
53	8335	4751	8·2890166	4611	8112	8·3080698	2395	3227	7
54	9450	5849	1248	5677	9163	1734	3416	4233	6
55	8·2760565	6947	2330	6742	8·3020213	2769	4436	5240	5
56	1680	8045	3411	7808	1263	3804	5457	6246	4
57	2794	9143	4492	8873	2313	4839	6477	7252	3
58	3909	8·2830240	5573	9938	3362	5873	7497	8258	2
59	5022	1337	6654	8·2961003	4411	6907	8516	9263	1
60	6136	2434	7734	2067	5460	7941	9536	8·3210269	0
″	55′	54′	53′	52′	51′	50′	49′	48′	″

LOG. COSINE 88°.

LOG. TAN. 1°.

″	4′	5′	6′	7′	8′	9′	10′	11′	″
0	8·2699563	8·2766912	8·2833234	8·2898559	8·2962917	8·3026335	8·3088842	8·3150462	60
1	8·2700694	8026	4331	9640	3981	7385	9876	1482	59
2	1825	9139	5428	8·2900720	5046	8433	8·3090910	2501	58
3	2955	8·2770253	6524	1800	6110	9482	1944	3520	57
4	4065	1365	7620	2879	7174	8·3030531	2977	4539	56
5	5215	2478	8716	3959	8237	1579	4010	5558	55
6	6345	3590	9811	5038	9300	2627	5043	6576	54
7	7474	4702	8·2840906	6117	8·2970363	3674	6076	7595	53
8	8603	5814	2001	7195	1426	4722	7109	8613	52
9	9732	6925	3096	8274	2489	5769	8141	9630	51
10	8·2710860	8036	4190	9352	3551	6816	9173	8·3160648	50
11	1989	9147	5284	8·2910430	4613	7862	8·3100205	1665	49
12	3116	8·2780258	6378	1507	5675	8909	1236	2682	48
13	4244	1368	7471	2584	6736	9955	2267	3699	47
14	5371	2478	8565	3661	7797	8·3041001	3298	4715	46
15	6498	3588	9658	4738	8858	2046	4329	5732	45
16	7625	4697	8·2850750	5815	9919	3092	5360	6748	44
17	8751	5806	1843	6891	8·2980980	4137	6390	7764	43
18	9877	6915	2935	7967	2040	5182	7420	8779	42
19	8·2721003	8024	4027	9042	3100	6226	8450	9795	41
20	2129	9132	5118	8·2920118	4159	7271	9479	8·3170810	40
21	3254	8·2790240	6210	1193	5219	8315	8·3110508	1825	39
22	4379	1348	7301	2268	6278	9359	1538	2839	38
23	5504	2455	8392	3342	7337	8·3050403	2566	3854	37
24	6628	3563	9482	4417	8395	1446	3595	4868	36
25	7752	4670	8·2860572	5491	9454	2489	4623	5882	35
26	8876	5776	1662	6565	8·2990512	3532	5651	6895	34
27	9999	6882	2752	7638	1570	4575	6679	7909	33
28	8·2731122	7988	3841	8711	2627	5617	7707	8922	32
29	2245	9094	4931	9784	3685	6659	8734	9935	31
30	3368	8·2800200	6019	8·2930857	4742	7701	9761	8·3180948	30
31	4490	1305	7108	1930	5799	8743	8·3120788	1960	29
32	5612	2410	8196	3002	6855	9784	1815	2973	28
33	6734	3515	9284	4074	7911	8·3060825	2841	3985	27
34	7856	4619	8·2870372	5145	8967	1866	3867	4997	26
35	8977	5723	1460	6217	8·3000023	2907	4893	6008	25
36	8·2740098	6827	2547	7288	1079	3947	5919	7019	24
37	1218	7930	3634	8359	2134	4987	6944	8031	23
38	2338	9034	4720	9429	3189	6027	7969	9041	22
39	3458	8·2810136	5807	8·2940500	4244	7067	8994	8·3190052	21
40	4578	1239	6893	1570	5298	8106	8·3130019	1062	20
41	5698	2342	7979	2640	6353	9145	1043	2073	19
42	6817	3444	9065	3709	7407	8·3070184	2068	3083	18
43	7936	4545	8·2880150	4779	8460	1223	3092	4092	17
44	9054	5647	1235	5848	9514	2261	4115	5102	16
45	8·2750173	6748	2320	6916	8·3010567	3299	5139	6111	15
46	1291	7849	3404	7985	1620	4337	6162	7120	14
47	2408	8950	4488	9053	2673	5375	7185	8129	13
48	3526	8·2820051	5572	8·2950121	3725	6412	8208	9137	12
49	4643	1151	6656	1189	4778	7449	9230	8·3200145	11
50	5760	2251	7740	2256	5830	8486	8·3140253	1154	10
51	6876	3350	8823	3324	6881	9523	1275	2161	9
52	7992	4450	9906	4391	7933	8·3080559	2296	3169	8
53	9108	5549	8·2890988	5457	8984	1596	3318	4176	7
54	8·2760224	6647	2071	6524	8·3020035	2631	4339	5183	6
55	1340	7746	3156	7590	1086	3667	5360	6190	5
56	2455	8844	4235	8656	2136	4703	6381	7197	4
57	3570	9942	5316	9721	3186	5738	7402	8203	3
58	4684	8·2831040	6397	8·2960787	4236	6773	8422	9210	2
59	5798	2137	7478	1852	5236	7807	9442	8·3210215	1
60	6912	3234	8559	2917	6335	8842	8·3150462	1221	0
″	55′	54′	53′	52′	51′	50′	49′	48′	″

LOG. COTAN. 88°.

LOG. SINE. 1°.

″	12′	13′	14′	15′	16′	17′	18′	19′	″
0	8·3210269	8·3270163	8·3329243	8·3387529	8·3445043	8·3501805	8·3557835	8·3613150	60
1	1274	1155	8·3330221	8494	5995	2745	8762	4066	59
2	2278	2146	1199	9459	6947	3685	9690	4982	58
3	3283	3137	2176	8·3390423	7899	4624	8·3560617	5897	57
4	4287	4127	3153	1387	8851	5563	1544	6813	56
5	5292	5118	4130	2351	9802	6502	2471	7728	55
6	6295	6108	5107	3315	8·3450753	7441	3398	8643	54
7	7299	7098	6084	4279	1704	8379	4324	9558	53
8	8303	8087	7060	5242	2655	9318	5251	8·3620472	52
9	9306	9077	8036	6205	3605	8·3510256	6177	1387	51
10	8·3220309	8·3280066	9012	7168	4555	1194	7103	2301	50
11	1311	1055	9988	8131	5505	2132	8029	3215	49
12	2314	2044	8·3340963	9093	6455	3069	8954	4129	48
13	3316	3032	1938	8·3400055	7405	4006	9880	5042	47
14	4318	4021	2913	1018	8354	4944	8·3570805	5956	46
15	5320	5009	3888	1979	9304	5881	1730	6869	45
16	6322	5997	4863	2941	8·3460253	6817	2654	7782	44
17	7323	6984	5837	3902	1201	7754	3579	8695	43
18	8324	7972	6811	4864	2150	8690	4503	9608	42
19	9325	8959	7785	5825	3098	9626	5427	8·3630520	41
20	8·3230326	9946	8759	6785	4047	8·3520562	6351	1433	40
21	1326	8·3290933	9732	7746	4995	1498	7275	2345	39
22	2326	1919	8·3350706	8706	5942	2433	8199	3257	38
23	3326	2906	1679	9666	6890	3369	9122	4169	37
24	4326	3892	2651	8·3410626	7837	4304	8·3580045	5080	36
25	5325	4878	3624	1586	8784	5239	0968	5991	35
26	6325	5863	4597	2546	9731	6173	1891	6903	34
27	7324	6849	5569	3505	8·3470678	7108	2814	7814	33
28	8322	7834	6541	4464	1625	8042	3736	8724	32
29	9321	8819	7512	5423	2571	8976	4658	9635	31
30	8·3240319	9804	8484	6382	3517	9910	5580	8·3640545	30
31	1317	8·3300788	9455	7340	4463	8·3530844	6502	1456	29
32	2315	1773	8·3360426	8298	5409	1778	7424	2366	28
33	3313	2757	1397	9256	6354	2711	8345	3275	27
34	4310	3740	2368	8·3420214	7300	3644	9266	4185	26
35	5308	4724	3338	1172	8245	4577	8·3590187	5095	25
36	6305	5708	4309	2129	9189	5510	1108	6004	24
37	7301	6691	5279	3086	8·3480134	6442	2029	6913	23
38	8298	7674	6248	4043	1079	7374	2949	7822	22
39	9294	8656	7218	5000	2023	8306	3870	8730	21
40	8·3250290	9639	8187	5957	2967	9238	4790	9639	20
41	1286	8·3310621	9156	6913	3911	8·3540170	5709	8·3650547	19
42	2282	1603	8·3370125	7869	4854	1102	6629	1455	18
43	3277	2585	1094	8825	5798	2033	7549	2363	17
44	4272	3567	2063	9781	6741	2964	8468	3271	16
45	5267	4548	3031	8·3430736	7684	3895	9387	4179	15
46	6262	5529	3999	1691	8627	4826	8·3600306	5086	14
47	7256	6510	4967	2646	9570	5756	1225	5993	13
48	8250	7491	5934	3601	8·3490512	6686	2143	6900	12
49	9244	8472	6902	4556	1454	7617	3061	7807	11
50	8·3260238	9452	7869	5510	2396	8546	3979	8713	10
51	1232	8·3320432	8836	6465	3338	9476	4897	9620	9
52	2225	1412	9803	7419	4280	8·3550406	5815	8·3660526	8
53	3218	2392	8·3380769	8372	5221	1335	6733	1432	7
54	4211	3371	1736	9326	6162	2264	7650	2338	6
55	5204	4350	2702	8·3440279	7103	3193	8567	3244	5
56	6196	5329	3668	1233	8044	4122	9484	4149	4
57	7188	6308	4633	2186	8985	5050	8·3610401	5054	3
58	8180	7287	5599	3138	9925	5979	1317	5959	2
59	9172	8265	6564	4091	8·3500865	6907	2234	6864	1
60	8·3270163	9243	7529	5043	1805	7835	3150	7769	0
″	47′	46′	45′	44′	43′	42′	41′	40′	″

LOG. COSINE. 88°.

″	12′	13′	14′	15′	16′	17′	18′	19′	″
0	8·3211221	8·3271143	8·3330249	8·3388563	8·3446105	8·3502895	8·3558953	8·3614297	60
1	2227	2134	1228	9528	7075	3835	9881	5213	59
2	3232	3126	2206	8·3390493	8010	4775	8·3560809	6129	58
3	4237	4117	3184	1458	8962	5715	1737	7045	57
4	5242	5108	4161	2423	9914	6655	2664	7961	56
5	6246	6099	5139	3387	8·3450866	7594	3592	8877	55
6	7251	7090	6116	4351	1817	8533	4519	9793	54
7	8255	8080	7093	5316	2769	9472	5446	8·3620708	53
8	9259	9070	8070	6279	3720	8·3510411	6373	1623	52
9	8·3220262	8·3280060	9046	7243	4671	1350	7299	2538	51
10	1266	1050	8·3340023	8206	5621	2288	8226	3453	50
11	2269	2039	0999	9169	6572	3226	9152	4367	49
12	3272	3028	1975	8·3400132	7522	4164	8·3570078	5281	48
13	4274	4017	2950	1095	8472	5102	1004	6196	47
14	5277	5006	3926	2058	9422	6040	1929	7110	46
15	6279	5995	4901	3020	8·3460372	6977	2855	8023	45
16	7281	6983	5876	3982	1321	7914	3780	8937	44
17	8283	7971	6851	4944	2271	8851	4705	9850	43
18	9285	8959	7826	5906	3220	9788	5630	8·5630763	42
19	8·3230286	9947	8800	6867	4169	8·3520725	6555	1676	41
20	1287	8·3290934	9774	7828	5117	1661	7479	2589	40
21	2288	1921	8·3350748	8789	6066	2597	8403	3502	39
22	3288	2908	1722	9750	7014	3533	9327	4414	38
23	4289	3895	2695	8·3410711	7962	4469	8·8580251	5327	37
24	5289	4882	3669	1671	8910	5405	1175	6239	36
25	6289	5868	4642	2631	9857	6340	2098	7150	35
26	7289	6854	5615	3591	8·3470805	7275	3022	8062	34
27	8288	7840	6587	4551	1752	8210	3945	8974	33
28	9287	8826	7560	5511	2699	9145	4868	9885	32
29	8·3240286	9811	8532	6470	3646	8·3530080	5790	8·3640796	31
30	1285	8·3300796	9504	7429	4592	1014	6713	1707	30
31	2284	1781	8·3360476	8388	5539	1948	7635	2617	29
32	3282	2766	1447	9347	6485	2882	8557	3528	28
33	4280	3751	2419	8·3420305	7431	3816	9479	4438	27
34	5278	4735	3390	1263	8377	4750	8·3590401	5348	26
35	6276	5719	4361	2221	9322	5683	1322	6258	25
36	7273	6703	5331	3179	8·3480268	6616	2243	7168	24
37	8270	7687	6302	4137	1213	7549	3165	8078	23
38	9267	8670	7272	5094	2158	8482	4086	8987	22
39	8·3250264	9653	8242	6052	3103	9414	5006	9896	21
40	1260	8·3310636	9212	7009	4047	8·3540347	5927	8·3650805	20
41	2257	1619	8·3370181	7965	4991	1279	6847	1714	19
42	3253	2601	1151	8922	5936	2211	7767	2623	18
43	4249	3584	2120	9876	6879	3143	8687	3531	17
44	5244	4566	3089	8·3430835	7823	4074	9607	4439	16
45	6240	5548	4058	1791	8767	5006	8.3600527	5347	15
46	7235	6529	5026	2746	9710	5937	1446	6255	14
47	8230	7511	5994	3702	8·3490653	6868	2365	7163	13
48	9224	8492	6963	4657	1596	7799	3284	8070	12
49	8·3260219	9473	7930	5612	2539	8729	4203	8978	11
50	1213	8·3320454	8898	6567	3481	9660	5121	9885	10
51	2207	1434	9866	7522	4423	8·3550590	6040	8·3660792	9
52	3201	2415	8·3380833	8476	5365	1520	6958	1698	8
53	4194	3395	1800	9431	6307	2450	7876	2605	7
54	5188	4375	2767	8·3440385	7249	3379	8794	3511	6
55	6181	5354	3733	1339	8191	4309	9711	4417	5
56	7173	6334	4700	2292	9132	5238	8·3610629	5323	4
57	8166	7313	5666	3246	8·3500073	6167	1546	6229	3
58	9158	8292	6632	4199	1014	7096	2463	7135	2
59	8·3270151	9271	7597	5152	1954	8024	3380	8040	1
60	1143	8·3330249	8563	6105	2895	8953	4297	8945	0
″	47′	46′	45′	44′	43′	42′	41′	40′	″

LOG. COTAN. 88°.

″	20′	21′	22′	23′	24′	25′	26′	27′	″
0	8·3667769	8·3721710	8·3774988	8·3827620	8·3879622	8·3931008	8·3981793	8·4031990	60
1	8674	2603	5870	8492	8·3880483	1859	2634	2822	59
2	9578	3496	6753	9364	1345	2710	3475	3653	58
3	8·3670482	4389	7635	8·3830235	2206	3561	4316	4485	57
4	1386	5282	8517	1106	3067	4412	5157	5316	56
5	2290	6174	9398	1978	3927	5263	5998	6147	55
6	3193	7067	8·3780280	2848	4788	6113	6839	6978	54
7	4097	7959	1161	3719	5648	6964	7679	7809	53
8	5000	8851	2042	4590	6509	7814	8519	8639	52
9	5903	6743	2924	5460	7369	8664	9359	9470	51
10	6806	8·3730635	3804	6330	8229	9513	8·3990199	8·4040300	50
11	7708	1526	4685	7201	9088	8·3940363	1039	1130	49
12	8611	2418	5566	8070	9948	1213	1879	1960	48
13	9513	3309	6446	8940	8·3890807	2062	2718	2790	47
14	8·3680415	4200	7326	9810	1666	2911	3557	3620	46
15	1317	5091	8206	8·3840679	2526	3760	4397	4449	45
16	2219	5981	9086	1548	3384	4609	5236	5279	44
17	3120	6872	9965	2417	4243	5457	6074	6108	43
18	4022	7762	8·3790845	3286	5102	6306	6913	6937	42
19	4923	8652	1724	4155	5960	7154	7751	7766	41
20	5824	9542	2603	5023	6818	8002	8590	8594	40
21	6725	8·3740431	3482	5892	7676	8850	9428	9423	39
22	7625	1321	4361	6760	8534	9698	8·4000266	8·4050251	38
23	8526	2210	5239	7628	9392	8·3950546	1104	1080	37
24	9426	3099	6117	8496	8·3900249	1393	1941	1908	36
25	8·3690326	3988	6996	9363	1107	2240	2779	2736	35
26	1226	4877	7874	8·3850231	1964	3088	3616	3563	34
27	2125	5766	8751	1098	2821	3935	4453	4391	33
28	3025	6654	9629	1965	3678	4781	5290	5218	32
29	3924	7542	8·3800507	2832	4534	5628	6127	6046	31
30	4823	8430	1384	3699	5391	6475	6964	6873	30
31	5722	9318	2261	4565	6247	7321	7801	7700	29
32	6621	8·3750206	3138	5432	7103	8167	8637	8527	28
33	7519	1094	4015	6298	7959	9013	9473	9353	27
34	8418	1981	4891	7164	8815	9859	8·4010309	8·4060180	26
35	9316	2868	5768	8030	9671	8·3960705	1145	1006	25
36	8·3700214	3755	6644	8896	8·3910526	1550	1981	1832	24
37	1111	4642	7520	9761	1382	2395	2816	2658	23
38	2009	5528	8396	8·3860627	2237	3241	3652	3484	22
39	2907	6415	9271	1492	3092	4086	4487	4310	21
40	3804	7301	8·3810147	2357	3947	4930	5322	5135	20
41	4701	8187	1022	3222	4801	5775	6157	5961	19
42	5598	9073	1897	4087	5656	6620	6992	6786	18
43	6494	9959	2772	4951	6510	7464	7826	7611	17
44	7391	8·3760844	3647	5816	7364	8308	8661	8436	16
45	8287	1729	4522	6680	8218	9152	9495	9261	15
46	9183	2615	5396	7544	9072	9996	8·4020329	8·4070085	14
47	8·3710079	3500	6271	8408	9926	8·3970840	1163	0910	13
48	0975	4384	7145	9271	8·3920779	1683	1997	1734	12
49	1870	5269	8019	8·3870135	1633	2527	2831	2558	11
50	2766	6153	8892	0998	2486	3370	3664	3382	10
51	3661	7038	9766	1861	3339	4213	4497	4206	9
52	4556	7922	8·3820639	2724	4191	5056	5331	5030	8
53	5451	8806	1513	3587	5044	5898	6164	5853	7
54	6346	9689	2386	4450	5897	6741	6996	6677	6
55	7240	8·3770573	3258	5312	6749	7583	7829	7500	5
56	8134	1456	4131	6174	7601	8425	8662	8323	4
57	9028	2339	5004	7037	8453	9268	9494	9146	3
58	9922	3222	5876	7898	9305	8·3980109	8·4030326	9969	2
59	8·3720816	4105	6748	8760	8·3930156	0951	1158	8·4080791	1
60	1710	4988	7620	9622	1008	1793	1990	1614	0
	39′	38′	37′	36′	35′	34′	33′	32′	″

LOG. COSINE 88°.

LOG. TAN. 1°.

″	20′	21′	22′	23′	24′	25′	26′	27′	″
0	8·3663945	8·3722915	8·3776223	8·3828886	8·3880918	8·3932336	8·3983152	8·4033381	60
1	9850	3809	7106	9758	1780	3187	3994	4213	59
2	8·3670755	4703	7989	8·3830631	2642	4039	4835	5045	58
3	1660	5596	8872	1503	3504	4891	5677	5877	57
4	2564	6489	9754	2374	4365	5742	6519	6709	56
5	3468	7383	8·3780636	3246	5227	6593	7360	7541	55
6	4372	8275	1519	4117	6088	7444	8201	8372	54
7	5276	9168	2400	4989	6948	8295	9042	9203	53
8	6180	8·3730061	3282	5860	7809	9145	9883	8·4040035	52
9	7083	0953	4164	6731	8670	9996	8·3990723	0866	51
10	7987	1845	5045	7601	9530	8·3940846	1564	1696	50
11	8890	2737	5926	8472	8·3890391	1696	2404	2527	49
12	9793	3629	6807	9342	1251	2546	3244	3358	48
13	8·3680696	4521	7688	8·3840213	2111	3396	4084	4188	47
14	1598	5412	8569	1083	2970	4246	4924	5018	46
15	2501	6304	9449	1953	3830	5095	5764	5848	45
16	3403	7195	8·3790329	2822	4689	5945	6603	6678	44
17	4305	8086	1209	3692	5548	6794	7442	7508	43
18	5207	8976	2089	4561	6408	7643	8282	8337	42
19	6108	9867	2969	5430	7266	8492	9121	9167	41
20	7010	8·3740757	3849	6299	8125	9340	9959	9996	40
21	7911	1647	4728	7168	8984	8·3950189	8·4000798	8·4050825	39
22	8812	2538	5607	8037	9842	1037	1637	1654	38
23	9713	3427	6486	8905	8·3900700	1885	2475	2483	37
24	8·3690614	4317	7365	9774	1558	2733	3313	3311	36
25	1514	5206	8244	8·3850642	2416	3581	4151	4140	35
26	2414	6096	9122	1510	3274	4429	4989	4968	34
27	3315	6985	8·3800001	2378	4131	5276	5827	5796	33
28	4215	7874	0879	3245	4989	6124	6664	6624	32
29	5114	8762	1757	4113	5846	6971	7502	7452	31
30	6014	9651	2634	4980	6703	7818	8339	8280	30
31	6913	8·3750539	3512	5847	7560	8665	9176	9107	29
32	7812	1428	4390	6714	8417	9511	8·4010013	9935	28
33	8711	2316	5267	7581	9273	8·3960358	0850	8·4060762	27
34	9610	3203	6144	8448	8·3910129	1204	1686	1589	26
35	8·3700509	4091	7021	9314	0986	2050	2523	2416	25
36	1407	4979	7898	8·3860180	1842	2897	3359	3242	24
37	2306	5866	8774	1046	2697	3742	4195	4069	23
38	3204	6753	9650	1912	3553	4588	5031	4895	22
39	4102	7640	8·3810527	2778	4409	5434	5867	5722	21
40	4999	8527	1403	3643	5264	6279	6702	6548	20
41	5897	9413	2278	4509	6119	7124	7538	7374	19
42	6794	8·3760299	3154	5374	6974	7969	8373	8199	18
43	7692	1186	4030	6239	7829	8814	9208	9025	17
44	8589	2072	4905	7104	8684	9659	8·4020043	9850	16
45	9485	2958	5780	7969	9538	8·3970503	0878	8·4070676	15
46	8·3710382	3843	6655	8833	8·3920393	1348	1713	1501	14
47	1278	4729	7530	9698	1247	2192	2547	2326	13
48	2175	5614	8404	8·3870562	2101	3036	3381	3151	12
49	3071	6499	9279	1426	2955	3880	4216	3975	11
50	3967	7384	8·3820153	2290	3808	4724	5050	4800	10
51	4862	8269	1027	3153	4662	5567	5884	5624	9
52	5758	9153	1901	4017	5515	6411	6717	6449	8
53	6653	8·3770038	2775	4880	6368	7254	7551	7273	7
54	7548	0922	3648	5743	7221	8097	8384	8097	6
55	8443	1806	4522	6606	8074	8940	9217	8920	5
56	9338	2690	5395	7469	8927	9782	8·4030050	9744	4
57	8·3720232	3574	6268	8332	9779	8·3980625	0883	8·4080567	3
58	1127	4457	7141	9194	8·3930631	1467	1716	1391	2
59	2021	5340	8014	8·3880056	1484	2310	2549	2214	1
60	2915	6223	8886	0918	2336	3152	3381	3037	0
″	39′	38′	37′	36′	35′	34′	33′	32′	″

LOG. COTAN. 88°.

″	28′	29′	30′	31′	32′	33′	34′	35′	″
0	8·4081614	8·4130676	8·4179190	8·4227168	8·4274621	8·4321561	8·4367999	8·4413944	60
1	2436	1489	9994	7963	5408	2339	8768	4706	59
2	3258	2302	8·4180798	8758	6194	3117	9538	5468	58
3	4080	3115	1602	9553	6980	3895	6·4370307	6229	57
4	4902	3927	2405	8·4230348	7766	4672	1077	6990	56
5	5723	4740	3209	1142	8552	5450	1846	7751	55
6	6545	5552	4012	1937	9338	6227	2615	8512	54
7	7366	6364	4815	2731	8·4280124	7004	3384	9273	53
8	8187	7176	5618	3525	0909	7781	4153	8·4420034	52
9	9008	7988	6421	4319	1694	8558	4921	0795	51
10	9829	8800	7223	5113	2480	9335	5690	1555	50
11	8·4090650	9611	8026	5907	3265	8·4330112	6458	2315	49
12	1471	8·4140422	8828	6700	4050	0888	7227	3076	48
13	2291	1234	9630	7494	4835	1665	7995	3836	47
14	3111	2045	8·4190432	8287	5619	2441	8763	4596	46
15	3931	2856	1234	9080	6404	3217	9531	5855	45
16	4751	3666	2036	9873	7188	3993	8·4380298	6115	44
17	5571	4477	2838	8·4240666	7972	4769	1066	6875	43
18	6391	5287	3639	1458	8756	5544	1833	7634	42
19	7210	6098	4441	2251	9540	6220	2601	8393	41
20	8029	6908	5242	3043	8·4290324	7095	3368	9152	40
21	8849	7718	6043	3836	1108	7871	4135	9911	39
22	9666	8528	6844	4628	1891	8646	4902	8·4430670	38
23	8·4100486	9337	7644	5420	2675	9421	5669	1429	37
24	1305	8·4150147	8445	6211	3458	8·4340196	6435	2187	36
25	2124	0956	9245	7003	4241	0970	7202	2946	35
26	2942	1765	8·4200046	7795	5024	1745	7968	3704	34
27	3760	2575	0846	8586	5807	2519	8734	4462	33
28	4578	3383	1646	9377	6590	3294	9501	5221	32
29	5396	4192	2440	8·4250168	7372	4068	8·4390266	5978	31
30	6214	5001	3245	0959	8154	4842	1032	6736	30
31	7032	5809	4045	1750	8937	5616	1798	7494	29
32	7849	6618	4844	2541	9719	6389	2564	8251	28
33	8667	7426	5644	3331	8·4300501	7163	3329	9009	27
34	9484	8234	6443	4122	1283	7937	4094	9766	26
35	8·4110301	9042	7242	4912	2064	8710	4859	8·4440523	25
36	1118	9850	8040	5702	2846	9483	5624	1280	24
37	1934	8·4160657	8839	6492	3627	8·4350256	6389	2037	23
38	2751	1465	9638	7282	4409	1029	7154	2794	22
39	3567	2272	8·4210436	8071	5190	1802	7919	3551	21
40	4383	3079	1234	8861	5971	2574	8683	4307	20
41	5200	3886	2032	9650	6751	3347	9447	5063	19
42	6015	4693	2830	8·4260439	7532	4119	8·4400212	5820	18
43	6831	5499	3628	1229	8313	4892	0976	6576	17
44	7647	6306	4426	2018	9093	5664	1740	7332	16
45	8462	7112	5223	2806	9873	6436	2503	8087	15
46	9278	7919	6020	3595	8·4310654	7207	3267	8843	14
47	8·4120093	8725	6818	4383	1434	7979	4031	9599	13
48	0908	9531	7615	5172	2213	8751	4794	8·4450354	12
49	1723	8·4170336	8412	5960	2993	9522	5557	1109	11
50	2537	1142	9208	6748	3773	8·4360293	6321	1865	10
51	3352	1948	8·4220005	7536	4552	1064	7083	2620	9
52	4166	2753	0801	8324	5332	1835	7846	3375	8
53	4981	3558	1598	9111	6111	2606	8609	4129	7
54	5795	4363	2394	9899	6890	3377	9372	4884	6
55	6609	5168	3190	8·4270686	7669	4148	8·4410134	5638	5
56	7422	5973	3986	1474	8447	4918	0896	6393	4
57	8236	6777	4782	2261	9226	5688	1659	7147	3
58	9050	7582	5577	3048	8·4320004	6459	2421	7901	2
59	9863	8386	6373	3834	0783	7229	3183	8655	1
60	8·4130676	9190	7168	4621	1561	7999	3944	9409	0
″	31′	30′	29′	28′	27′	26′	25′	24′	″

LOG. COSINE 88°.

″	28′	29′	30′	31′	32′	33′	34′	35′	″
0	8·4083037	8·4132132	8·4180679	8·4228690	8·4276176	8·4323150	8·4369622	8·4415603	60
1	3859	2945	1483	9485	6963	3929	8·4370393	6365	59
2	4682	3759	2288	8·4230281	7750	4707	1163	7127	58
3	5505	4572	3092	1076	8537	5486	1933	7889	57
4	6327	5385	3896	1872	9324	6264	2703	8651	56
5	7149	6198	4700	2667	8·4280110	7042	3473	9413	55
6	7971	7011	5504	3462	0897	7820	4242	8·4420174	54
7	8793	7823	6307	4257	1683	8598	5012	0936	53
8	9615	8636	7111	5051	2469	9375	5781	1697	52
9	8·4090436	9448	7914	5846	3255	8·4330153	6550	2458	51
10	1258	8·4140261	8717	6640	4041	0930	7320	3219	50
11	2079	1073	9520	7434	4826	1707	8089	3980	49
12	2900	1885	8·4190323	8229	5612	2484	8857	4741	48
13	3721	2696	1126	9023	6397	3261	9626	5502	47
14	4542	3508	1929	9816	7182	4038	8·4380395	6262	46
15	5362	4319	2731	8·4240610	7968	4815	1163	7023	45
16	6183	5131	3533	1404	8752	5591	1931	7783	44
17	7003	5942	4336	2197	9537	6368	2700	8543	43
18	7823	6753	5138	2990	8·4290322	7144	3468	9303	42
19	8643	7564	5940	3783	1106	7920	4235	8·4430063	41
20	9463	8374	6741	4576	1891	8696	5003	0822	40
21	8·4100283	9185	7543	5369	2675	9472	5771	1582	39
22	1103	9995	8344	6162	3459	8·4340248	6538	2341	38
23	1922	8·4150805	9146	6954	4243	1023	7306	3101	37
24	2741	1616	9947	7747	5027	1799	8073	3860	36
25	3560	2425	8·4200748	8539	5811	2574	8840	4619	35
26	4379	3235	1549	9331	6594	3349	9607	5378	34
27	5198	4045	2349	8·4250123	7377	4124	8·4390374	6137	33
28	6017	4854	3150	0915	8161	4899	1140	6895	32
29	6835	5664	3950	1706	8944	5674	1907	7654	31
30	7653	6473	4750	2498	9727	6448	2673	8412	30
31	8472	7282	5550	3289	8·4300510	7223	3440	9171	29
32	9290	8091	6350	4080	1292	7997	4206	9929	28
33	8·4110107	8900	7150	4872	2075	8771	4972	8·4440687	27
34	0925	9708	7950	5662	2857	9545	5738	1444	26
35	1743	8·4160517	8749	6453	3639	8·4350319	6503	2202	25
36	2560	1325	9549	7244	4422	1093	7269	2960	24
37	3377	2133	8·4210348	8034	5204	1867	8034	3717	23
38	4194	2941	1147	8825	5985	2640	8800	4475	22
39	5011	3749	1946	9615	6767	3413	9565	5232	21
40	5828	4556	2745	8·4260405	7549	4187	8·4400330	5989	20
41	6645	5364	3543	1195	8330	4960	1095	6746	19
42	7461	6171	4342	1985	9111	5733	1860	7503	18
43	8278	6979	5140	2774	9892	6506	2624	8259	17
44	9094	7786	5938	3564	8·4310673	7278	3389	9016	16
45	9910	8593	6736	4353	1454	8051	4153	9772	15
46	8·4120726	9399	7534	5142	2235	8823	4918	8·4450529	14
47	1541	8·4170206	8332	5932	3016	9595	5682	1285	13
48	2357	1012	9130	6720	3796	8·4360367	6446	2041	12
49	3172	1819	9927	7509	4576	1139	7209	2797	11
50	3988	2625	8·4220725	8298	5356	1911	7973	3552	10
51	4803	3431	1522	9086	6136	2683	8737	4308	9
52	5618	4237	2319	9875	6916	3455	9500	5063	8
53	6432	5043	3116	8·4270663	7696	4226	8·4410263	5819	7
54	7247	5848	3912	1451	8476	4997	1027	6574	6
55	8062	6654	4709	2239	9255	5768	1790	7329	5
56	8876	7459	5505	3027	8·4320034	6540	2553	8084	4
57	9690	8264	6302	3814	0814	7310	3315	8839	3
58	8·4130504	9069	7098	4602	1593	8081	4078	9594	2
59	1318	9874	7894	5389	2372	8852	4841	8·4460348	1
60	2132	8·4180679	8690	6176	3150	9622	5603	1103	0
″	31′	30′	29′	28′	27′	26′	25′	24′	″

LOG. COTAN. 88°.

″	36′	37′	38′	39′	40′	41′	42′	43′	″
0	8·4459409	8·4504402	8·4548934	8·4593013	8·4636649	8·4679850	8·4722626	8·4764984	60
1	8·4460163	5148	9672	3744	7372	8·4680567	3335	5686	59
2	0916	5894	8·4550410	4474	8096	1283	4044	6388	58
3	1670	6640	1148	5205	8819	1999	4753	7091	57
4	2423	7385	1886	5936	9542	2715	5462	7793	56
5	3176	8131	2624	6666	8·4640265	3431	6171	8495	55
6	3929	8876	3362	7396	0988	4147	6880	9197	54
7	4682	9621	4099	8126	1711	4862	7589	9899	53
8	5435	8·4510366	4837	8856	2434	5578	8297	8·4770600	52
9	6188	1111	5574	9586	3156	6293	9006	1302	51
10	6940	1856	6311	8·4600316	3879	7009	9714	2003	50
11	7693	2601	7048	1046	4601	7724	8·4730422	2705	49
12	8445	3345	7785	1775	5323	8439	1130	3406	48
13	9197	4090	8522	2505	6046	9154	1838	4107	47
14	9949	4834	9259	3234	6768	9869	2546	4808	46
15	8·4470701	5578	9996	3963	7489	8·4690584	3254	5509	45
16	1453	6322	8·4560732	4692	8211	1298	3962	6210	44
17	2205	7066	1468	5421	8933	2013	4669	6910	43
18	2956	7810	2205	6150	9654	2727	5377	7611	42
19	3707	8553	2941	6878	8·4650376	3441	6084	8311	41
20	4459	9297	3677	7607	1097	4156	6791	9012	40
21	5210	8·4520040	4412	8335	1818	4870	7498	9712	39
22	5961	0784	5148	9064	2539	5583	8205	8·4780412	38
23	6712	1527	5884	9792	3260	6297	8912	1112	37
24	7462	2270	6619	8·4610520	3981	7011	9618	1812	36
25	8213	3013	7354	1248	4702	7725	8·4740325	2511	35
26	8963	3755	8090	1975	5422	8438	1032	3211	34
27	9714	4498	8825	2703	6143	9151	1738	3911	33
28	8·4480464	5240	9560	3431	6863	9865	2444	4610	32
29	1214	5983	8·4570295	4158	7583	8·4700578	3150	5309	31
30	1964	6725	1029	4886	8303	1291	3856	6009	30
31	2714	7467	1764	5613	9023	2003	4562	6706	29
32	3463	8209	2498	6340	9743	2716	5268	7407	28
33	4213	8951	3233	7067	8·4660463	3429	5974	8105	27
34	4962	9693	3967	7794	1182	4141	6679	8804	26
35	5712	8·4530434	4701	8520	1902	4854	7385	9503	25
36	6461	1176	5435	9247	2621	5566	8090	8·4790201	24
37	7210	1917	6169	9973	3340	6278	8795	0900	23
38	7959	2659	6902	8·4620700	4059	6990	9500	1598	22
39	8708	3400	7636	1426	4778	7702	8·4750205	2296	21
40	9456	4141	8369	2152	5497	8414	0910	2994	20
41	8·4490205	4881	9103	2878	6216	9126	1615	3692	19
42	0953	5622	9836	3604	6935	9837	2320	4390	18
43	1701	6363	8·4580569	4330	7653	8·4710549	3024	5088	17
44	2450	7103	1302	5055	8372	1260	3729	5785	16
45	3198	7844	2035	5781	9090	1971	4433	6483	15
46	3945	8584	2768	6506	9808	2682	5137	7180	14
47	4693	9324	3500	7231	8·4670526	3393	5841	7878	13
48	5441	8·4540064	4233	7957	1244	4104	6545	8575	12
49	6188	0804	4965	8682	1962	4815	7249	9272	11
50	6936	1543	5697	9406	2680	5526	7953	9969	10
51	7683	2283	6429	8·4630131	3397	6236	8656	8·4800666	9
52	8430	3023	7161	0856	4115	6947	9360	1362	8
53	9177	3762	7893	1580	4832	7657	8·4760063	2059	7
54	9924	4501	8625	2305	5549	8367	0766	2755	6
55	8·4500671	5240	9357	3029	6266	9077	1470	3452	5
56	1417	5979	8·4590088	3753	6983	9787	2173	4148	4
57	2164	6718	0819	4477	7700	8·4720497	2876	4844	3
58	2910	7457	1551	5201	8417	1207	3578	5540	2
59	3656	8195	2282	5925	9134	1916	4281	6236	1
60	4402	8934	3013	6649	9850	2626	4984	6932	0
″	23′	22′	21′	20′	19′	18′	17′	16′	″

LOG. COSINE 88°.

″	36′	37′	38′	39′	40′	41′	42′	43′	″
0	8·4461103	8·4506131	8·4550699	8·4594814	8·4638486	8·4681725	8·4724538	8·4766933	60
1	1857	6878	1438	5545	9211	2442	5248	7636	59
2	2611	7624	2176	6277	9935	3159	5957	8339	58
3	3365	8371	2915	7008	8·4640659	3875	6667	9042	57
4	4119	9117	3654	7739	1382	4592	7377	9745	56
5	4873	9863	4392	8470	2106	5309	8086	8·4770448	55
6	5627	8·4510609	5130	9201	2830	6025	8796	1150	54
7	6380	1354	5868	9932	3553	6741	9505	1853	53
8	7133	2100	6607	8·4600662	4276	7458	8·4730214	2555	52
9	7887	2846	7344	1393	5000	8174	0923	3257	51
10	8640	3591	8082	2123	5723	8890	1632	3959	50
11	9393	4336	8820	2853	6446	9605	2341	4661	49
12	8·4470146	5081	9558	3584	7168	8·4690321	3050	5363	48
13	0898	5826	8·4560295	4314	7891	1037	3758	6065	47
14	1651	6571	1032	5043	8614	1752	4467	6766	46
15	2404	7316	1769	5773	9336	2468	5175	7468	45
16	3156	8061	2506	6503	8·4650059	3183	5884	8169	44
17	3908	8805	3243	7232	0781	3898	6592	8871	43
18	4660	9549	3980	7962	1503	4613	7300	9572	42
19	5412	8·4520294	4717	8691	2225	5328	8008	8·4780273	41
20	6164	1038	5453	9420	2947	6043	8715	0974	40
21	6916	1782	6190	8·4610149	3669	6757	9423	1675	39
22	7667	2526	6926	0878	4390	7472	8·4740131	2375	38
23	8419	3269	7662	1607	5112	8186	0838	3076	37
24	9170	4013	8398	2336	5833	8900	1545	3776	36
25	9921	4757	9134	3064	6555	9615	2253	4477	35
26	8·4480672	5500	9870	3792	7276	8·4700329	2960	5177	34
27	1423	6243	8·4570606	4521	7997	1043	3667	5877	33
28	2174	6986	1341	5249	8718	1756	4374	6577	32
29	2925	7729	2077	5977	9439	2470	5080	7277	31
30	3675	8472	2812	6705	8·4660159	3184	5787	7977	30
31	4426	9215	3547	7433	0880	3897	6494	8677	29
32	5176	9957	4282	8160	1600	4611	7200	9376	28
33	5926	8·4530700	5017	8888	2321	5324	7906	8·4790076	27
34	6676	1442	5752	9615	3041	6037	8612	0775	26
35	7426	2184	6487	8·4620343	3761	6750	9319	1475	25
36	8176	2926	7221	1070	4481	7463	8·4750025	2174	24
37	8925	3668	7956	1797	5201	8176	0730	2873	23
38	9675	4410	8690	2524	5921	8888	1436	3572	22
39	8·4490424	5152	9424	3251	6640	9601	2142	4271	21
40	1173	5893	8·4580158	3978	7360	8·4710313	2847	4969	20
41	1923	6635	0892	4704	8079	1026	3553	5668	19
42	2672	7376	1626	5431	8798	1738	4258	6366	18
43	3420	8117	2360	6157	9517	2450	4963	7065	17
44	4169	8859	3094	6883	8·4670236	3162	5668	7763	16
45	4918	9599	3827	7609	0955	3874	6373	8461	15
46	5666	8·4540340	4560	8335	1674	4586	7078	9159	14
47	6415	1081	5293	9061	2393	5297	7783	9857	13
48	7163	1822	6027	9787	3111	6009	8487	8·4800555	12
49	7911	2562	6760	8·4630512	3830	6720	9192	1252	11
50	8659	3302	7492	1238	4548	7431	9896	1950	10
51	9407	4043	8225	1963	5266	8142	8·4760600	2648	9
52	8·4500154	4783	8958	2689	5984	8853	1304	3345	8
53	0902	5523	9690	3414	6702	9564	2008	4042	7
54	1649	6262	8·4590422	4139	7420	8·4720275	2712	4739	6
55	2397	7002	1155	4864	8138	0986	3416	5436	5
56	3144	7742	1887	5588	8855	1696	4120	6133	4
57	3891	8481	2619	6313	9573	2407	4823	6830	3
58	4638	9220	3351	7038	8·4680290	3117	5527	7527	2
59	5385	9960	4082	7762	1008	3827	6230	8223	1
60	6131	8·4550699	4814	8486	1725	4538	6933	8920	0
″	23′	22′	21′	20′	19′	18′	17′	16′	″

LOG. COTAN. 88°.

″	44′	45′	46′	47′	48′	49′	50′	51′	″
0	8·4806932	8·4848479	8·4889632	8·4930398	8·4970784	8·5010798	8·5050447	8·5089736	60
1	7628	9168	8·4890314	1074	1454	1462	1105	8·5090388	59
2	8323	9857	0997	1750	2124	2126	1762	1040	58
3	9019	8·4850546	1679	2426	2794	2790	2420	1691	57
4	9714	1235	2361	3102	3463	3453	3077	2343	56
5	8·4810410	1923	3043	3778	4133	4116	3735	2994	55
6	1105	2612	3726	4453	4802	4780	4392	3646	54
7	1800	3300	4407	5129	5472	5443	5049	4297	53
8	2495	3989	5089	5804	6141	6106	5706	4948	52
9	3190	4677	5771	6480	6810	6769	6363	5599	51
10	3884	5365	6453	7155	7479	7432	7020	6250	50
11	4579	6053	7134	7830	8148	8095	7677	6901	49
12	5273	6741	7816	8505	8817	8757	8333	7552	48
13	5968	7429	8497	9180	9485	9420	8990	8202	47
14	6662	8116	9178	9855	8·4980154	8·5020082	9646	8853	46
15	7356	8804	9859	8·4940530	0823	0745	8·5060303	9503	45
16	8050	9491	8·4900540	1204	1491	1407	0959	8·5100154	44
17	8744	8·4860179	1221	1879	2159	2069	1615	0804	43
18	9438	0866	1902	2553	2827	2731	2271	1454	42
19	8·4820132	1553	2582	3228	3495	3393	2927	2104	41
20	0825	2240	3263	3902	4163	4055	3583	2754	40
21	1519	2927	3943	4576	4831	4717	4239	3404	39
22	2212	3614	4624	5250	5499	5378	4894	4054	38
23	2905	4300	5304	5924	6167	6040	5550	4703	37
24	3599	4987	5984	6597	6834	6701	6205	5353	36
25	4292	5673	6664	7271	7502	7363	6861	6002	35
26	4985	6360	7344	7945	8169	8024	7516	6652	34
27	5677	7046	8024	8618	8836	8685	8171	7301	33
28	6370	7732	8703	9292	9504	9346	8826	7950	32
29	7063	8418	9383	9965	8·4990171	8·5030007	9481	8599	31
30	7755	9104	8·4910063	8·4950638	0838	0668	8·5070136	9248	30
31	8448	9790	0742	1311	1504	1329	0791	9897	29
32	9140	8·4870476	1421	1984	2171	1989	1446	8·5110546	28
33	9832	1161	2100	2657	2838	2650	2100	1195	27
34	8·4830524	1847	2779	3330	3504	3310	2755	1843	26
35	1216	2532	3458	4002	4171	3971	3409	2492	25
36	1908	3217	4137	4675	4837	4631	4063	3140	24
37	2600	3903	4816	5347	5503	5291	4717	3789	23
38	3291	4588	5495	6020	6169	5951	5371	4437	22
39	3983	5273	6173	6692	6835	6611	6025	5085	21
40	4674	5957	6852	7364	7501	7271	6679	5733	20
41	5365	6642	7530	8036	8167	7931	7333	6381	19
42	6057	7327	8208	8708	8833	8590	7987	7029	18
43	6748	8011	8886	9380	9499	9250	8640	7676	17
44	7439	8696	9564	8·4960051	8·5000164	9909	9294	8324	16
45	8129	9380	8·4920242	0723	0829	8·5040569	9947	8972	15
46	8820	8·4880064	0920	1394	1495	1228	8·5080601	9619	14
47	9511	0748	1598	2066	2160	1887	1254	8·5120266	13
48	8·4840201	1432	2275	2737	2825	2546	1907	0914	12
49	0892	2116	2953	3408	3490	3205	2560	1561	11
50	1582	2800	3630	4079	4155	3864	3213	2208	10
51	2272	3484	4307	4750	4820	4523	3866	2855	9
52	2962	4167	4984	5421	5485	5181	4518	3502	8
53	3652	4851	5661	6092	6149	5840	5171	4148	7
54	4342	5534	6338	6763	6814	6498	5823	4795	6
55	5032	6217	7015	7433	7478	7157	6476	5442	5
56	5721	6900	7692	8104	8142	7815	7128	6088	4
57	6411	7583	8368	8774	8806	8473	7780	6735	3
58	7100	8266	9045	9444	9471	9131	8432	7381	2
59	7790	8949	9721	8·4970114	8·5010135	9789	9084	8027	1
60	8479	9632	8·4930398	0784	0798	8·5050447	9736	8673	0
″	15′	14′	13′	12′	11′	10′	9′	8′	″

LOG. COSINE 88°.

″	44′	45′	46′	47′	48′	49′	50′	51′	″
0	8·4808920	8·4850505	8·4891696	8·4932502	8·4972928	8·5012982	8·5052671	8·5092001	60
1	9616	1195	2380	3179	3598	3646	3329	2653	59
2	8·4810312	1884	3063	3855	4269	4311	3987	3305	58
3	1008	2574	3746	4532	4939	4975	4646	3958	57
4	1704	3263	4429	5208	5610	5639	5304	4610	56
5	2400	3953	5112	5885	6280	6303	5962	5262	55
6	3096	4642	5794	6561	6950	6967	6620	5914	54
7	3792	5331	6477	7237	7620	7631	7277	6566	53
8	4487	6020	7159	7914	8290	8295	7935	7218	52
9	5183	6709	7842	8590	8959	8958	8593	7870	51
10	5878	7397	8524	9266	9629	9622	9250	8521	50
11	6574	8086	9206	9941	8·4980299	8·5020285	9908	9173	49
12	7269	8775	9888	8·4940617	0968	0949	8·5060565	9824	48
13	7964	9463	8·4900570	1293	1638	1612	1222	8·5100475	47
14	8659	8·4860151	1252	1968	2307	2275	1879	1127	46
15	9353	0839	1934	2643	2976	2938	2536	1778	45
16	8·4820048	1528	2615	3319	3645	3601	3193	2429	44
17	0743	2216	3297	3994	4314	4264	3850	3080	43
18	1437	2903	3978	4669	4983	4927	4507	3731	42
19	2131	3591	4660	5344	5652	5589	5164	4381	41
20	2826	4279	5341	6019	6320	6252	5820	5032	40
21	3520	4966	6022	6694	6989	6914	6477	5683	39
22	4214	5654	6703	7368	7657	7576	7133	6333	38
23	4908	6341	7384	8043	8325	8239	7789	6983	37
24	5602	7028	8065	8717	8994	8901	8445	7634	36
25	6295	7716	8745	9392	9662	9563	9101	8284	35
26	6989	8403	9426	8·4950066	8·4990330	8·5030225	9757	8934	34
27	7682	9089	8·4910106	0740	0998	0887	8·5070413	9584	33
28	8376	9776	0787	1414	1666	1548	1069	8·5110234	32
29	9069	8·4870463	1467	2088	2333	2210	1724	0883	31
30	9762	1149	2147	2762	3001	2871	2380	1533	30
31	8·4830455	1836	2827	3435	3668	3533	3035	2183	29
32	1148	2522	3507	4109	4336	4194	3691	2832	28
33	1841	3209	4187	4783	5003	4855	4346	3482	27
34	2533	3895	4866	5456	5670	5517	5001	4131	26
35	3226	4581	5546	6129	6337	6178	5656	4780	25
36	3919	5267	6226	6802	7004	6838	6311	5429	24
37	4611	5952	6905	7476	7671	7499	6966	6078	23
38	5303	6638	7584	8148	8338	8160	7621	6727	22
39	5995	7324	8263	8821	9005	8821	8275	7376	21
40	6687	8009	8942	9494	9671	9481	8930	8025	20
41	7379	8695	9621	8·4960167	8·5000338	8·5040142	9584	8673	19
42	8071	9380	8·4920300	0889	1004	0802	8·5080239	9322	18
43	8763	8·4880065	0979	1512	1671	1462	0893	9970	17
44	9454	0750	1658	2184	2337	2122	1547	8·5120618	16
45	8·4840146	1435	2336	2856	3003	2782	2201	1267	15
46	0837	2120	3015	3529	3669	3442	2855	1915	14
47	1528	2805	3693	4201	4335	4102	3509	2563	13
48	2220	3489	4371	4873	5000	4762	4163	3211	12
49	2911	4174	5049	5544	5666	5421	4817	3859	11
50	3602	4858	5727	6216	6332	6081	5470	4506	10
51	4292	5543	6405	6888	6997	6740	6124	5154	9
52	4983	6227	7083	7559	7663	7400	6777	5801	8
53	5674	6911	7761	8231	8328	8059	7430	6449	7
54	6364	7595	8438	8902	8993	8718	8084	7096	6
55	7055	8279	9116	9573	9658	9377	8737	7743	5
56	7745	8962	9793	8·4970244	8·5010323	8·5050036	9390	8391	4
57	8435	9646	8·4930471	0915	0988	0695	8·5090042	9038	3
58	9125	8·4890330	1148	1586	1653	1353	0695	9685	2
59	9815	1013	1825	2257	2317	2012	1348	8·5130332	1
60	8·4850505	1696	2502	2928	2982	2671	2001	0978	0
″	15′	14′	13′	12′	11′	10′	9′	8′	″

LOG. COTAN. 88°.

″	52′	53′	54′	55′	56′	57′	58′	59′	″
0	8·5128673	8·5167264	8·5205514	8·5243430	8·5281017	8·5318281	8·5355228	8·5391863	60
1	9319	7904	6148	4059	1641	8900	5842	2471	59
2	9965	8544	6783	4688	2264	9518	6455	3079	58
3	8·5130611	9184	7417	5317	2888	8·5320136	7068	3687	57
4	1256	9824	8052	5946	3511	0754	7680	4295	56
5	1902	8·5170464	8686	6574	4135	1372	8293	4902	55
6	2548	1104	9320	7203	4758	1990	8906	5510	54
7	3193	1743	9954	7833	5381	2608	9518	6117	53
8	3838	2383	8·5210588	8460	6004	3226	8·5360131	6725	52
9	4484	3023	1222	9088	6627	3844	0743	7332	51
10	5129	3662	1856	9717	7250	4461	1356	7939	50
11	5774	4301	2490	8·5250345	7873	5079	1968	8546	49
12	6419	4941	3123	0973	8495	5696	2580	9153	48
13	7064	5580	3757	1601	9118	6313	3192	9760	47
14	7708	6219	4390	2229	9741	6931	3804	8·5400367	46
15	8353	6858	5024	2857	8·5290363	7548	4416	0974	45
16	8997	7497	5657	3485	0985	8165	5028	1581	44
17	9642	8135	6290	4112	1608	8782	5640	2187	43
18	8·5140286	8774	6923	4740	2230	9399	6251	2794	42
19	0931	9413	7556	5367	2852	8·5330015	6863	3400	41
20	1575	8·5180051	8189	5995	3474	0632	7474	4007	40
21	2219	0689	8822	6622	4096	1249	8086	4613	39
22	2863	1328	9455	7249	4718	1865	8697	5219	38
23	3507	1966	8·5220087	7877	5339	2482	9308	5825	37
24	4150	2604	0720	8504	5961	3098	9920	6431	36
25	4794	3242	1352	9131	6583	3714	8·5370531	7037	35
26	5438	3880	1985	9757	7204	4330	1142	7643	34
27	6081	4518	2617	8·5260384	7826	4946	1752	8249	33
28	6725	5156	3249	1011	8447	5562	2363	8854	32
29	7368	5793	3881	1637	9068	6178	2974	9460	31
30	8011	6431	4513	2264	9689	6794	3585	8·5410066	30
31	8654	7068	5145	2890	8·5300310	7410	4195	0671	29
32	9297	7706	5777	3517	0931	8026	4806	1276	28
33	9940	8343	6408	4143	1552	8641	5416	1882	27
34	8·5150583	8980	7040	4769	2173	9257	6026	2487	26
35	1226	9617	7672	5395	2793	9872	6636	3092	25
36	1869	8·5190254	8303	6021	3414	8·5340487	7247	3697	24
37	2511	0891	8934	6647	4034	1103	7857	4302	23
38	3154	1528	9566	7273	4655	1718	8466	4907	22
39	3796	2164	8·5230197	7898	5275	2333	9076	5511	21
40	4438	2801	0828	8524	5895	2948	9686	6116	20
41	5080	3438	1459	9149	6516	3563	8·5380296	6721	19
42	5722	4074	2090	9775	7136	4177	0905	7325	18
43	6364	4710	2720	8·5270400	7756	4792	1515	7929	17
44	7006	5347	3351	1025	8375	5407	2124	8534	16
45	7648	5983	3982	1651	8995	6021	2734	9138	15
46	8290	6619	4612	2276	9615	6636	3343	9742	14
47	8931	7255	5243	2901	8·5310235	7250	3952	8·5420346	13
48	9573	7891	5873	3525	0854	7864	4561	0950	12
49	8·5160214	8526	6503	4150	1473	8478	5170	1554	11
50	0856	9162	7133	4775	2093	9092	5779	2158	10
51	1497	9798	7763	5400	2712	9706	6388	2762	9
52	2138	8·5200433	8393	6024	3331	8·5350320	6997	3365	8
53	2779	1069	9023	6648	3950	0934	7605	3969	7
54	3420	1704	9653	7273	4569	1548	8214	4572	6
55	4061	2339	8·5240283	7897	5188	2161	8822	5176	5
56	4701	2974	0912	8521	5807	2775	9431	5779	4
57	5342	3609	1542	9145	6426	3389	8·5390039	6382	3
58	5983	4244	2171	9769	7044	4002	0647	6986	2
59	6623	4879	2800	8·5280393	7663	4615	1255	7589	1
60	7264	5514	3430	1017	8281	5228	1863	8192	0
″	7′	6′	5′	4′	3′	2′	1′	0′	″

LOG. COSINE 88°.

″	52′	53′	54′	55′	56′	57′	58′	59′	″
0	8·5130978	8·5169610	8·5207902	8·5245860	8·5283490	8·5320797	8·5357787	8·5394466	60
1	1625	8·5170251	8537	6490	4114	1416	8401	5075	59
2	2272	0892	9173	7120	4739	2035	9015	5683	58
3	2918	1533	9808	7749	5363	2654	9629	6292	57
4	3564	2173	8·5210443	8379	5987	3273	8·5360242	6900	56
5	4211	2814	1078	9008	6611	3892	0856	7509	55
6	4857	3455	1713	9638	7235	4510	1469	8117	54
7	5503	4095	2348	8·5250267	7859	5129	2082	8725	53
8	6149	4735	2982	0896	8483	5747	2696	9333	52
9	6795	5375	3617	1525	9106	6366	3309	9941	51
10	7441	6016	4251	2154	9730	6984	3922	8·5400549	50
11	8087	6656	4886	2783	8·5290353	7602	4535	1157	49
12	8732	7296	5520	3412	0977	8220	5148	1765	48
13	9378	7935	6154	4041	1600	8838	5761	2372	47
14	8·5140023	8575	6789	4669	2223	9456	6373	2980	46
15	0668	9215	7423	5298	2847	8·5330074	6986	3587	45
16	1314	9854	8057	5926	3470	0692	7599	4195	44
17	1959	8·5180494	8690	6555	4093	1310	8211	4802	43
18	2604	1133	9324	7183	4716	1927	8823	5409	42
19	3249	1772	9958	7811	5338	2545	9436	6017	41
20	3894	2412	8·5220591	8439	5961	3162	8·5370048	6624	40
21	4539	3051	1225	9067	6584	3779	0660	7231	39
22	5183	3690	1858	9695	7206	4397	1272	7838	38
23	5828	4329	2492	8·5260323	7829	5014	1884	8445	37
24	6472	4967	3125	0951	8451	5631	2496	9051	36
25	7117	5606	3758	1579	9073	6248	3108	9658	35
26	7761	6245	4391	2206	9696	6865	3719	8·5410264	34
27	8405	6883	5024	2834	8·5300318	7482	4331	0871	33
28	9049	7522	5657	3461	0940	8098	4942	1477	32
29	9693	8160	6290	4088	1562	8715	5554	2084	31
30	8·6150337	8798	6922	4716	2183	9331	6165	2690	30
31	0981	9436	7555	5343	2805	9948	6777	3296	29
32	1625	8·5190074	8187	5970	3427	8·5340564	7388	3902	28
33	2268	0712	8820	6597	4048	1181	7999	4508	27
34	2912	1350	9452	7223	4670	1797	8610	5114	26
35	3555	1988	8·5230084	7850	5291	2413	9221	5720	25
36	4199	2626	0717	8477	5912	3029	9832	6326	24
37	4842	3263	1349	9103	6534	3645	8·5380442	6931	23
38	5485	3901	1980	9730	7155	4261	1053	7537	22
39	6128	4538	2612	8·5270356	7776	4876	1664	8142	21
40	6771	5175	3244	0983	8397	5492	2274	8748	20
41	7414	5813	3876	1609	9018	6108	2884	9353	19
42	8057	6450	4507	2235	9638	6723	3495	9958	18
43	8699	7087	5139	2861	8·5310259	7339	4105	8·5420563	17
44	9342	7724	5770	3487	0880	7954	4715	1168	16
45	9984	8361	6401	4113	1500	8569	5325	1773	15
46	8·5160627	8997	7033	4739	2121	9184	5935	2378	14
47	1269	9634	7664	5364	2741	9799	6545	2983	13
48	1911	8·5200271	8295	5990	3361	8·5350414	7155	3588	12
49	2553	0907	8926	6615	3981	1029	7765	4193	11
50	3195	1543	9557	7241	4601	1644	8374	4797	10
51	3837	2180	8·5240187	7866	5221	2259	8984	5402	9
52	4479	2816	0818	8491	5841	2873	9593	6006	8
53	5121	3452	1449	9116	6461	3488	8·5390203	6610	7
54	5762	4088	2079	9741	7081	4102	0812	7214	6
55	6404	4724	2709	8·5280366	7700	4717	1421	7819	5
56	7045	5360	3340	0991	8320	5331	2030	8423	4
57	7687	5995	3970	1616	8939	5945	2639	9027	3
58	8328	6631	4600	2241	9559	6559	3248	9631	2
59	8969	7267	5230	2865	8·5320178	7173	3857	8·5430234	1
60	9610	7902	5860	3490	0797	7787	4466	0838	0
″	7′	6′	5′	4′	3′	2′	1′	0′	″

LOG. COTAN. 88°.

′	2°	diff.	3°	diff.	4°	diff.	5°	diff.	6°	diff.	
0	8·5428192	36026	8·7188002	24038	8·8435845	18029	8·9402960	14416	9·0192346	12002	60
1	64218	35730	8·7212040	23906	53874	17953	17376	14367	9·0204348	11970	59
2	99948	35438	35946	23775	71827	17880	31743	14320	16318	11936	58
3	8·5535386	35150	59721	23645	89707	17805	46063	14272	28254	11903	57
4	70536	34868	83366	23516	8·8507512	17733	60335	14226	40157	11870	56
5	8·5605404	34590	8·7306882	23390	25245	17660	74561	14178	52027	11838	55
6	39994	34316	30272	23263	42905	17588	88739	14132	63865	11804	54
7	74310	34047	53535	23140	60493	17517	8·9502871	14086	75669	11773	53
8	8·5708357	33782	76675	23016	78010	17447	16957	14039	87442	11740	52
9	42139	33521	99691	22895	95457	17376	30996	13995	99182	11708	51
10	75660	33263	8·7422586	22774	8·8612833	17306	44991	13949	9·0310890	11677	50
11	8·5808923	33010	45360	22655	30139	17237	58940	13903	22567	11645	49
12	41933	32761	68015	22538	47376	17169	72843	13860	34212	11613	48
13	74694	32515	90553	22420	64545	17101	86703	13814	45825	11582	47
14	8·5907209	32274	8·7512973	22305	81646	17034	8·9600517	13771	57407	11551	46
15	39483	32034	35278	22191	98680	16966	14288	13726	68958	11519	45
16	71517	31800	57469	22077	8·8715646	16900	28014	13683	80477	11489	44
17	8·6003317	31569	79546	21966	32546	16835	41697	13640	91966	11458	43
18	34886	31340	·7601512	21854	49381	16769	55337	13597	9·0403424	11428	42
19	66226	31115	23366	21745	66150	16704	68934	13553	14852	11397	41
20	97341	30894	45111	21636	82854	16639	82487	13512	26249	11368	40
21	8·6128235	30675	66747	21528	99493	16576	95999	13469	37617	11337	39
22	58910	30459	88275	21422	8·8816069	16512	8·9709468	13427	48954	11307	38
23	89369	30247	8·7709697	21317	32581	16450	22895	13385	60261	11277	37
24	8·6219616	30037	31014	21212	49031	16387	36280	13344	71538	11248	36
25	49653	29831	52226	21108	65418	16325	49624	13302	82786	11219	35
26	79484	29627	73334	21006	81743	16264	62926	13262	94005	11189	34
27	8·6309111	29426	94340	20904	98007	16202	76188	13220	9·0505194	11160	33
28	38537	29227	8·7815244	20804	8·8914209	16142	89408	13181	16354	11131	32
29	67764	29032	36048	20705	30351	16082	8·9802589	13140	27485	11103	31
30	96796	28838	56753	20606	46433	16022	15729	13100	38588	11073	30
31	8·6425634	28648	77359	20508	62455	15963	28829	13060	49661	11045	29
32	54282	28460	97867	20411	78418	15904	41889	13021	60706	11017	28
33	82742	28274	8·7918278	20316	94322	15846	54910	12981	71723	10988	27
34	8·6511016	28091	38594	20220	8·9010168	15787	67891	12943	82711	10961	26
35	39107	27910	58814	20127	25955	15730	80834	12903	93672	10932	25
36	67017	27731	78941	20033	41685	15673	93737	12865	9·0604604	10905	24
37	94748	27555	98974	19941	57358	15617	8·9906602	12827	15509	10877	23
38	8·6622303	27381	8·8018915	19849	72975	15560	19429	12788	26386	10849	22
39	49684	27209	38764	19759	88535	15504	32217	12751	37235	10822	21
40	76893	27039	58523	19669	8·9104039	15448	44968	12713	48057	10795	20
41	8·6703932	26872	78192	19580	19487	15394	57681	12675	58852	10767	19
42	30804	26706	97772	19492	34831	15338	70356	12638	69619	10741	18
43	57510	26542	8·8117264	19404	50219	15285	82994	12601	80360	10714	17
44	84052	26381	36668	19317	65504	15230	95595	12565	91074	10687	16
45	8·6810433	26221	55985	19232	80734	15177	9·0008160	12527	9·0701761	10660	15
46	36654	26064	75217	19146	95911	15123	20687	12492	12421	10634	14
47	62718	25907	94363	19062	8·9211034	15071	33179	12455	23055	10608	13
48	88625	25754	8·8213425	18979	26105	15018	45634	12419	33663	10581	12
49	8·6914379	25601	32404	18895	41123	14966	58053	12383	44244	10555	11
50	39980	25451	51299	18813	56089	14914	70436	12348	54799	10530	10
51	65431	25303	70112	18732	71003	14863	82784	12312	65329	10503	9
52	90734	25155	88844	18651	85866	14812	95096	12278	75832	10478	8
53	8·7015889	25010	8·8307495	18571	8·9300678	14761	9·0107374	12242	86310	10452	7
54	40899	24867	26066	18491	15439	14711	19616	12207	86762	10427	6
55	65766	24724	44557	18412	30150	14661	31823	12173	9·0807189	10401	5
56	90490	24585	62969	18335	44811	14611	43996	12139	17590	10376	4
57	8·7115075	24445	81304	18257	59422	14561	56135	12104	27966	10351	3
58	39520	24309	99561	18180	73983	14513	68239	12070	38317	10326	2
59	63829	24173	8·8417741	18104	88496	14464	80309	12037	48643	10302	1
60	88002		35845		8·9402960		92346		58945		0
′	87°	diff.	86°	diff.	85°	diff.	84°	diff.	83°	diff.	′

LOG. COSINE.

′	2°	*diff.*	3°	*diff.*	4°	*diff.*	5°	*diff.*	6°	*diff.*	′
0	8·5430838	36071	8·7193958	24105	8·8446437	18117	8·9419518	14526	9·0216202	12136	60
1	66909	35774	8·7218063	23972	64554	18043	34044	14479	28338	12103	59
2	8·5502683	35483	42035	23842	82597	17969	48523	14431	40441	12069	58
3	38166	35196	65877	23712	8·8500566	17895	62954	14384	52510	12038	57
4	73362	34914	89589	23585	18461	17822	77338	14338	64548	12004	56
5	8·5608276	34636	8.7313174	23457	36283	17751	91676	14291	76552	11972	55
6	42912	34363	36631	23333	54034	17679	8·9505967	14244	88524	11940	54
7	77275	34093	59964	23208	71713	17608	20211	14199	9·0300464	11909	53
8	8·5711368	33829	83172	23086	89321	17538	34410	14154	12373	11876	52
9	45197	33569	8·7406258	22964	8·8606859	17468	48564	14108	24249	11844	51
10	78766	33311	29222	22845	24327	17398	62672	14063	36093	11813	50
11	8·5812077	33059	52067	22725	41725	17330	76735	14019	47906	11782	49
12	45136	32809	74792	22608	59055	17262	90754	13974	59688	11751	48
13	77945	32564	97400	22492	76317	17194	8·9604728	13931	71439	11720	47
14	8·5910509	32323	8·7519892	22377	93511	17127	18659	13886	83159	11689	46
15	42832	32085	42269	22262	8·8710638	17061	32545	13843	94848	11658	45
16	74917	31850	64531	22150	27699	16995	46388	13800	9·0406506	11628	44
17	8·6006767	31619	86681	22038	44694	16929	60188	13756	18134	11597	43
18	38386	31391	8·7608719	21928	61623	16864	73944	13714	29731	11568	42
19	69777	31166	30647	21818	78487	16799	87658	13672	41299	11537	41
20	8.6100943	30946	52465	21710	95286	16736	8·9701330	13629	52836	11507	40
21	31889	30727	74175	21602	8·8812022	16672	14959	13588	64343	11478	39
22	62616	30511	95777	21497	28694	16609	28547	13545	75821	11449	38
23	93127	30300	8·7717274	21391	45303	16547	42092	13505	87270	11419	37
24	8·6223427	30091	38665	21287	61850	16484	55597	13463	98689	11389	36
25	53518	29834	59952	21184	78334	16423	69060	13423	9·0510078	11361	35
26	83402	29681	81136	21082	94757	16362	82483	13382	21439	11332	34
27	8·6313083	29480	8·7802218	20981	8·8911119	16301	95865	13341	32771	11303	33
28	42563	29282	23199	20880	27420	16240	8·9809206	13301	44074	11275	32
29	71845	29086	44079	20782	43660	16182	22507	13262	55349	11246	31
30	8·6400931	28894	64861	20683	59842	16121	35769	13222	66595	11218	30
31	29825	28703	85544	20586	75963	16063	48991	13182	77813	11189	29
32	58528	28516	8·7906130	20490	92026	16004	62173	13144	89002	11162	28
33	87044	28331	26620	20394	8·9008030	15947	75317	13104	9·0600164	11133	27
34	8·6515375	28147	47014	20299	23977	15889	88421	13066	11297	11106	26
35	43522	27968	67313	20206	39866	15831	8·9901487	13027	22403	11079	25
36	71490	27789	87519	20113	55697	15775	14514	12989	33482	11051	24
37	99279	27612	8·8007632	20021	71472	15718	27503	12951	44533	11023	23
38	8·6626891	27440	27653	19930	87190	15663	40454	12913	55556	10997	22
39	54331	27267	47583	19839	8·9102853	15607	53367	12876	66553	10969	21
40	81598	27099	67422	19750	18460	15552	66243	12838	77522	10943	20
41	8·6708697	26931	87172	19662	34012	15497	79081	12802	88465	10916	19
42	35628	26765	8·8106834	19573	49509	15443	91883	12764	99381	10889	18
43	62393	26603	26407	19487	64952	15388	9·0004647	12728	9·0710270	10863	17
44	88996	26441	45894	19400	80340	15335	17375	12691	21133	10836	16
45	8·6815437	26282	65294	19314	95675	15282	30066	12655	31969	10810	15
46	41719	26125	84608	19230	8·9210957	15229	42721	12619	42779	10784	14
47	67844	25969	8·8203838	19146	26186	15177	55340	12584	53563	10758	13
48	93813	25816	22984	19062	41363	15124	67924	12547	64321	10732	12
49	8·6919629	25663	42046	18980	56487	15073	80471	12513	75053	10707	11
50	45292	25514	61026	18898	71560	15021	92984	12477	85760	10681	10
51	70806	25366	79924	18817	86581	14971	9·0105461	12442	96441	10655	9
52	96172	25218	98741	18737	8·9301552	14919	17903	12407	9·0807096	10630	8
53	8·7021390	25075	8·8317478	18656	16471	14869	30310	12372	17726	10605	7
54	46465	24930	36134	18578	31340	14820	42682	12339	28331	10580	6
55	71395	24790	54712	18499	46160	14769	55021	12304	38911	10555	5
56	96185	24649	73211	18422	60929	14721	67325	12269	49466	10530	4
57	8·7120834	24511	91633	18344	75650	14671	79594	12237	59996	10505	3
58	45345	24374	8·8409977	18268	90321	14623	91831	12202	70501	10480	2
59	69719	24239	28245	18192	8·9404944	14574	9·0204033	12169	80981	10457	1
60	93958		46437		19518		16202		91438		0
′	87°	*diff.*	86°	*diff.*	85°	*diff.*	84°	*diff.*	83°	*diff.*	′

LOG. COTAN.

′	7°	*diff.*	8°	*diff.*	9°	*diff.*	10°	*diff.*	11°	*diff.*	′
0	9·0858945	10276	9·1435553	8979	9·1943324	7969	9·2396702	7159	9·2805988	6495	60
1	69221	10252	44532	8961	51293	7954	9·2403861	7146	12483	6484	59
2	79473	10227	53493	8942	59247	7939	11007	7134	18967	6474	58
3	89700	10203	62435	8923	67186	7924	18141	7123	25441	6464	57
4	99903	10179	71358	8904	75110	7909	25264	7110	31905	6454	56
5	9·0910082	10155	80262	8896	83019	7894	32374	7098	38359	6444	55
6	20237	10130	89148	8867	90913	7880	39472	7086	44803	6434	54
7	30367	10107	98015	8849	98793	7865	46558	7074	51237	6424	53
8	40474	10082	9·1506864	8830	9·2006658	7851	53632	7063	57661	6415	52
9	50556	10059	15694	8813	14509	7836	60695	7051	64076	6404	51
10	60615	10036	24507	8794	22345	7822	67746	7038	70480	6395	50
11	70651	10011	33301	8775	30167	7807	74764	7027	76875	6385	49
12	80662	9989	42076	8758	37974	7792	81811	7016	83260	6376	48
13	80651	9965	50834	8740	45766	7779	88827	7003	89636	6365	47
14	9·1000616	9942	59574	8722	53545	7764	95830	6992	96001	6356	46
15	10558	9919	68296	8704	61309	7750	9·2502822	6981	9·2902357	6347	45
16	20477	9896	77000	8686	69059	7736	09803	6969	08704	6336	44
17	30373	9873	85686	8668	76795	7721	16772	6957	15040	6327	43
18	40246	9850	94354	8651	84516	7708	23729	6946	21367	6318	42
19	50096	9828	9·1603005	8634	92224	7693	30675	6934	27685	6308	41
20	59924	9805	11639	8615	99917	7680	37609	6923	33993	6298	40
21	69729	9783	20254	8599	9·2107597	7666	44532	6912	40291	6289	39
22	79512	9760	28853	8581	15263	7651	51444	6900	46580	6279	38
23	89272	9738	37434	8564	22914	7638	58344	6889	52859	6270	37
24	99010	9716	45998	8546	30552	7624	65233	6877	59129	6261	36
25	9·1108726	9694	54544	8530	38176	7611	72110	6867	65390	6251	35
26	18420	9672	63074	8512	45787	7597	78977	6855	71641	6242	34
27	28092	9650	71586	8495	53384	7583	85832	6844	77883	6233	33
28	37742	9628	80081	8478	60967	7569	92676	6833	84116	6223	32
29	47370	9607	88559	8462	68536	7556	99509	6821	90339	6214	31
30	56977	9585	97021	8444	76092	7543	9·2606330	6811	96553	6205	30
31	66562	9563	9·1705465	8428	83635	7529	13141	6800	9·3002758	6195	29
32	76125	9542	13893	8412	91164	7516	19941	6788	08953	6187	28
33	85667	9521	22305	8394	98680	7502	26729	6778	15140	6177	27
34	95188	9500	30699	8378	9·2206182	7489	33507	6767	21317	6168	26
35	9·1204688	9479	39077	8362	13671	7476	40274	6756	27485	6159	25
36	14167	9457	47439	8345	21147	7462	47030	6745	33644	6150	24
37	23624	9437	55784	8328	28609	7450	53775	6734	39794	6140	23
38	33061	9416	64112	8313	36059	7436	60509	6723	45934	6132	22
39	42477	9395	72425	8296	43495	7423	67232	6713	52066	6123	21
40	51872	9374	80721	8280	50918	7410	73945	6702	58189	6114	20
41	61246	9354	89001	8264	58328	7397	80647	6691	64303	6104	19
42	70600	9334	97265	8247	65725	7385	87338	6681	70407	6096	18
43	79934	9313	9·1805512	8232	73110	7371	94019	6670	76503	6087	17
44	89247	9292	13744	8216	80481	7358	9·2700689	6659	82590	6078	16
45	98539	9273	21960	8200	87839	7346	07348	6649	88668	6069	15
46	9·1 07812	9252	30160	8184	95185	7333	13997	6638	94737	6061	14
47	17064	9233	38344	8168	9·2300518	7320	20635	6628	9·3100798	6051	13
48	26297	9212	46512	8153	09838	7307	27263	6617	06849	6043	12
49	35509	9193	54665	8137	17145	7295	33880	6607	12892	6034	11
50	44702	9173	62802	8121	24440	7282	40487	6596	18926	6025	10
51	53875	9153	70923	8106	31722	7270	47083	6586	24951	6017	9
52	63028	9133	79029	8091	38992	7257	53669	6576	30968	6008	8
53	72161	9114	87120	8075	46249	7245	60245	6566	36976	5999	7
54	81275	9095	95195	8059	53494	7232	66811	6555	42975	5990	6
55	90370	9075	9·1903254	8045	60726	7220	73366	6545	48965	5982	5
56	99415	9056	11299	8029	67946	7207	79911	6534	54947	5974	4
57	9·1408501	9036	19328	8014	75153	7196	86445	6525	60921	5964	3
58	17537	9018	27342	7999	82349	7183	92970	6514	66885	5956	2
59	26555	8998	35341	7983	89532	7170	99484	6504	72841	5948	1
60	35553		43324		96702		9·2805988		78789		0
′	82°	*diff.*	81°	*diff.*	80°	*diff.*	79°	*diff.*	78°	*diff.*	′

LOG. COSINE.

′	7°	*diff.*	8°	*diff.*	9°	*diff.*	10°	*diff.*	11°	*diff.*	′
0	9·0891438	10431	9·1478025	9157	9·1997125	8169	9·2463188	7381	9·2886523	6740	60
1	9·0901869	10408	87182	9139	9·2005294	8155	70569	7370	93263	6730	59
2	12277	10383	96321	9120	13449	8139	77939	7358	99993	6720	58
3	22660	10360	9·1505441	9102	21588	8126	85297	7346	9·2906713	6711	57
4	33020	10335	14543	9084	29714	8111	92643	7335	13424	6702	56
5	43355	10312	23627	9065	37825	8097	99978	7323	20126	6691	55
6	53667	10288	32692	9047	45922	8082	9·2507301	7311	26817	6683	54
7	63955	10264	41739	9030	54004	8068	14612	7300	33500	6672	53
8	74219	10241	50769	9011	62072	8054	21912	7288	40172	6664	52
9	84460	10218	59780	8993	70126	8039	29200	7277	46836	6653	51
10	94678	10194	68773	8975	78165	8026	36477	7266	53489	6645	50
11	9.1004872	10172	77748	9958	86191	8012	43743	7254	60134	6635	49
12	15044	10148	86706	8940	94203	7997	50997	7243	66769	6626	48
13	25192	10125	95646	8923	9·2102200	7984	58240	7232	73395	6616	47
14	35317	10103	9·1604569	8904	10184	7969	65472	7220	80011	6607	46
15	45420	10080	13473	8888	18153	7956	72692	7209	86618	6598	45
16	55500	10057	22361	8870	26109	7942	79901	7198	93216	6588	44
17	65557	10034	31231	8852	34051	7929	87099	7186	99804	6579	43
18	75591	10013	40083	8836	41980	7914	94285	7176	9·3006383	6571	42
19	85604	9990	48919	8818	49894	7901	9·2601461	7164	12954	6560	41
20	95594	9968	57737	8801	57795	7888	08625	7154	19514	6552	40
21	9·1105562	9946	66538	8784	65683	7873	15779	7142	26066	6543	39
22	15508	9923	75322	8767	73556	7861	22921	7132	32609	6534	38
23	25431	9902	84089	8750	81417	7847	30053	7120	39143	6524	37
24	35333	9880	92839	8733	89264	7833	37173	7110	45667	6516	36
25	45213	7859	9·1701572	8717	97097	7820	44283	7099	52183	6506	35
26	55072	9837	10289	8700	9·2204917	7807	51382	7088	58689	6498	34
27	64909	9815	18989	8683	12724	7794	58470	7077	65187	6488	33
28	74724	9794	27672	8666	20518	7780	65547	7066	71675	6480	32
29	84518	9773	36338	8650	28298	7767	72613	7056	78155	6471	31
30	94291	9752	44988	8634	36065	7754	79669	7045	84626	6462	30
31	9·1204043	9730	53622	8617	43819	7742	86714	7035	91088	6453	29
32	13773	9709	62239	8601	51561	7728	93749	7023	97541	6444	28
33	23482	9689	70840	8585	59289	7715	9·2700772	7014	9·3103985	6436	27
34	33171	9668	79425	8568	67004	7102	07786	7002	10421	6427	26
35	42839	9647	87993	8553	74706	7689	14788	6992	16848	6418	25
36	52486	9626	96546	8536	82395	7676	21780	6982	23266	6409	24
37	62112	9606	9·1805082	8520	90071	7664	28762	6971	29675	6401	23
38	71718	9585	13602	8504	97735	7651	35733	6961	36076	6392	22
39	81303	9565	22106	8489	9·2305386	7638	42694	6950	42468	6383	21
40	90868	9545	30595	8473	13024	7626	49644	6940	48851	6375	20
41	9.1300413	9524	39068	8457	20650	7612	56584	6930	55226	6366	19
42	09937	9505	47525	8441	28262	7601	63514	6920	61592	6358	18
43	19442	9484	55966	8426	35863	7588	70434	6909	67950	6349	17
44	28926	9465	64392	8410	43451	7575	77343	6899	74299	6341	16
45	38391	9444	72802	8394	51026	7563	84242	6889	80640	6332	15
46	47835	9425	81196	8379	58589	7550	91131	6878	86972	6323	14
47	57260	9405	89575	8364	66139	7539	98009	6869	93295	6316	13
48	66665	9386	97939	8348	73678	7525	9·2804878	6858	99611	6307	12
49	76051	9366	9·1906287	8334	81203	7514	11736	6849	9·3205918	6298	11
50	85417	9347	14621	8318	88717	7501	18585	6838	12216	6290	10
51	94764	9328	22934	8302	96218	7490	25423	6828	18506	6282	9
52	9·1404092	9308	31241	8288	9·2403708	7477	32251	6819	24788	6273	8
53	13400	9289	39529	8273	11185	7465	39070	6808	31061	6266	7
54	22689	9270	47802	8257	18650	7453	45878	6799	37327	6257	6
55	31959	9251	56059	8243	26103	7440	52677	6789	43584	6248	5
56	41210	9232	64302	8228	33543	7429	59466	6779	49832	6241	4
57	50442	9213	72530	8213	40972	7417	66245	6769	56073	6232	3
58	59655	9194	80743	8198	48389	7405	73014	6759	62305	6224	2
59	68849	9176	88941	8184	55794	7394	79773	6750	68529	6316	1
60	78025		97125		63188		86523		74745		0
′	82°	*diff.*	81°	*diff.*	80°	*diff.*	79°	*diff.*	78°	*diff.*	′

LOG. COTAN.

′	12°	diff.	13°	diff.	14°	diff.	15°	diff.	16°	diff.	′
0	9·3178789	5939	9·3520880	5469	9·3836752	5063	9·4129962	4712	9·4403381	4403	60
1	84728	5931	26349	5461	41815	5058	34674	4707	07784	4398	59
2	90659	5922	31810	5454	46873	5051	39381	4701	12182	4394	58
3	96581	5914	37264	5446	51924	5045	44082	4696	16576	4389	57
4	9·3202495	5905	42710	5440	56969	5039	48778	4690	20965	4384	56
5	08400	5897	48150	5432	62008	5032	52468	4684	25349	4379	55
6	14297	5889	53582	5425	67040	5027	58152	4680	29728	4375	54
7	20186	5880	59007	5419	72067	5020	62832	4674	34103	4369	53
8	26066	5872	64426	5410	77087	5014	67506	4668	38472	4365	52
9	31938	5864	69836	5404	82101	5008	72174	4663	42837	4360	51
10	37802	5855	75240	5397	87109	5002	76837	4658	47197	4356	50
11	43657	5848	80637	5390	92111	4995	81495	4653	51553	4351	49
12	49505	5839	86027	5382	97106	4990	86148	4647	55904	4346	48
13	55344	5830	91409	5376	9·3902096	4983	90795	4641	60250	4341	47
14	61174	5823	96785	5369	07079	4978	95436	4637	64591	4336	46
15	66997	5814	9·3602154	5361	12057	4971	9·4200073	4631	68927	4332	45
16	72811	5806	07515	5355	17028	4965	04704	4626	73259	4327	44
17	78617	5799	12870	5347	21993	4959	09330	4620	77586	4323	43
18	84416	5790	18217	5341	26952	4953	13950	4616	81909	4318	42
19	90206	5782	23558	5334	31905	4947	18566	4610	86227	4313	41
20	95988	5773	28892	5327	36852	4942	23176	4604	90540	4309	40
21	9·3301761	5766	34219	5320	41794	4935	27780	4600	94849	4304	39
22	07527	5758	39539	5313	46729	4929	32380	4594	99153	4299	38
23	13285	5750	44852	5306	51658	4923	36974	4589	9·4503452	4295	37
24	19035	5742	50158	5300	56581	4918	41563	4584	07747	4290	36
25	24777	5734	55458	5292	61499	4911	46147	4579	12037	4285	35
26	30511	5726	60750	5286	66410	4905	50726	4573	16322	4281	34
27	36237	5718	66036	5279	71315	4900	55299	4568	20603	4276	33
28	41955	5710	71315	5272	76215	4894	59867	4563	24879	4272	32
29	47665	5703	76587	5266	81109	4887	64430	4558	29151	4267	31
30	53368	5694	81853	5258	85996	4882	68988	4553	33418	4263	30
31	59062	5687	87111	5252	90878	4876	73541	4548	37681	4258	29
32	64749	5679	92363	5245	95754	4871	78089	4542	41939	4253	28
33	70428	5671	97608	5239	9·4000625	4864	82631	4538	46192	4249	27
34	76099	5663	9·3702847	5232	05489	4859	87169	4532	50441	4245	26
35	81762	5656	08079	5225	10348	4853	91701	4527	54686	4240	25
36	87418	5647	13304	5219	15201	4847	96228	4522	58926	4235	24
37	93065	5641	18523	5212	20048	4841	9·4300750	4517	63121	4231	23
38	98706	5632	23735	5205	24889	4835	05267	4512	67392	4226	22
39	9·3404338	5625	28940	5199	29724	4830	09779	4507	71618	4222	21
40	09963	5617	34139	5192	34554	4824	14286	4502	75840	4218	20
41	15580	5610	39331	5186	39378	4818	18788	4497	80058	4213	19
42	21190	5602	44517	5179	44196	4813	23285	4492	84271	4209	18
43	26792	5594	49696	5172	49009	4807	27777	4487	88480	4204	17
44	32386	5587	54868	5166	53816	4801	32264	4482	92684	4200	16
45	37973	5579	60034	5160	58617	4796	36746	4477	96884	4195	15
46	43552	5572	65194	5153	63413	4790	41223	4471	9·4601079	4191	14
47	49124	5564	70347	5146	68203	4784	45694	4467	05270	4186	13
48	54688	5557	75493	5140	72987	4779	50161	4462	09456	4182	12
49	60245	5549	80633	5134	77766	4773	54623	4457	13638	4178	11
50	65794	5542	85767	5127	82539	4767	59080	4452	17816	4173	10
51	71336	5534	90894	5121	87306	4762	63532	4448	21989	4169	9
52	76870	5527	96015	5114	92068	4756	67980	4442	26158	4165	8
53	82397	5520	9·3801129	5108	96824	4751	72422	4437	30323	4160	7
54	87917	5512	06237	5102	9·4101575	4745	76859	4433	34483	4156	6
55	93429	5505	11339	5095	06320	4739	81292	4427	38639	4151	5
56	98934	5498	16434	5089	11059	4734	85719	4423	42790	4148	4
57	9·3504432	5490	21523	5082	15793	4729	90142	4418	46938	4143	3
58	09922	5483	26605	5077	20522	4723	94560	4413	51081	4138	2
59	15405	5475	31682	5070	25245	4717	98973	4408	55219	4134	1
60	20880		36752		29962		9·4403381		59353		0
′	77°	diff.	76°	diff.	75°	diff.	74°	diff.	73°	diff.	′

LOG. COSINE.

′	12°	*diff.*	13°	*diff.*	14°	*diff.*	15°	*diff.*	16°	*diff.*	′
0	9·3274745	6208	9.3633641	5760	9·3967711	5378	9·4280525	5050	9.4574964	4766	60
1	80953	6200	39401	5754	73089	5374	85575	5046	79730	4761	59
2	87153	6192	45155	5746	78463	5367	90621	5040	84491	4757	58
3	93345	6183	50901	5740	83830	5361	95661	5036	89248	4753	57
4	99528	6176	56641	5733	89191	5356	9·4300697	5030	94001	4748	56
5	9·3305704	6168	62374	5726	94547	5349	05727	5026	98749	4743	55
6	11872	6159	68100	5719	99896	5344	10753	5020	9·4603492	4740	54
7	18031	6152	73819	5713	9·4005240	5338	15773	5016	08232	4735	53
8	24183	6144	79532	5706	10578	5332	20789	5010	12967	4730	52
9	30327	6136	85238	5699	15910	5327	25799	5005	17697	4726	51
10	36463	6128	90937	5692	21237	5321	30804	5001	22423	4722	50
11	42591	6120	96629	5686	26558	5315	35805	4995	27145	4718	49
12	48711	6112	9·3702315	5679	31873	5309	40800	4991	31863	4713	48
13	54823	6104	07994	5673	37192	5304	45791	4985	36576	4709	47
14	60927	6097	13667	5666	42486	5298	50776	4981	41285	4705	46
15	67024	6089	19333	5659	47784	5292	55757	4976	45990	4700	45
16	73113	6081	24992	5653	53076	5287	60733	4971	50690	4696	44
17	79194	6073	30645	5646	58363	5281	65704	4966	55386	4692	43
18	85267	6066	36291	5639	63644	5275	70670	4961	60078	4687	42
19	91333	6058	41930	5633	68919	5270	75631	4956	64765	4683	41
20	97391	6050	47563	5627	74189	5264	80587	4951	69448	4679	40
21	9·3403441	6043	53190	5620	79453	5259	85538	4947	74127	4675	39
22	09484	6035	58810	5613	84712	5253	90485	4941	78802	4671	38
23	15519	6027	64423	5607	89965	5247	95426	4937	83473	4666	37
24	21546	6020	70030	5601	95212	5242	9·4400363	4932	88139	4662	36
25	27566	6012	75631	5594	9·4100454	5236	05295	4927	92801	4658	35
26	33578	6006	81225	4588	05690	5231	10222	4923	97459	4653	34
27	39583	5997	86813	5581	10921	5225	15145	4917	9·4702112	4650	33
28	45580	5990	92394	5575	16146	5220	20062	4913	06762	4645	32
29	51570	5982	97969	5568	21366	5215	24975	4908	11407	4641	31
30	57552	5975	9.3803537	5563	26581	5208	29883	4903	16048	4637	30
31	63527	5967	09100	5555	31789	5204	34786	4899	20685	4633	29
32	69494	5960	14655	5550	36993	5198	39685	4894	25318	4629	28
33	75454	5953	20205	5543	42191	5192	44579	4889	29947	4625	27
34	81407	5945	25748	5537	47383	5187	49468	4884	34572	4620	26
35	87352	5938	31285	5531	52570	5182	54352	4880	39192	4616	25
36	93290	5930	36816	5524	57752	5176	59232	4875	43808	4613	24
37	99220	5923	42340	5518	62928	5171	64107	4871	48421	4608	23
38	9·3505143	5916	47858	5512	68099	5166	68978	4865	53029	4604	22
39	11059	5909	53370	5506	73265	5160	73843	4861	57633	4600	21
40	16968	5901	58876	5500	78425	5155	78704	4857	62233	4596	20
41	22869	5894	64376	5493	83580	5149	83561	4852	66829	4592	19
42	28763	5887	69869	5487	88729	5145	88413	4847	71421	4588	18
43	34650	5880	75356	5481	93874	5139	93260	4842	76009	4583	17
44	40530	5872	80837	5475	99013	5133	98102	4838	80592	4580	16
45	46402	5865	86312	5469	9·4204146	5129	9·4502940	4834	85172	4576	15
46	52267	5859	91781	5463	09275	5123	07774	4828	89748	4571	14
47	58126	5851	97244	5456	14398	5117	12602	4825	94319	4568	13
48	63977	5844	9·3902700	5451	19515	5113	17427	4819	98887	4564	12
49	69821	5837	08151	5444	24628	5107	22246	4815	9·4803451	4560	11
50	75658	5829	13595	5439	29735	5103	27061	4811	08011	4555	10
51	81487	5823	19034	5432	34838	5097	31872	4806	12566	4552	9
52	87310	5816	24466	5427	39935	5091	36678	4801	17118	4548	8
53	93126	5809	29893	5420	45026	5087	41479	4797	21666	4544	7
54	98935	5801	35313	5414	50113	5081	46276	4793	26210	4540	6
55	9·3604736	5795	40727	5409	55194	5077	51069	4788	30750	4536	5
56	10531	5788	46136	5402	60271	5071	55857	4784	35286	4532	4
57	16319	5781	51538	5397	65342	5066	60641	4779	39818	4528	3
58	22100	5774	56935	5391	70408	5061	65420	4774	44346	4524	2
59	27874	5767	62326	5385	75469	5056	70194	4770	48870	4520	1
60	33641		67711		80525		74964		53390		0
′	77°	*diff.*	76°	*diff.*	75°	*diff.*	74°	*diff.*	73°	*diff.*	′

LOG. COTAN.

LOG. SINE.

′	17°	diff.	18°	diff.	19°	diff.	20°	diff.	21°	diff.	′
0	9·4659353	4130	9·4899824	3886	9·5126419	3667	9·5340517	3469	9·5543292	3289	60
1	63483	4126	9·4903710	3882	30086	3664	43986	3466	46581	3287	59
2	67609	4121	07592	3879	33750	3660	47452	3463	49868	3284	58
3	71730	4118	11471	3874	37410	3657	50915	3460	53152	3281	57
4	75848	4112	15345	3871	41067	3654	54375	3457	56433	3278	56
5	79960	4109	19216	3867	44721	3650	57832	3454	59711	3276	55
6	84069	4104	23083	3863	48371	3646	61286	3451	62987	3272	54
7	88173	4100	26946	3860	52017	3643	64737	3447	66259	3270	53
8	92273	4096	30806	3855	55660	3640	68184	3445	69529	3267	52
9	96369	4092	34661	3852	59300	3636	71629	3441	72796	3264	51
10	9·4700461	4087	38513	3848	62936	3633	75070	3438	76060	3261	50
11	04548	4083	42361	3844	66569	3629	78508	3435	79321	3258	49
12	08631	4079	46205	3841	70198	3626	81943	3432	82579	3256	48
13	12710	4075	50046	3837	73824	3623	85375	3429	85835	3253	47
14	16785	4071	53883	3833	77447	3619	88804	3426	89088	3250	46
15	20856	4066	57716	3829	81066	3616	92230	3423	92338	3247	45
16	24922	4063	61545	3825	84682	3613	95653	3420	95585	3244	44
17	28985	4058	65370	3822	88295	3609	99073	3416	98829	3242	43
18	33043	4054	69192	3818	91904	3606	9·5402489	3414	9·5602071	3239	42
19	37097	4049	73010	3814	95510	3602	05903	3411	05310	3236	41
20	41146	4046	76824	3811	99112	3599	09314	3407	08546	3233	40
21	45192	4042	80635	3807	9·5202711	3596	12721	3405	11779	3231	39
22	49234	4037	84442	3803	06307	3592	16126	3401	15010	3227	38
23	53271	4033	88245	3800	09899	3589	19527	3399	18237	3225	37
24	57304	4030	92045	3795	13488	3586	22926	3395	21462	3223	36
25	61334	4025	95840	3793	17074	3582	26321	3392	24685	3219	35
26	65359	4021	99633	3788	20656	3579	29713	3390	27904	3217	34
27	69380	4016	9·5003421	3785	24235	3576	33103	3386	31121	3214	33
28	73396	4013	07206	3781	27811	3572	36489	3384	34335	3211	32
29	77409	4009	10987	3777	31383	3570	39873	3380	37546	3208	31
30	81418	4005	14764	3774	34953	3565	43253	3377	40754	3206	30
31	85423	4000	18538	3770	38518	3563	46630	3375	43960	3203	29
32	89423	3997	22308	3767	42081	3559	50005	3371	47163	3200	28
33	93420	3992	26075	3763	45640	3556	53376	3369	50363	3198	27
34	97412	3989	29838	3759	49196	3553	56745	3365	53561	3195	26
35	9·4801401	3984	33597	3756	52749	3549	60110	3362	56756	3192	25
36	05385	3981	37353	3752	56298	3546	63472	3360	59948	3189	24
37	09366	3976	41105	3748	59844	3543	66832	3357	63137	3187	23
38	13342	3973	44853	3745	63387	3540	70189	3353	66324	3184	22
39	17315	3968	48598	3741	66927	3536	73542	3351	69508	3181	21
40	21283	3965	52339	3738	70463	3534	76893	3347	72689	3179	20
41	25248	3960	56077	3734	73997	3529	80240	3345	75868	3176	19
42	29208	3957	59811	3731	77526	3527	83585	3342	79044	3173	18
43	33165	3952	63542	3727	81053	3524	86927	3339	82217	3170	17
44	37117	3949	67269	3723	84577	3520	90266	3336	85387	3168	16
45	41066	3944	70992	3720	88097	3517	93602	3333	88555	3166	15
46	45010	3941	74712	3716	91614	3514	96935	3330	91721	3162	14
47	48951	3937	78428	3713	95128	3510	9·5500265	3327	94883	3160	13
48	52888	3932	82141	3709	98638	3508	03592	3324	98043	3157	12
49	56820	3929	85850	3706	9·5302146	3504	06916	3321	9·5701200	3155	11
50	60749	3925	89556	3702	05650	3501	10237	3319	04355	3151	10
51	64674	3921	93258	3698	09151	3498	13556	3315	07506	3150	9
52	68595	3917	96956	3695	12649	3494	16871	3313	10656	3146	8
53	72512	3914	9·5100651	3692	16143	3492	20184	3310	13802	3144	7
54	76426	3909	04343	3688	19635	3488	23494	3307	16946	3141	6
55	80335	3905	08031	3685	23123	3485	26801	3304	20087	3139	5
56	84240	3902	11716	3681	26608	3482	30105	3301	23226	3136	4
57	88142	3898	15397	3677	30090	3479	33406	3298	26362	3133	3
58	92040	3894	19074	3675	33569	3475	36704	3295	29495	3131	2
59	95934	3890	22749	3670	37044	3473	39999	3293	32626	3128	1
60	99824		26419		40517		43292		35754		0
′	72°	diff.	71°	diff.	70°	diff.	69°	diff.	68°	diff.	′

LOG. COSINE.

′	17°	*diff.*	18°	*diff.*	19°	*diff.*	20°	*diff.*	21°	*diff.*	′
0	9·4853390	4517	9·5117760	4297	9·5369719	4102	9·5610659	3929	9·5841774	3775	60
1	57907	4512	22057	4294	73821	4099	14588	3927	45549	3772	59
2	62419	4509	26351	4290	77920	4097	18515	3924	49321	3770	58
3	66928	4505	30641	4286	82017	4093	22439	3921	53091	3768	57
4	71433	4500	34927	4283	86110	4090	26360	3918	56859	3765	56
5	75933	4497	39210	4280	90200	4087	30278	3916	60624	3762	55
6	80430	4494	43490	4276	94287	4084	34194	3913	64386	3761	54
7	84924	4489	47766	4273	98371	4082	38107	3911	68147	3757	53
8	89413	4485	52039	4270	9·5402453	4078	42018	3907	71904	3756	52
9	93898	4482	56309	4266	06531	4075	45925	3906	75660	3753	51
10	98380	4478	60575	4263	10606	4072	49831	3902	79413	3750	50
11	9·4902858	4474	64838	4259	14678	4069	53733	3900	83163	3749	49
12	07332	4470	69097	4256	18747	4066	57633	3897	86912	3745	48
13	11802	4467	73353	4253	22813	4064	61530	3894	90657	3744	47
14	16269	4462	77606	4249	26877	4060	65424	3892	94401	3741	46
15	20731	4459	81855	4246	30937	4057	69316	3889	98142	3739	45
16	25190	4456	86101	4243	34994	4054	73205	3886	9·5901881	3736	44
17	29646	4451	90344	4239	39048	4052	77091	3884	05617	3734	43
18	34097	4448	94583	4236	43100	4048	80975	3881	09351	3731	42
19	38545	4443	98819	4233	47148	4045	84856	3879	13082	3730	41
20	42988	4441	9·5203052	4230	51193	4043	88735	3876	16812	3727	40
21	47429	4436	07282	4226	55236	4040	92611	3873	20539	3724	39
22	51865	4433	11508	4222	59276	4036	96484	3871	24263	3722	38
23	56298	4429	15730	4220	63312	4034	9·5700355	3868	27985	3720	37
24	60727	4425	19950	4216	67346	4031	04223	3865	31705	3718	36
25	65152	4422	24166	4213	71377	4028	08088	3863	35423	3715	35
26	69574	4417	28379	4210	75405	4025	11951	3860	39138	3713	34
27	73991	4415	32589	4206	79430	4022	15811	3858	42851	3710	33
28	78406	4410	36795	4204	83452	4019	19669	3855	46561	3708	32
29	82816	4407	40999	4200	87471	4016	23524	3853	50269	3706	31
30	87223	4403	45199	4196	91487	4013	27377	3850	53975	3704	30
31	91626	4400	49395	4194	95500	4011	31227	3847	57679	3701	29
32	96026	4396	53589	4190	99511	4008	35074	3845	61380	3699	28
33	9·5000422	4392	57779	4187	9·5503519	4004	38919	3842	65079	3697	27
34	04814	4389	61966	4184	07523	4002	42761	3840	68776	3694	26
35	09203	4385	66150	4181	11525	3999	46601	3837	72470	3692	25
36	13588	4381	70331	4177	15524	3997	50438	3834	76162	3690	24
37	17969	4378	74508	4174	19521	3993	54272	3832	79852	3688	23
38	22347	4374	78682	4171	23514	3990	58104	3830	83540	3685	22
39	26721	4371	82853	4168	27504	3988	61934	3827	87225	3683	21
40	31092	4367	87021	4165	31492	3985	65761	3824	90908	3680	20
41	35459	4363	91186	4161	35477	3982	69585	3822	94588	3679	19
42	39822	4360	95347	4158	39459	3979	73407	3819	98267	3676	18
43	44182	4356	99505	4156	43438	3977	77226	3817	9·6001943	3674	17
44	48538	4353	9·5303661	4152	47415	3973	81043	3815	05617	3672	16
45	52891	4349	07813	4148	51388	3971	84858	3811	09289	3669	15
46	57240	4346	11961	4146	55359	3968	88669	3810	12958	3667	14
47	61586	4342	16107	4143	59327	3965	92479	3807	16625	3665	13
48	65928	4339	20250	4139	63292	3963	96286	3804	20290	3663	12
49	70267	4335	24389	4137	67255	3959	9·5800090	3802	23953	3660	11
50	74602	4331	28526	4133	71214	3957	03892	3799	27613	3658	10
51	78933	4328	32659	4130	75171	3954	07691	3797	31271	3656	9
52	83261	4325	36789	4127	79125	3952	11488	3794	34927	3654	8
53	87586	4321	40916	4124	83077	3948	15282	3792	38581	3652	7
54	91907	4317	45040	4121	87025	3946	19074	3790	42233	3649	6
55	96224	4315	49161	4117	90971	3943	22864	3787	45882	3647	5
56	9·5100539	4310	53278	4115	94914	3940	26651	3784	49529	3645	4
57	04849	4307	57393	4112	98854	3938	30435	3782	53174	3643	3
58	09156	4304	61505	4108	9·5602792	3935	34217	3780	56817	3640	2
59	13460	4300	65613	4106	06727	3932	37997	3777	60457	3639	1
60	17760		69719		10659		41774		64096		0
′	72°	*diff.*	71°	*diff.*	70°	*diff.*	69°	*diff.*	68°	*diff.*	′

LOG. COTAN.

′	22°	*diff.*	23°	*diff.*	24°	*diff.*	25°	*diff.*	26°	*diff.*	′
0	9·5735754	3126	9·5918780	2975	9·6093133	2836	9·6259483	2708	9·6418420	2589	60
1	8880	3123	9·5921755	2973	5969	2834	9·6262191	2706	9·6421009	2587	59
2	9·5742003	3120	4728	2970	8803	2832	4897	2704	3596	2586	58
3	5123	3117	7698	2968	9·6101635	2830	7601	2702	6182	2583	57
4	8240	3116	9·5930666	2965	4465	2828	9·6270303	2700	8765	2582	56
5	9·5751356	3112	3631	2963	7293	2825	3003	2698	9·6431347	2579	55
6	4468	3110	6594	2961	9·6110118	2823	5701	2696	3926	2578	54
7	7578	3107	9555	2958	2941	2821	8397	2693	6504	2576	53
8	9·5760685	3105	9·5942513	2956	5762	2818	9·6281090	2692	9080	2574	52
9	3790	3102	5469	2953	8580	2817	3782	2690	9·6441654	2572	51
10	6892	3099	8422	2951	9·6121397	2814	6472	2688	4226	2570	50
11	9991	3097	9·5951373	2949	4211	2812	9160	2685	6796	2569	49
12	9·5773088	3095	4322	2946	7023	2810	9·6291845	2684	9365	2566	48
13	6183	3092	7268	2944	9833	2808	4529	2682	9·6451931	2565	47
14	9275	3089	9·5960212	2942	9·6132641	2805	7211	2679	4496	2562	46
15	9·5782364	3086	3154	2939	5446	2804	9890	2678	7058	2561	45
16	5450	3085	6093	2937	8250	2801	9·6302568	2675	9619	2559	44
17	8535	3081	9030	2935	9·6141051	2799	5243	2674	9·6462178	2557	43
18	9·5791616	3079	9·5971965	2932	3850	2797	7917	2672	4735	2555	42
19	4695	3077	4897	2930	6647	2794	9·6310589	2669	7290	2554	41
20	7772	3073	7827	2927	9441	2793	3258	2668	9844	2551	40
21	9·5800845	3072	9·5980754	2925	9·6152234	2790	5926	2665	9·6472395	2550	39
22	3917	3069	3679	2923	5024	2788	8591	2664	4945	2547	38
23	6986	3066	6602	2921	7812	2787	9·6321255	2661	7492	2546	37
24	9·5810052	3064	9523	2918	9·6160599	2783	3916	2660	9·6480038	2544	36
25	3116	3061	9·5992441	2916	3382	2782	6576	2657	2582	2542	35
26	6177	3059	5357	2913	6164	2780	9233	2656	5124	2541	34
27	9236	3056	8270	2911	8944	2777	9·6331889	2653	7665	2538	33
28	9·5822292	3053	9·6001181	2909	9·6171721	2775	4542	2652	9·6490203	2537	32
29	5345	3052	4090	2907	4496	2774	7194	2650	2740	2534	31
30	8397	3048	6997	2904	7270	2771	9844	2647	5274	2533	30
31	9·5831445	3046	9901	2902	9·6180041	2768	9·6342491	2646	7807	2531	29
32	4491	3044	9·6012803	2900	2809	2767	5137	2643	9·6500338	2530	28
33	7535	3041	5703	2897	5576	2765	7780	2642	2868	2527	27
34	9·5840576	3039	8600	2895	8341	2762	9·6350422	2640	5395	2525	26
35	3615	3036	9·6021495	2893	9·6191103	2761	3062	2637	7920	2524	25
36	6651	3034	4388	2890	3864	2758	5699	2636	9·6510444	2522	24
37	9685	3031	7278	2888	6622	2756	8335	2634	2966	2520	23
38	9·5852716	3029	9·6030166	2886	9378	2754	9·6360969	2632	5486	2518	22
39	5745	3026	3052	2884	9·6202132	2752	3601	2630	8004	2517	21
40	8771	3024	5936	2881	4884	2750	6231	2628	9·6520521	2514	20
41	9·5861795	3021	8817	2879	7634	2748	8859	2625	3035	2513	19
42	4816	3019	9·6041696	2877	9·6210382	2745	9·6371484	2624	5548	2511	18
43	7835	3016	4573	2875	3127	2744	4108	2623	8059	2509	17
44	9·5870851	3014	7448	2872	5871	2741	6731	2620	9·6530568	2507	16
45	3865	3011	9·6050320	2870	8612	2739	9351	2618	3075	2506	15
46	6876	3009	3190	2867	9·6221351	2737	9·6381969	2616	5581	2503	14
47	9885	3007	6057	2866	4088	2736	4585	2614	8084	2502	13
48	9·5882892	3004	8923	2863	6824	2733	7199	2613	9·6540586	2500	12
49	5896	3001	9·6061786	2861	9557	2730	9812	2610	3086	2498	11
50	8897	3000	4647	2859	9·6232287	2729	9·6392422	2608	5584	2497	10
51	9·5891897	2996	7506	2856	5016	2727	5030	2607	8081	2494	9
52	4893	2995	9·6070362	2854	7743	2725	7637	2604	9·6550575	2493	8
53	7888	2992	3216	2852	9·6240468	2722	9·6400241	2603	3068	2491	7
54	9·5900880	2989	6068	2850	3190	2721	2844	2601	5559	2489	6
55	3869	2987	8918	2847	5911	2718	5445	2599	8048	2488	5
56	6856	2985	9·6081765	2846	8629	2717	8044	2596	9·6560536	2485	4
57	9841	2982	4611	2843	9·6251346	2714	9·6410640	2595	3021	2484	3
58	9·5912823	2980	7454	2840	4060	2712	3235	2593	5505	2482	2
59	5803	2977	9·6090294	2839	6772	2711	5828	2592	7987	2481	1
60	8780		3133		9483		8420		9·6570468		0
′	67°	*diff.*	66°	*diff.*	65°	*diff.*	64°	*diff.*	63°	*diff.*	′

LOG. COSINE.

′	22°	diff.	23°	diff.	24°	diff.	25°	diff.	26°	diff.	′
0	9·6064096	3636	9·6278519	3512	9·6485831	3399	9·6686725	3298	9·6881819	3205	60
1	7732	3634	9·6282031	3509	9230	3398	9·6690023	3296	5023	3204	59
2	9·6071366	3631	5540	3508	9·6492628	3395	3319	3294	8227	3203	58
3	4997	3630	9048	3505	6023	3394	6613	3293	9·6891430	3201	57
4	8627	3627	9·6292553	3504	9417	3392	9906	3291	4631	3200	56
5	9·6082254	3626	6057	3501	9·6502809	3390	9·6703197	3239	7831	3199	55
6	5880	3623	9558	3500	6199	3388	6486	3288	9·6901030	3196	54
7	9503	3621	9·6303058	3498	9587	3387	9774	3286	4226	3196	53
8	9·6093124	3618	6556	3496	9·6512974	3385	9·6713060	3285	7422	3194	52
9	6742	3617	9·6310052	3493	6359	3383	6345	3283	9·6910616	3193	51
10	9·6100359	3614	3545	3492	9742	3381	9628	3282	3809	3191	50
11	3973	3613	7037	3490	9·6523123	3380	9·6722910	3280	7000	3189	49
12	7586	3610	9·6320527	3488	6503	3378	6190	3278	9·6920189	3189	48
13	9·6111196	3608	4015	3486	9881	3376	9468	3277	3378	3187	47
14	4804	3605	7501	3484	9·6533257	3374	9·6732745	3275	6565	3185	46
15	8409	3604	9·6330985	3483	6631	3373	6020	3274	9750	3184	45
16	9·6122013	3602	4468	3480	9·6540004	3371	9294	3272	9·6932934	3183	44
17	5615	3599	7948	3478	3375	3369	9·6742566	3270	6117	3181	43
18	9214	3598	9·6341426	3477	6744	3368	5836	3269	9298	3180	42
19	9·6132812	3595	4903	3475	9·6550112	3365	9105	3267	9·6942478	3178	41
20	6407	3593	8378	3472	3477	3364	9·6752372	3266	5656	3177	40
21	9·6140000	3591	9·6351850	3471	6841	3363	5638	3265	8833	3176	39
22	3591	3539	5321	3469	9·6560204	3360	8903	3262	9·6952009	3174	38
23	7180	3586	8790	3467	3564	3359	9·6762165	3261	5183	3172	37
24	9·6150766	3585	9·6362257	3465	6923	3357	5426	3260	8355	3172	36
25	4351	3583	5722	3463	9·6570280	3356	8686	3258	9·6961527	3170	35
26	7934	3580	9185	3461	3636	3353	9·6771644	3257	4697	3168	34
27	9·6161514	3579	9·6372646	3460	6989	3352	5201	3255	7865	3167	33
28	5093	3576	6106	3457	9·6580341	3351	8456	3253	9·6971032	3166	32
29	8669	3574	9563	3456	3692	3349	9·6781709	3252	4198	3165	31
30	9·6172243	3572	9·6383019	3454	7041	3346	4961	3250	7363	3163	30
31	5815	3570	6473	3452	9·6590387	3346	8211	3249	9·6980526	3161	29
32	9385	3568	9925	3450	3733	3343	9·6791460	3248	3687	3160	28
33	9·6182953	3566	9·6393375	3448	7076	3342	4708	3245	6847	3159	27
34	6519	3564	6823	3446	9·6600418	3340	7953	3245	9·6990006	3158	26
35	9·6190083	3562	9·6400269	3445	3758	3339	9·6801198	3242	3164	3156	25
36	3645	3560	3714	3442	7097	3337	4440	3242	6320	3154	24
37	7205	3557	7156	3441	9·6610434	3335	7682	3239	9474	3154	23
38	9·6200762	3556	9·6410597	3439	3769	3334	9·6810921	3239	9·7002628	3152	22
39	4318	3554	4036	3437	7103	3331	4160	3236	5780	3150	21
40	7872	3451	7473	3435	9·6620434	3331	7396	3236	8930	3150	20
41	9·6211423	3550	9·6420908	3434	3765	3328	9·6820632	3233	9·7012080	3147	19
42	4973	3547	4342	3431	7093	3327	3865	3233	5227	3147	18
43	8520	3546	7773	3430	9·6630420	3325	7098	3230	8374	3145	17
44	9·6222066	3543	9·6431203	3428	3745	3324	9·6830328	3229	9·7021519	3144	16
45	5609	3541	4631	3426	7069	3322	3557	3228	4663	3142	15
46	9150	3540	8057	3424	9·6640391	3320	6785	3226	7805	3141	14
47	9·6232690	3537	9·6441491	3422	3711	3319	9·6840011	3225	9·7030946	3140	13
48	6227	3536	4903	3421	7030	3316	3236	3223	4086	3139	12
49	9763	3533	8324	3419	9·6650346	3316	6459	3222	7225	3137	11
50	9·6243296	3531	9·6451743	3417	3662	3313	9681	3220	9·7040362	3135	10
51	6827	3529	5160	3415	6975	3313	9·6852901	3219	3497	3135	9
52	9·6250356	3528	8575	3413	9·6660288	3310	6120	3218	6632	3133	8
53	3884	3525	9·6461988	3412	3596	3309	9338	3215	9765	3132	7
54	7409	3523	5400	3410	6907	3307	9·6862553	3215	9·7052897	3130	6
55	9·6260932	3522	8810	3407	9·6670214	3305	5768	3213	6027	3129	5
56	4454	3519	9·6472217	3407	3519	3304	8981	3211	9156	3128	4
57	7973	3518	5624	3404	6823	3303	9·6872192	3210	9·7062284	3126	3
58	9·6271491	3515	9028	3403	9·6680126	3300	5402	3209	5410	3125	2
59	5006	3513	9·6482431	3400	3426	3299	8611	3207	8535	3124	1
60	8519		5831		6725		9·6881818		9·7071659		0
′	67°	diff.	66°	diff.	65°	diff.	64°	diff.	63°	diff.	′

LOG. COTAN.

′	27°	diff.	28°	diff.	29°	diff.	30°	diff.	31°	diff.	′
0	9·6570468	2478	9·6716093	2375	9·6855712	2279	9·6989700	2187	9·7118393	2102	60
1	2946	2477	8468	2373	7991	2276	9·6991887	2186	9·7120495	2101	59
2	5423	2475	9·6720841	2372	9·6860267	2275	4073	2185	2596	2099	58
3	7898	2473	3213	2370	2542	2274	6258	2183	4695	2097	57
4	9·6580371	2471	5583	2369	4816	2272	8441	2181	6792	2097	56
5	2842	2470	7952	2367	7088	2271	9·7000622	2180	8889	2094	55
6	5312	2468	9·6730319	2365	9359	2269	2802	2179	9·7130983	2094	54
7	7780	2466	2684	2363	9·6871628	2267	4981	2177	3077	2092	53
8	9·6590246	2464	5047	2362	3895	2266	7158	2176	5169	2091	52
9	2710	2463	7409	2360	6161	2264	9334	2174	7260	2089	51
10	5173	2460	9769	2359	8425	2263	9·7011508	2173	9349	2088	50
11	7633	2460	9·6742128	2357	9·6880688	2261	3681	2171	9·7141437	2087	49
12	9·6600093	2457	4485	2355	2949	2260	5852	2170	3524	2085	48
13	2550	2455	6840	2354	5209	2258	8022	2168	5609	2084	47
14	5005	2454	9194	2352	7467	2256	9·7020190	2167	7693	2083	46
15	7459	2452	9·6751546	2350	9723	2255	2357	2166	9776	2081	45
16	9911	2450	3896	2349	9·6891678	2254	4523	2164	9·7151857	2080	44
17	9·6612361	2449	6245	2347	4232	2252	6687	2162	3937	2078	43
18	4810	2447	8592	2345	6484	2250	8849	2162	6015	2077	42
19	7257	2445	9·6760937	2344	8734	2249	9·7031011	2159	8092	2076	41
20	9702	2443	3281	2342	9·6900983	2248	3170	2159	9·7160168	2075	40
21	9·6622145	2441	5623	2340	3231	2245	5329	2157	2243	2073	39
22	4586	2440	7963	2339	5476	2245	7486	2155	4316	2071	38
23	7026	2438	9·6770302	2338	7721	2243	9641	2154	6387	2071	37
24	9464	2436	2640	2335	9964	2241	9·7041795	2152	8458	2068	36
25	9·6631900	2435	4975	2334	9·6912205	2240	3947	2152	9·7170526	2068	35
26	4335	2433	7309	2333	4445	2238	6099	2149	2594	2066	34
27	6768	2431	9642	2330	6683	2236	8248	2149	4660	2065	33
28	9199	2429	9·6781972	2329	8919	2236	9·7050397	2146	6725	2064	32
29	9·6641628	2428	4301	2328	9·6921155	2233	2543	2146	8789	2062	31
30	4056	2426	6629	2326	3388	2232	4689	2144	9·7180851	2061	30
31	6482	2424	8955	2324	5620	2231	6833	2142	2912	2059	29
32	8906	2423	9·6791279	2323	7851	2229	8975	2141	4971	2059	28
33	9·6651329	2420	3602	2321	9·6930080	2228	9·7061116	2140	7030	2056	27
34	3749	2419	5923	2320	2308	2226	3256	2138	9086	2056	26
35	6168	2418	8243	2317	4534	2224	5394	2137	9·7191142	2054	25
36	8586	2415	9·6800560	2317	6758	2223	7531	2136	3196	2053	24
37	9·6661001	2414	2877	2314	8981	2222	9667	2134	5249	2051	23
38	3415	2413	5191	2313	9·6941203	2220	9·7071801	2132	7300	2050	22
39	5828	2410	7504	2312	3423	2219	3933	2131	9350	2049	21
40	8238	2409	9816	2310	5642	2217	6064	2130	9·7201399	2048	20
41	9·6670647	2407	9·6812126	2308	7859	2215	8194	3129	3447	2046	19
42	3054	2405	4434	2307	9·6950074	2214	9·7080323	2127	5493	2045	18
43	5459	2404	6741	2305	2288	2213	2450	2125	7538	2043	17
44	7863	2402	9046	2303	4501	2211	4575	2124	9581	2042	16
45	9·6680265	2400	9·6821349	2302	6712	2210	6699	2123	9·7211623	2041	15
46	2665	2399	3651	2301	8922	2208	8822	2121	3664	2040	14
47	5064	2397	5952	2298	9·6961130	2206	9·7090943	2120	5704	2038	13
48	7461	2395	8250	2298	3336	2205	3063	2119	7742	2037	12
49	9856	2394	9·6830548	2295	5541	2204	5182	2117	9779	2035	11
50	9·6692250	2392	2843	2294	7745	2202	7299	2116	9·7221814	2034	10
51	4642	2390	5137	2293	9947	2201	9415	2114	3848	2033	9
52	7032	2388	7430	2290	9·6972148	2199	9·7101529	2113	5881	2032	8
53	9420	2387	9720	2290	4347	2198	3642	2111	7913	2030	7
54	9·6701807	2385	9·6842010	2287	6545	2196	5753	2110	9943	2029	6
55	4192	2384	4297	2286	8741	2195	7863	2109	9·7231972	2028	5
56	6576	2382	6583	2285	9·6980936	2193	9972	2108	4000	2026	4
57	8958	2380	8868	2283	3129	2192	9·7112080	2106	6026	2025	3
58	9·6711338	2378	9·6851151	2281	5321	2190	4186	2104	8051	2024	2
59	3716	2377	3432	2280	7511	2189	6290	2103	9·7240075	2022	1
60	6093		5712		9700		8393		2097		0
′	62°	diff.	61°	diff.	60°	diff.	59°	diff.	58°	diff.	′

LOG. COSINE.

′	27°	*diff.*	28°	*diff.*	29°	*diff.*	30°	*diff.*	31°	*diff.*	′
0	9·7071659	3122	9·7256744	3047	9·7437520	2979	9·7614394	2917	9·7787737	2862	60
1	4781	3121	9791	3046	9 7440499	2977	7311	2916	9·7790599	2860	59
2	7902	3120	9·7262837	3044	3476	2977	9·7620227	2915	3459	2859	58
3	9·7081022	3119	5881	3044	6453	2975	3142	2914	6318	2859	57
4	4141	3117	8925	3042	9428	2975	6056	2913	9177	2857	56
5	7258	3116	9·7271967	3041	9·7452403	2973	8969	2912	9·7802034	2857	55
6	9·7090374	3114	5008	3040	5376	2973	9·7631881	2911	4891	2856	54
7	3488	3113	8048	3039	8349	2971	4792	2910	7747	2855	53
8	6601	3112	9·7281087	3037	9·7461320	2970	7702	2910	9·7810602	2854	52
9	9713	3111	4124	3037	4290	2969	9·7640612	2908	3456	2853	51
10	9·7102824	3109	7161	3035	7259	2968	3520	2907	6309	2853	50
11	5933	3108	9·7290196	3034	9·7470227	2967	6427	2907	9162	2851	49
12	9041	3107	3230	3033	3194	2966	9334	2905	9·7822013	2851	48
13	9·7112148	3106	6263	3032	6160	2965	9·7652239	2904	4864	2849	47
14	5254	3104	9295	3030	9125	2964	5143	2904	7713	2849	46
15	8358	3103	9·7302325	3029	9·7482089	2963	8047	2902	9·7830562	2848	45
16	9·7121461	3101	5354	3029	5052	2961	9·7660949	2902	3410	2848	44
17	4562	3100	8383	3027	8013	2961	3851	2900	6258	2846	43
18	7662	3099	9·7311410	3026	9·7490974	2960	6751	2900	9104	2845	42
19	9·7130761	3098	4436	3024	3934	2958	9651	2899	9·7841949	2845	41
20	3859	3097	7460	3024	6892	2958	9·7672550	2898	4794	2844	40
21	6956	3095	9·7320484	3022	9850	2956	5448	2896	7638	2843	39
22	9·7140051	3094	3506	3021	9·7502806	2956	8344	2896	9·7850481	2842	38
23	3145	3092	6527	3020	5762	2954	9·7681240	2895	3323	2841	37
24	6237	3092	9547	3019	8716	2953	4135	2894	6164	2840	36
25	9329	3090	9·7332566	3018	9·7511669	2953	7029	2893	9004	2840	35
26	9·7152419	3089	5584	3017	4622	2951	9922	2892	9·7861844	2838	34
27	5508	3087	8601	3015	7573	2950	9·7692814	2891	4682	2838	33
28	8595	3087	9·7341616	3015	9·7520523	2949	5705	2891	7520	2837	32
29	9·7161682	3085	4631	3013	3472	2948	8596	2889	9·7870357	2836	31
30	4767	3084	7644	3012	6420	2948	9·7701485	2888	3193	2835	30
31	7851	3082	9·7350656	3011	9368	2946	4373	2888	6028	2835	29
32	9·7170933	3081	3667	3010	9·7532314	2945	7261	2886	8863	2833	28
33	4014	3080	6677	3008	5259	2944	9·7710147	2886	9·7881696	2833	27
34	7094	3079	9685	3008	8203	2943	3033	2884	4529	2832	26
35	9·7180173	3078	9·7362693	3006	9·7541146	2942	5917	2884	7361	2831	25
36	3251	3076	5699	3006	4088	2941	8801	2883	9·7890192	2831	24
37	6327	3075	8705	3004	7029	2940	9·7721684	2882	3023	2829	23
38	9402	3074	9·7371709	3003	9969	2939	4566	2881	5852	2829	22
39	9·7192476	3073	4712	3002	9·7552908	2938	7447	2880	8681	2827	21
40	5549	3071	7714	3001	5846	2937	9·7730327	2879	9·7901508	2827	20
41	8620	3070	9·7380715	2999	8783	2935	3206	2878	4335	2826	19
42	9·7201690	3069	3714	2999	9·7561718	2935	6084	2877	7161	2826	18
43	4759	3068	6713	2997	4653	2934	8961	2877	9987	2824	17
44	7827	3066	9710	2997	7587	2933	9·7741838	2875	9·7912811	2824	16
45	9·7210893	3065	9·7392707	2995	9·7570520	2932	4713	2875	5635	2823	15
46	3958	3064	5702	2994	3452	2931	7588	2874	8458	2822	14
47	7022	3063	8696	2993	6383	2930	9·7750462	2872	9·7921280	2821	13
48	9·7220085	3062	9·7401689	2992	9313	2929	3334	2872	4101	2820	12
49	3147	3060	4681	2991	9·7582242	2928	6206	2871	6921	2820	11
50	6207	3059	7672	2990	5170	2926	9077	2870	9741	2819	10
51	9266	3058	9·7410662	2988	8096	2926	9·7761947	2869	9·7932560	2818	9
52	9·7232324	3057	3650	2988	9·7591022	2925	4816	2869	5378	2817	8
53	5391	3055	6638	2986	3947	2924	7685	2867	8195	2816	7
54	8436	3054	9624	2985	6871	2923	9·7770552	2866	9·7941011	2816	6
55	9·7241490	3053	9·7422609	2985	9794	2922	3418	2866	3827	2814	5
56	4543	3052	5594	2983	9·7602716	2921	6284	2865	6641	2814	4
57	7595	3051	8577	2982	5637	2920	9149	2863	9455	2813	3
58	9·7250646	3049	9·7431559	2981	8557	2919	9·7782012	2863	9·7952268	2813	2
59	3695	3049	4540	2980	9·7611476	2918	4875	2862	5081	2811	1
60	6744		7520		4394		7737		7892		0
′	62°	*diff.*	61°	*diff.*	60°	*diff.*	59°	*diff.*	58°	*diff.*	′

LOG. COTAN.

′	32°	*diff.*	33°	*diff.*	34°	*diff.*	35°	*diff.*	36°	*diff.*	′
0	9·7242097	2021	9·7361088	1944	9·7475617	1872	9·7585913	1804	9·7692187	1738	60
1	4118	2020	3032	1944	7489	1871	7717	1802	3925	1737	59
2	6138	2018	4976	1942	9360	1870	9519	1802	5662	1736	58
3	8156	2018	6918	1941	9·7481230	1869	9·7591321	1800	7398	1736	57
4	9·7250174	2015	8859	1940	3099	1868	3121	1799	9134	1734	56
5	2189	2015	9·7370799	1938	4967	1866	4920	1798	9·7700868	1733	55
6	4204	2013	2737	1938	6833	1865	6718	1797	2601	1731	54
7	6217	2012	4675	1936	8698	1864	8515	1796	4332	1731	53
8	8229	2011	6611	1935	9·7490562	1863	9·7600311	1795	6063	1730	52
9	9·7260240	2009	8546	1933	2425	1862	2106	1793	7793	1729	51
10	2249	2008	9·7380479	1933	4287	1861	3899	1793	9522	1727	50
11	4257	2007	2412	1931	6148	1859	5692	1791	9·7711249	1727	49
12	6264	2005	4343	1930	8007	1859	7483	1791	2976	1726	48
13	8269	2004	6273	1928	9866	1857	9274	1789	4702	1724	47
14	9·7270273	2003	8201	1928	9·7501723	1856	9·7611063	1788	6426	1724	46
15	2276	2002	9·7390129	1926	3579	1855	2851	1787	8150	1722	45
16	4278	2000	2055	1925	5434	1853	4638	1786	9872	1721	44
17	6278	1999	3980	1924	7287	1853	6424	1784	9·7721593	1721	43
18	8277	1998	5904	1923	9140	1851	8208	1784	3314	1719	42
19	9·7280275	1996	7827	1921	9·7510991	1851	9992	1783	5033	1718	41
20	2271	1996	9748	1920	2842	1849	9·7621775	1781	6751	1717	40
21	4267	1993	9·7401668	1919	4691	1847	3556	1781	8468	1717	39
22	6260	1993	3587	1918	6538	1847	5337	1779	9·7730185	1715	38
23	8253	1991	5505	1916	8385	1846	7116	1778	1900	1714	37
24	9·7290244	1990	7421	1916	9·7520231	1844	8894	1777	3614	1713	36
25	2234	1989	9337	1914	2075	1844	9·7630671	1776	5327	1712	35
26	4223	1988	9·7411251	1913	3919	1842	2447	1775	7039	1710	34
27	6211	1986	3164	1911	5761	1841	4222	1774	8749	1710	33
28	8197	1985	5075	1911	7602	1840	5996	1773	9·7740459	1709	32
29	9·7300182	1983	6986	1909	9442	1838	7769	1771	2168	1708	31
30	2165	1983	8895	1908	9·7531280	1838	9540	1771	3876	1707	30
31	4148	1981	9·7420803	1907	3118	1836	9·7641311	1769	5583	1705	29
32	6129	1980	2710	1906	4954	1836	3080	1769	7288	1705	28
33	8109	1978	4616	1904	6790	1834	4849	1767	8993	1704	27
34	9·7310087	1977	6520	1803	8624	1833	6616	1766	9·7750697	1702	26
35	2064	1976	8423	1902	9·7540457	1831	8382	1765	2399	1702	25
36	4040	1975	9·7430325	1901	2288	1831	9·7650147	1764	4101	1700	24
37	6015	1974	2226	1900	4119	1830	1911	1763	5801	1700	23
38	7989	1972	4126	1898	5949	1828	3674	1762	7501	1698	22
39	9961	1971	6024	1897	7777	1827	5436	1761	9199	1698	21
40	9·7321932	1970	7921	1896	9604	1827	7197	1760	9·7760897	1696	20
41	3902	1968	9817	1895	9·7551431	1825	8957	1758	2593	1696	19
42	5870	1967	9·7441712	1894	3256	1824	9·7660715	1758	4289	1694	18
43	7837	1966	3606	1892	5080	1822	2473	1756	5983	1693	17
44	9803	1965	5498	1892	6902	1822	4229	1756	7676	1693	16
45	9·7331768	1963	7390	1890	8724	1820	5985	1754	9369	1691	15
46	3731	1962	9280	1889	9·7560544	1820	7739	1753	9·7771060	1690	14
47	5693	1961	9·7451169	1887	2364	1818	9492	1752	2750	1689	13
48	7654	1960	3056	1887	4182	1817	9·7671244	1752	4439	1689	12
49	9614	1958	4943	1885	5999	1816	2996	1750	6128	1687	11
50	9·7341572	1957	6828	1884	7815	1815	4746	1748	7815	1686	10
51	3529	1956	8712	1883	9630	1814	6494	1748	9501	1685	9
52	5485	1955	9·7460595	1882	9·7571444	1812	8242	1747	9·7781186	1684	8
53	7440	1953	2477	1881	3256	1812	9989	1746	2870	1683	7
54	9393	1952	4358	1879	5068	1810	9·7681735	1745	4553	1682	6
55	9·7351345	1951	6237	1878	6878	1809	3480	1743	6235	1681	5
56	3296	1950	8115	1877	8687	1808	5223	1743	7916	1680	4
57	5246	1949	9992	1876	9·7580495	1807	6966	1441	9596	1679	3
58	7195	1947	9·7471868	1875	2302	1806	8707	1741	9·7791275	1678	2
59	9142	1946	3743	1874	4108	1805	9·7690448	1739	2953	1677	1
60	9·7361088		5617		5913		2187		4630		0
′	57°	*diff.*	56°	*diff.*	55°	*diff.*	54°	*diff*	53°	*diff.*	′

LOG. COSINE.

′	32°	*diff.*	33°	*diff.*	34°	*diff.*	35°	*diff.*	36°	*diff.*	′
0	9·7957892	2811	9·8125174	2765	9·8289874	2725	9·8452268	2688	9·8612610	2657	60
1	9·7960703	2810	7939	2765	9·8292599	2724	4956	2688	5267	2656	59
2	3513	2809	9·8130704	2764	5323	2724	7644	2688	7923	2655	58
3	6322	2808	3468	2763	8047	2722	9·8460332	2686	9·8620578	2655	57
4	9130	2808	6231	2762	9·8300769	2723	3018	2687	3233	2654	56
5	9·7971938	2807	8993	2762	3492	2721	5705	2685	5887	2654	55
6	4745	2806	9·8141755	2761	6213	2721	8390	2685	8541	2654	54
7	7551	2805	4516	2761	8934	2720	9·8471075	2685	9·8631195	2653	53
8	9·7980356	2804	7277	2759	9·8311654	2720	3760	2684	3848	2652	52
9	3160	2804	9·8150036	2759	4374	2719	6444	2683	6500	2652	51
10	5964	2803	2795	2759	7093	2718	9127	2683	9152	2651	50
11	8767	2802	5554	2757	9811	2718	9·8481810	2682	9·8641803	2651	49
12	9·7991569	2801	8311	2757	9·8322529	2717	4492	2682	4454	2651	48
13	4370	2800	9·8161068	2756	5246	2717	7174	2681	7105	2650	47
14	7170	2800	3824	2756	7963	2716	9855	2681	9755	2649	46
15	9970	2799	6580	2755	9·8330679	2715	9·8492536	2680	9·8652404	2649	45
16	9·8002769	2798	9335	2754	3394	2715	5216	2680	5053	2649	44
17	5567	2798	9·8172089	2753	6109	2714	7896	2679	7702	2648	43
18	8365	2796	4842	2753	8823	2713	9·8500575	2678	9·8660350	2647	42
19	9·8011161	2796	7595	2752	9·8341536	2713	3253	2678	2997	2647	41
20	3957	2795	9·8180347	2751	4249	2712	5931	2677	5644	2647	40
21	6752	2794	3098	2751	6961	2712	8608	2677	8291	2646	39
22	9546	2794	5849	2750	9673	2711	9·8511285	2676	9·8670937	2646	38
23	9·8022340	2793	8599	2749	9·8352384	2710	3961	2676	3583	2645	37
24	5133	2792	9·8191348	2748	5094	2710	6637	2675	6228	2645	36
25	7925	2791	4096	2748	7804	2709	9312	2675	8873	2644	35
26	9·8030716	2790	6844	2748	9·8360513	2708	9·8521987	2674	9·8681517	2643	34
27	3506	2790	9592	2746	3221	2708	4661	2674	4160	2644	33
28	6296	2789	9·8202338	2746	5929	2707	7335	2673	6804	2642	32
29	9085	2788	5084	2745	8636	2707	9·8530008	2672	9446	2643	31
30	9·8041873	2788	7829	2745	9·8371343	2706	2680	2672	9·8692089	2642	30
31	4661	2786	9·8210574	2743	4049	2706	5352	2671	4731	2641	29
32	7447	2786	3317	2743	6755	2705	8023	2671	7372	2641	28
33	9·8050233	2786	6060	2743	9460	2704	9·8540694	2671	9·8700013	2640	27
34	3019	2784	8803	2742	9·8382164	2703	3365	2669	2653	2640	26
35	5803	2784	9·8221545	2741	4867	2704	6034	2670	5293	2640	25
36	8587	2783	4286	2740	7571	2702	8704	2668	7933	2639	24
37	9·8061370	2782	7026	2740	9·8390273	2702	9·8551372	2669	9·8710572	2638	23
38	4152	2781	9766	2739	2975	2701	4041	2667	3210	2638	22
39	6933	2781	9·8232505	2739	5676	2701	6708	2668	5848	2638	21
40	9714	2780	5244	2737	8377	2700	9376	2666	8486	2637	20
41	9·8072494	2779	7981	2738	9·8401077	2699	9·8562042	2666	9·8721123	2637	19
42	5273	2779	9·8240719	2736	3776	2699	4708	2666	3760	2636	18
43	8052	2777	3455	2736	6475	2699	7374	2665	6396	2636	17
44	9·8080829	2777	6191	2735	9174	2697	9·8570039	2665	9032	2636	16
45	3606	2777	8926	2734	9·8411871	2698	2704	2664	9·8731668	2634	15
46	6383	2775	9·8251660	2734	4569	2696	5368	2663	4302	2635	14
47	9158	2775	4394	2733	7265	2696	8031	2663	6937	2634	13
48	9·8091933	2774	7127	2733	9961	2696	9·8580694	2663	9571	2633	12
49	4707	2773	9860	2732	9·8422657	2694	3357	2662	9·8742204	2634	11
50	7480	2773	9·8262592	2731	5351	2695	6019	2661	4838	2632	10
51	9·8100253	2772	5323	2730	8046	2693	8680	2661	7470	2632	9
52	3025	2771	8053	2730	9·8430739	2693	9·8591341	2661	9·8750102	2632	8
53	5796	2770	9·8270783	2730	3432	2693	4002	2659	2734	2631	7
54	8566	2770	3513	2728	6125	2692	6661	2660	5365	2631	6
55	9·8111336	2769	6241	2728	8817	2691	9321	2659	7996	2631	5
56	4105	2768	8969	2727	9·8441508	2691	9·8601980	2658	9·8760627	2630	4
57	6873	2768	9·8281696	2727	4199	2690	4638	2658	3257	2629	3
58	9641	2767	4423	2726	6889	2690	7296	2658	5886	2629	2
59	9·8122408	2766	7149	2725	9579	2689	9954	2656	8515	2629	1
60	5174		9874		9·8452268		9·8612610		9·8771144		0
′	57°	*diff.*	56°	*diff.*	55°	*diff.*	54°	*diff.*	53°	*diff.*	′

LOG. COTAN.

′	37°	*diff.*	38°	*diff.*	39°	*diff.*	40°	*diff.*	41°	*diff.*	′
0	9·7794630	1676	9·7893420	1616	9·7988718	1560	9·8080675	1505	9·8169429	1453	60
1	6306	1675	5036	1616	9·7990278	1558	2180	1504	9·8170882	1452	59
2	7981	1674	6652	1614	1836	1558	3684	1504	2334	1451	58
3	9655	1673	8266	1614	3394	1557	5188	1502	3785	1450	57
4	9·7801328	1672	9880	1613	4951	1556	6690	1502	5235	1450	56
5	3000	1671	9·7901493	1611	6507	1555	8192	1500	6685	1448	55
6	4671	1670	3104	1611	8062	1554	9692	1500	8133	1448	54
7	6341	1669	4715	1610	9616	1553	9·8091192	1499	9581	1447	53
8	8010	1667	6325	1608	9·8001169	1552	2691	1498	9·8181028	1446	52
9	9677	1667	7933	1608	2721	1551	4189	1497	2474	1445	51
10	9·7811344	1666	9541	1607	4272	1551	5686	1496	3919	1445	50
11	3010	1665	9·7911148	1606	5823	1549	7182	1496	5364	1443	49
12	4675	1664	2754	1605	7372	1549	8678	1494	6807	1443	48
13	6339	1663	4359	1604	8921	1547	9·8100172	1494	8250	1442	47
14	8002	1662	5963	1603	9·8010468	1547	1666	1493	9692	1441	46
15	9664	1660	7566	1602	2015	1546	3159	1491	9·8191133	1440	45
16	9·7821324	1660	9168	1601	3561	1545	4650	1491	2573	1439	44
17	2984	1659	9·7920769	1600	5106	1543	6141	1490	4012	1438	43
18	4643	1658	2369	1599	6649	1543	7631	1490	5450	1438	42
19	6301	1657	3968	1598	8192	1543	9121	1488	6888	1437	41
20	7958	1656	5566	1597	9735	1541	9·8110609	1487	8325	1436	40
21	9614	1654	7163	1597	9·8021276	1540	2096	1487	9761	1435	39
22	9·7831268	1654	8760	1595	2816	1539	3583	1486	9·8201196	1434	38
23	2922	1653	9·7930355	1594	4355	1539	5069	1485	2630	1433	37
24	4575	1652	1949	1594	5894	1537	6554	1484	4063	1433	36
25	6227	1651	3543	1592	7431	1537	8038	1483	5496	1431	35
26	7878	1650	5135	1592	8968	1536	9521	1482	6927	1431	34
27	9528	1649	6727	1590	9·8030504	1534	9·8121003	1481	8358	1430	33
28	9·7841177	1647	8317	1590	2038	1534	2484	1481	9788	1429	32
29	2824	1647	9907	1589	3572	1533	3965	1479	9·8211217	1429	31
30	4471	1646	9·7941496	1587	5105	1532	5444	1479	2646	1427	30
31	6117	1645	3083	1587	6637	1531	6923	1478	4073	1427	29
32	7762	1644	4670	1586	8168	1531	8401	1477	5500	1426	28
33	9406	1643	6256	1585	9699	1529	9878	1476	6926	1425	27
34	9·7851049	1642	7841	1584	9·8041228	1529	9·8131354	1475	8351	1424	26
35	2691	1641	9425	1583	2757	1527	2829	1474	9775	1423	25
36	4332	1640	9·7951008	1582	4284	1527	4303	1474	9·8221198	1423	24
37	5972	1639	2590	1581	5811	1525	5777	1473	2621	1421	23
38	7611	1638	4171	1580	7336	1525	7250	1471	4042	1421	22
39	9249	1637	5751	1579	8861	1524	8721	1471	5463	1420	21
40	9·7860886	1636	7330	1579	9·8050385	1523	9·8140192	1470	6883	1419	20
41	2522	1635	8909	1577	1908	1522	1662	1469	8302	1419	19
42	4157	1634	9·7960486	1576	3430	1521	3131	1469	9721	1417	18
43	5791	1633	2062	1576	4951	1521	4600	1467	9·8231138	1417	17
44	7424	1632	3638	1574	6472	1519	6067	1467	2555	1416	16
45	9056	1631	5212	1574	7991	1519	7534	1465	3971	1415	15
46	9·7870687	1630	6786	1573	9510	1517	8999	1465	5386	1414	14
47	2317	1629	8359	1571	9·8061027	1517	9·8150464	1464	6800	1413	13
48	3946	1628	9930	1571	2544	1516	1928	1463	8213	1413	12
49	5574	1628	9·7971501	1570	4060	1515	3391	1463	9626	1411	11
50	7202	1626	3071	1569	5575	1514	4854	1461	9·8241037	1411	10
51	8828	1625	4640	1568	7089	1513	6315	1461	2448	1410	9
52	9·7880453	1624	6208	1567	8602	1512	7776	1459	3858	1409	8
53	2077	1624	7775	1566	9·8070114	1512	9235	1459	5267	1409	7
54	3701	1622	9341	1565	1626	1510	9·8160694	1458	6676	1407	6
55	5323	1621	9·7980906	1564	3136	1510	2152	1457	8083	1407	5
56	6944	1621	2470	1564	4646	1508	3609	1457	9490	1406	4
57	8565	1619	4034	1562	6154	1508	5066	1455	9·8250896	1405	3
58	9·7890184	1618	5596	1562	7662	1507	6521	1454	2301	1404	2
59	1802	1618	7158	1560	9169	1506	7975	1454	3705	1404	1
60	3420		8718		9·8080675		9429		5109		0
′	52°	*diff.*	51°	*diff.*	50°	*diff.*	49°	*diff.*	48°	*diff.*	′

LOG. COSINE.

′	37°	*diff.*	38°	*diff.*	39°	*diff.*	40°	*diff.*	41°	*diff.*	′
0	9·8 71144	2628	9·8928098	2604	9·9083692	2583	9·9238135	2566	9·9391631	2551	60
1	3772	2628	9·8930702	2604	6275	2583	9·9240701	2565	4182	2551	59
2	6400	2627	3306	2603	8858	2582	3266	2565	6733	2551	58
3	9027	2627	5909	2602	9·9091440	2582	5831	2565	9284	2551	57
4	9·8781654	2627	8511	2603	4022	2581	8396	2564	9·9401835	2550	56
5	4281	2626	9·8941114	2601	6603	2582	9·9250960	2564	4385	2551	55
6	6907	2626	3715	2602	9185	2581	3524	2564	6936	2550	54
7	9533	2625	6317	2601	9·9101766	2581	6088	2564	9486	2550	53
8	9·8792158	2624	8918	2601	4347	2580	8652	2563	9·9412036	2549	52
9	4782	2625	9·8951519	2600	6927	2580	9·9261215	2563	4585	2550	51
10	7407	2624	4119	2600	9507	2580	3778	2563	7135	2549	50
11	9·8800031	2623	6719	2600	9·9112087	2579	6341	2563	9684	2549	49
12	2654	2623	9319	2599	4666	2579	8904	2562	9·9422233	2549	48
13	5277	2623	9·8961918	2599	7245	2579	9·9271466	2562	4782	2549	47
14	7900	2622	4517	2599	9824	2579	4028	2562	7331	2548	46
15	9·8810522	2622	7116	2598	9·9122403	2578	6590	2562	9879	2549	45
16	3144	2621	9714	2598	4981	2578	9152	2561	9·9432428	2548	44
17	5765	2621	9·8972312	2598	7559	2578	9·9281713	2561	4976	2548	43
18	8386	2621	4910	2597	9·9130137	2577	4274	2561	7524	2548	42
19	9·8821007	2620	7507	2597	2714	2577	6835	2561	9·9440072	2547	41
20	3627	2619	9·8980104	2596	5291	2577	9396	2560	2619	2547	40
21	6246	2620	2700	2596	7868	2576	9·9291956	2560	5166	2548	39
22	8866	2618	5296	2596	9·9140444	2576	4516	5560	7714	2547	38
23	9·8831484	2619	7892	2595	3020	2576	7076	2560	9·9450261	2546	37
24	4103	2618	9·8990487	2595	5596	2575	9636	2559	2807	2547	36
25	6721	2617	3082	2595	8171	2576	9·9302195	2560	5354	2546	35
26	9338	2618	5677	2594	9·9150747	2575	4755	2559	7900	2547	34
27	9·8841956	2616	8271	2594	3322	2574	7314	2558	9·9460447	2546	33
28	4572	2617	9·9000865	2594	5896	2575	9872	2559	2993	2546	32
29	7189	2616	3459	2593	8471	2574	9·9312431	2558	5539	2545	31
30	9805	2615	6052	2593	9·9161045	2673	4989	2558	8084	2546	30
31	9·8852420	2615	8645	2592	3618	2574	7547	2558	9·9470630	2545	29
32	5035	2615	9·9011237	2593	6192	2573	9·9320105	2557	3175	2545	28
33	7650	2614	3830	2592	8765	2573	2662	2558	5720	2545	27
34	9·8860264	2614	6422	2591	9·9171338	2573	5220	2557	8265	2545	26
35	2878	2614	9013	2591	3911	2572	7777	2557	9·9480810	2545	25
36	5492	2613	9·9021604	2591	6483	2572	9·9330334	2556	3355	2544	24
37	8105	2613	4195	2591	9055	2572	2890	2556	5899	2544	23
38	9·8870718	2612	6786	2590	9·9181627	2571	5446	2557	8443	2544	22
39	3330	2612	9376	2590	4198	2571	8003	2556	9·9490987	2544	21
40	5942	2612	9·9031966	2589	6769	2571	9·9340559	2555	3531	2544	20
41	8554	2611	4555	2589	9340	2571	3114	2556	6075	2544	19
42	9·8881165	2610	7144	2589	9·9191911	2570	5670	2555	8619	2543	18
43	3775	2611	9733	2588	4481	2570	8225	2555	9·9501162	2543	17
44	6386	2610	9·9042321	2589	7051	2570	9·9350780	2555	3705	2543	16
45	8996	2609	4910	2587	9621	2570	3335	2554	6248	2543	15
46	9·8891605	2609	7497	2588	9·9202191	2569	5889	2555	8791	2543	14
47	4214	2609	9·9050085	2587	4760	2569	8444	2554	9·9511334	2542	13
48	6823	2609	2672	2587	7329	2569	9·9360998	2554	3876	2543	12
49	9432	2608	5259	2586	9898	2568	3552	2553	6419	2542	11
50	9·8902040	2607	7845	2586	9·9212466	2568	6105	2554	8961	2542	10
51	4647	2607	9·9060431	2586	5034	2568	8659	2553	9·9521503	2542	9
52	7254	2607	3017	2586	7602	2568	9·9371212	2553	4045	2542	8
53	9861	2607	5603	2585	9·9220170	2567	3765	2553	6587	2541	7
54	9·8912468	2606	8188	2585	2737	2567	6318	2553	9128	2542	6
55	5074	2605	9·9070773	2584	5304	2567	8871	2552	9·9531670	2541	5
56	7679	2606	3357	2584	7871	2566	9·9381423	2552	4211	2541	4
57	9·8920285	2605	5941	2584	9·9230437	2567	3975	2552	6752	2541	3
58	2890	2604	8525	2584	3004	2566	6527	2552	9293	2541	2
59	5494	2604	9·9081109	2583	5570	2565	9079	2552	9·9541834	2540	1
60	8098		3692		8135		9·9391631		4374		0
′	52°	*diff.*	51°	*diff.*	50°	*diff.*	49°	*diff.*	48°	*diff.*	′

LOG. COTAN.

LOG. SINE.

′	42°	*diff.*	43°	*diff.*	44°	*diff.*	45°	*diff.*	46°	*diff.*	′
0	9·8255109	1403	9·8337833	1355	9·8417713	1308	9·8494850	1263	9·8569341	1220	60
1	6512	1401	9188	1353	9021	1307	6113	1262	9·8570561	1218	59
2	7913	1401	9·8340541	1353	9·8420328	1306	7375	1262	1779	1219	58
3	9314	1401	1894	1352	1634	1305	8637	1260	2998	1217	57
4	9·8260715	1399	3246	1351	2939	1305	9897	1260	4215	1217	56
5	2114	1398	4597	1351	4244	1304	9·8501157	1260	5432	1216	55
6	3512	1398	5948	1349	5548	1303	2417	1258	6648	1215	54
7	4910	1397	7297	1349	6851	1303	3675	1258	7863	1215	53
8	6307	1396	8646	1348	8154	1302	4933	1257	9078	1214	52
9	7703	1395	9994	1347	9456	1301	6190	1256	9·8580292	1213	51
10	9098	1395	9·8351341	1347	9·8430757	1300	7446	1256	1505	1213	50
11	9·8270493	1394	2688	1345	2057	1299	8702	1255	2718	1211	49
12	1887	1392	4033	1345	3356	1299	9957	1254	3929	1212	48
13	3279	1392	5378	1344	4655	1298	9·8511211	1254	5141	1210	47
14	4671	1392	6722	1344	5953	1297	2465	1252	6351	1210	46
15	6063	1390	8066	1342	7250	1297	3717	1252	7561	1209	45
16	7453	1390	9408	1342	8547	1295	4969	1251	8770	1208	44
17	8843	1388	9·8360750	1341	9842	1295	6220	1251	9978	1208	43
18	9·8280231	1388	2091	1340	9·8441137	1295	7471	1250	9·8591186	1207	42
19	1619	1387	3431	1340	2432	1293	8721	1249	2393	1206	41
20	3006	1387	4771	1338	3725	1293	9970	1248	3599	1205	40
21	4393	1385	6109	1338	5018	1292	9·8521218	1248	4804	1205	39
22	5778	1385	7447	1337	6310	1291	2466	1247	6009	1204	38
23	7163	1384	8784	1337	7601	1290	3713	1246	7213	1203	37
24	8547	1383	9·8370121	1335	8891	1290	4959	1245	8416	1203	36
25	9930	1382	1456	1335	9·8450181	1289	6204	1245	9619	1202	35
26	9·8291312	1382	2791	1334	1470	1288	7449	1244	9·8600821	1201	34
27	2694	1381	4125	1333	2758	1287	8693	1243	2022	1201	33
28	4075	1379	5458	1332	4045	1287	9936	1243	3223	1200	32
29	5454	1379	6790	1332	5332	1296	9·8531179	1242	4423	1199	31
30	6833	1679	8122	1331	6618	1285	2421	1241	5622	1199	30
31	8212	1377	9453	1330	7903	1285	3662	1240	6821	1197	29
32	9589	1377	9·8380783	1329	9188	1283	4902	1240	8018	1197	28
33	9·8300966	1376	2112	1329	9·8460471	1283	6142	1239	9215	1197	27
34	2342	1375	3441	1328	1754	1282	7381	1238	9·8610412	1196	26
35	3717	1374	4769	1327	3036	1282	8619	1237	1608	1195	25
36	5091	1373	6096	1326	4318	1281	9856	1237	2803	1194	24
37	6464	1373	7422	1325	5599	1280	9·8541093	1236	3997	1193	23
38	7837	1372	8747	1325	6879	1279	2329	1235	5190	1193	22
39	9209	1371	9·8390072	1324	8158	1278	3564	1235	6383	1193	21
40	9·8310580	1370	1396	1323	9436	1278	4799	1234	7576	1191	20
41	1950	1370	2719	1322	9·8470714	1277	6033	1233	8767	1191	19
42	3320	1368	4041	1322	1991	1276	7266	1233	9958	1190	18
43	4688	1368	5363	1321	3267	1276	8499	1231	9·8621148	1190	17
44	6056	1367	6684	1320	4543	1274	9730	1231	2338	1188	16
45	7423	1366	8004	1319	5817	1274	9·8550961	1231	3526	1188	15
46	8789	1366	9323	1319	7091	1274	2192	1229	4714	1188	14
47	9·8320155	1364	9·8400642	1317	8365	1272	3421	1229	5902	1186	13
48	1519	1364	1959	1317	9637	1272	4650	1228	7088	1186	12
49	2883	1363	3276	1317	9·8480909	1271	5878	1228	8274	1186	11
50	4246	1363	4593	1315	2180	1270	7106	1226	9460	1184	10
51	5609	1361	5908	1315	3450	1270	8332	1226	9·8630644	1184	9
52	6970	1361	7223	1314	4720	1269	9558	1226	1828	1183	8
53	8331	1360	8537	1313	5989	1268	9·8560784	1224	3011	1183	7
54	9691	1359	9850	1312	7257	1267	2008	1224	4194	1182	6
55	9·8331050	1358	9·8411162	1312	8524	1267	3232	1223	5376	1181	5
56	2408	1358	2474	1311	9791	1266	4455	1223	6557	1180	4
57	3766	1356	3785	1310	9·8491057	1265	5678	1222	7737	1180	3
58	5122	1356	5095	1309	2322	1264	6900	1221	8917	1179	2
59	6478	1355	6404	1309	3586	1264	8121	1220	9·8640096	1179	1
60	7833		7713		4850		9341		1275		0
′	47°	*diff.*	46°	*diff.*	45°	*diff.*	44°	*diff.*	43°	*diff.*	′

LOG. COSINE.

′	42°	*diff.*	43°	*diff.*	44°	*diff.*	45°	*diff.*	46°	*diff.*	′
0	9·9544374	2541	9·9696559	2532	9·9848372	2528	10·0000000	2527	10·0151628	2528	60
1	6915	2540	9091	2533	9·9850900	2528	2527	2526	4156	2529	59
2	9455	2540	9·9701624	2533	3428	2528	5053	2527	6685	2528	58
3	9·9551995	2540	4157	2532	5956	2528	7580	2527	9213	2528	57
4	4535	2540	6689	2532	8484	2528	10·0010107	2526	10 0161741	2529	56
5	7075	2540	9221	2533	9·9861012	2528	2633	2527	4270	2528	55
6	9615	2539	9·9711754	2532	3540	2528	5160	2526	6798	2529	54
7	9·9562154	2540	4286	2532	6068	2528	7686	2527	9327	2528	53
8	4694	2539	6818	2532	8596	2527	10·0020213	2527	10·0171855	2529	52
9	7233	2539	9350	2532	9·9871123	2528	2740	2526	4384	2529	51
10	9772	2539	9·9721882	2531	3651	2528	5266	2527	6913	2528	50
11	9·9572311	2539	4413	2532	6179	2527	7793	2527	9441	2529	49
12	4850	2539	6945	2532	8706	2528	10·0030320	2526	10·0181970	2529	48
13	7389	2538	9477	2531	9·9881234	2527	2846	2527	4499	2529	47
14	9927	2538	9·9732008	2531	3761	2528	5373	2527	7028	2529	46
15	9·9582465	2539	4539	2532	6289	2527	7900	2527	9557	2529	45
16	5004	2538	7071	2531	8816	2528	10·0040427	2526	10·0192086	2529	44
17	7542	2538	9602	2531	9·9891344	2527	2953	2527	4615	2529	43
18	9·9590080	2538	9·9742133	2531	3871	2528	5480	2527	7144	2530	42
19	2618	2537	4664	2531	6399	2527	8007	2527	9674	2529	41
20	5155	2538	7195	2531	8926	2527	10·0050534	2526	10·0202203	2529	40
21	7693	2537	9726	2531	9·9901453	2528	3060	2527	4732	2530	39
22	9·9600230	2537	9·9752257	2530	3981	2527	5587	2527	7262	2529	38
23	2767	2538	4787	2531	6508	2527	8114	2527	9791	2530	37
24	5305	2537	7318	2531	9035	2527	10·0060641	2527	10·0212321	2530	36
25	7842	2536	9849	2530	9·9911562	2527	3168	2527	4851	2529	35
26	9·9610378	2537	9·9762379	2530	4089	2527	5695	2527	7380	2530	34
27	2915	2537	4909	2531	6616	2527	8222	2527	9910	2530	33
28	5452	2536	7440	2530	9143	2527	10·0070749	2527	10·0222440	2530	32
29	7988	2537	9970	2530	9·9921670	2527	3276	2527	4970	2530	31
30	9·9620525	2536	9·9772500	2530	4197	2527	5803	2527	7500	2530	30
31	3061	2536	5030	2530	6724	2527	8330	2527	10·0230030	2530	29
32	5597	2536	7560	2530	9251	2527	10 0080857	2527	2560	2531	28
33	8133	2536	9·9780090	2530	9·9931778	2527	3384	2527	5091	2530	27
34	9·9630669	2535	2620	2529	4305	2527	5911	2527	7621	2530	26
35	3204	2536	5149	2530	6832	2527	8438	2527	10·0240151	2531	25
36	5740	2535	7679	2530	9359	2527	10·0090965	2527	2682	2531	24
37	8275	2536	9·9790209	2529	9·9941886	2527	3492	2527	5213	2530	23
38	9·9640811	2535	2738	2530	4413	2527	6019	2528	7743	2531	22
39	3346	2535	5268	2529	6940	2526	8547	2527	10·0250274	2531	21
40	5881	2535	7797	2529	9466	2527	10·0101074	2527	2805	2531	20
41	8416	2535	9·9800326	2530	9·9951993	2527	3601	2528	5336	2531	19
42	9·9650951	2535	2856	2529	4520	2527	6129	2527	7867	2531	18
43	3486	2534	5385	2529	7047	2526	8656	2528	10·0260398	2531	17
44	6020	2535	7914	2529	9573	2527	10·0111184	2527	2929	2532	16
45	8555	2534	9·9810443	2529	9·9962100	2527	3711	2528	5461	2531	15
46	9·9661089	2534	2972	2529	4627	2527	6239	2527	7992	2531	14
47	3623	2534	5501	2529	7154	2526	8766	2528	10·0270523	2532	13
48	6157	2535	8030	2529	9680	2527	10·0121294	2527	3055	2532	12
49	8692	2533	9·9820559	2528	9·9972207	2527	3821	2528	5587	2531	11
50	9·9671225	2534	3087	2529	4734	2526	6349	2528	8118	2532	10
51	3759	2534	5616	2529	7260	2527	8877	2527	10·0280650	2532	9
52	6293	2534	8145	2528	9787	2527	10·0131404	2528	3182	2532	8
53	8827	2533	9·9830673	2529	9·9982314	2526	3932	2528	5714	2532	7
54	9·9681360	2533	3202	2528	4840	2527	6460	2528	8246	2533	6
55	3893	2534	5730	2529	7367	2526	8988	2528	10·0290779	2532	5
56	6427	2533	8259	2528	9893	2527	10·0141516	2528	3311	2532	4
57	8960	2533	9·9840787	2528	9·9992420	2527	4044	2528	5843	2533	3
58	9·9691493	2533	3315	2529	4947	2526	6572	2528	8376	2533	2
59	4026	2533	5844	2528	7473	2527	9100	2528	10·0300909	2532	1
60	6559		8372		10·0000000		10·0151628		3441		0
′	47°	*diff.*	46°	*diff.*	45°	*diff.*	44°	*diff.*	43°	*diff.*	′

LOG. COTAN.

′	47°	*diff.*	48°	*diff.*	49°	*diff.*	50°	*diff.*	51°	*diff.*	′
0	9·8641275	1177	9·8710735	1137	9·8777799	1097	9·8842540	1059	9·8905026	1023	60
1	2452	1177	1872	1136	8896	1098	3599	1060	6049	1022	59
2	3629	1177	3008	1136	9994	1096	4659	1058	7071	1021	58
3	4806	1175	4144	1135	9·8781090	1096	5717	1058	8092	1021	57
4	5981	1175	5279	1135	2186	1095	6775	1057	9113	1020	56
5	7156	1175	6414	1134	3281	1095	7832	1057	9·8910133	1020	55
6	8331	1173	7548	1133	4376	1094	8889	1056	1153	1019	54
7	9504	1173	8681	1132	5470	1093	9945	1055	2172	1019	53
8	9·8650677	1172	9813	1132	6563	1093	9·8851000	1055	3191	1017	52
9	1849	1172	9·8720945	1131	7656	1092	2055	1054	4208	1018	51
10	3021	1171	2076	1131	8748	1092	3109	1053	5226	1016	50
11	4192	1170	3207	1130	9840	1090	4162	1053	6242	1016	49
12	5362	1169	4337	1129	9·8790930	1091	5215	1052	7258	1016	48
13	6531	1169	5466	1128	2021	1089	6267	1052	8274	1015	47
14	7700	1168	6594	1128	3110	1089	7319	1051	9289	1014	46
15	8868	1168	7722	1127	4199	1088	8370	1050	9·8920303	1013	45
16	9·8660036	1167	8849	1127	5287	1088	9420	1050	1316	1013	44
17	1203	1166	9976	1126	6375	1087	9·8860470	1049	2329	1013	43
18	2369	1165	9·8731102	1125	7462	1086	1519	1049	3342	1012	42
19	3534	1165	2227	1125	8548	1086	2568	1048	4354	1011	41
20	4699	1164	3352	1124	9634	1085	3616	1047	5365	1010	40
21	5863	1163	4476	1123	9·8800719	1084	4663	1047	6375	1010	39
22	7026	1163	5599	1123	1803	1084	5710	1046	7385	1010	38
23	8189	1162	6722	1122	2887	1083	6756	1045	8395	1009	37
24	9351	1161	7844	1121	3970	1082	7801	1045	9404	1008	36
25	9·8670512	1161	8965	1120	5052	1082	8846	1044	9·8930412	1007	35
26	1673	1160	9·8740085	1120	6134	1081	9890	1044	1419	1007	34
27	2833	1159	1205	1120	7215	1081	9·8870934	1043	2426	1007	33
28	3992	1159	2325	1118	8296	1080	1977	1042	3433	1006	32
29	5151	1158	3443	1118	9376	1079	3019	1042	4439	1005	31
30	6309	1157	4561	1118	9·8810455	1079	4061	1041	5444	1004	30
31	7466	1157	5679	1116	1534	1078	5102	1040	6448	1004	29
32	8623	1156	6795	1117	2612	1077	6142	1040	7452	1004	28
33	9779	1155	7912	1115	3689	1077	7182	1039	8456	1002	27
34	9·8680934	1154	9027	1115	4766	1076	8221	1039	9458	1003	26
35	2088	1154	9·8750142	1114	5842	1076	9260	1038	9·8940461	1001	25
36	3242	1154	1256	1113	6918	1074	9·8880298	1037	1462	1001	24
37	4396	1152	2369	1113	7992	1075	1335	1037	2463	1001	23
38	5548	1152	3482	1112	9067	1073	2372	1036	3464	999	22
39	6700	1151	4594	1112	9·8820140	1073	3408	1036	4463	1000	21
40	7851	1151	5706	1110	1213	1072	4444	1035	5463	998	20
41	9002	1150	6816	1111	2285	1072	5479	1034	6461	998	19
42	9·8690152	1149	7927	1109	3357	1071	6513	1034	7459	998	18
43	1301	1148	9036	1109	4428	1071	7547	1033	8457	996	17
44	2449	1148	9·8760145	1108	5499	1069	8580	1032	9453	997	16
45	3597	1147	1253	1108	6568	1070	9612	1032	9·8950450	995	15
46	4744	1147	2361	1107	7638	1068	9·8890644	1031	1445	995	14
47	5891	1146	3468	1106	8706	1068	1675	1031	2440	995	13
48	7037	1145	4574	1106	9774	1067	2706	1030	3435	994	12
49	8182	1144	5680	1105	9·8830841	1067	3736	1029	4429	993	11
50	9326	1144	6785	1104	1908	1066	4765	1029	5422	992	10
51	9·8700470	1143	7889	1104	2974	1065	5794	1028	6414	992	9
52	1613	1143	8993	1103	4039	1065	6822	1028	7406	992	8
53	2756	1142	9·8770096	1102	5104	1064	7850	1027	8398	991	7
54	3898	1141	1198	1102	6168	1064	8877	1026	9389	990	6
55	5039	1140	2300	1101	7232	1062	9903	1026	9·8960379	990	5
56	6179	1140	3401	1100	8294	1063	9·8900929	1025	1369	989	4
57	7319	1139	4501	1100	9357	1061	1954	1025	2358	988	3
58	8458	1139	5601	1099	9·8840418	1061	2979	1024	3346	988	2
59	9597	1138	6700	1099	1479	1061	4003	1023	4334	987	1
60	9·8710735		7799		2540		5026		5321		0
′	42°	*diff.*	41°	*diff.*	40°	*diff.*	39°	*diff.*	38°	*diff.*	′

LOG. COSINE.

′	47°	*diff.*	48°	*diff.*	49°	*diff.*	50°	*diff.*	51°	*diff.*	′
0	10·0303441	2533	10·0455626	2540	10·0608369	2552	10·0761865	2565	10·0916308	2583	60
1	5974	2533	8166	2541	10·0610921	2552	4430	2566	8891	2584	59
2	8507	2533	10·0460707	2541	3473	2552	6996	2567	10·0921475	2584	58
3	10·0311040	2533	3248	2541	6025	2552	9563	2566	4059	2584	57
4	3573	2534	5789	2541	8577	2552	10·0772129	2567	6643	2584	56
5	6107	2533	8330	2542	10·0621129	2553	4696	2567	9227	2585	55
6	8640	2533	10·0470872	2541	3682	2553	7263	2567	10·0931812	2585	54
7	10·0321173	2534	3413	2542	6235	2553	9830	2568	4397	2586	53
8	3707	2534	5955	2542	8788	2553	10·0782398	2568	6983	2586	52
9	6241	2534	8497	2542	10·0631341	2554	4966	2568	9569	2586	51
10	8775	2533	10·0481039	2542	3895	2553	7534	2568	10·0942155	2586	50
11	10·0331308	2535	3581	2543	6448	2554	10·0790102	2569	4741	2587	49
12	3843	2534	6124	2542	9002	2554	2671	2569	7328	2587	48
13	6377	2534	8666	2543	10·0641556	2555	5240	2569	9915	2588	47
14	8911	2534	10·0491209	2543	4111	2554	7809	2570	10·0952503	2587	46
15	10·0341445	2535	3752	2543	6665	2555	10·0800379	2570	5090	2589	45
16	3980	2534	6295	2543	9220	2555	2949	2570	7679	2588	44
17	6514	2535	8838	2543	10·0651775	2555	5519	2570	10·0960267	2589	43
18	9049	2535	10·0501381	2544	4330	2556	8089	2571	2856	2589	42
19	10·0351584	2535	3925	2544	6886	2555	10·0810660	2571	5445	2589	41
20	4119	2535	6469	2544	9441	2556	3231	2571	8034	2590	40
21	6654	2535	9013	2544	10·0661997	2557	5802	2571	10·0970624	2590	39
22	9189	2536	10·0511557	2544	4554	2556	8373	2572	3214	2591	38
23	10·0361725	2535	4101	2544	7110	2556	10·0820945	2572	5805	2591	37
24	4260	2536	6645	2545	9666	2557	3517	2572	8396	2591	36
25	6796	2535	9190	2545	10·0672223	2557	6089	2573	10·0980987	2591	35
26	9331	2536	10·0521735	2545	4780	2558	8662	2573	3578	2592	34
27	10·0371867	2536	4280	2545	7338	2557	10·0831235	2573	6170	2593	33
28	4403	2536	6825	2545	9895	2558	3808	2574	8763	2592	32
29	6939	2536	9370	2546	10·0682453	2558	6382	2573	10·0991355	2593	31
30	9475	2537	10·0531916	2545	5011	2558	8955	2574	3948	2593	30
31	10·0382012	2536	4461	2546	7569	2559	10·0841529	2575	6541	2594	29
32	4548	2537	7007	2546	10·0690128	2558	4104	2574	9135	2594	28
33	7085	2537	9553	2547	2686	2559	6678	2575	10·1001729	2594	27
34	9622	2536	10·0542100	2546	5245	2560	9253	2576	4323	2595	26
35	10·0392158	2537	4646	2547	7805	2559	10·0851829	2575	6918	2595	25
36	4695	2538	7193	2546	10·0700364	2560	4404	2576	9513	2595	24
37	7233	2537	9739	2547	2924	2560	6980	2576	10·1012108	2596	23
38	9770	2537	10·0552286	2548	5484	2560	9556	2576	4704	2596	22
39	10·0402307	2538	4834	2547	8044	2560	10·0862132	2577	7300	2596	21
40	4845	2537	7381	2547	10·0710604	2561	4709	2577	9896	2597	20
41	7382	2538	9928	2548	3165	2561	7286	2577	10·1022493	2597	19
42	9920	2538	10·0562476	2548	5726	2561	9863	2578	5090	2598	18
43	10·0412458	2538	5024	2548	8287	2561	10·0872441	2578	7688	2598	17
44	4996	2539	7572	2549	10·0720848	2562	5019	2578	10·1030286	2598	16
45	7535	2538	10·0570121	2548	3410	2562	7597	2579	2884	2599	15
46	10·0420073	2538	2669	2549	5972	2562	10·0880176	2579	5483	2599	14
47	2611	2539	5218	2549	8534	2562	2755	2579	8082	2599	13
48	5150	2539	7767	2549	10·0731096	2563	5334	2579	10·1040681	2600	12
49	7689	2539	10·0580316	2549	3659	2563	7913	2580	3281	2600	11
50	10·0430228	2539	2865	2550	6222	2563	10·0890493	2580	5881	2600	10
51	2767	2539	5415	2549	8785	2563	3073	2580	8481	2601	9
52	5306	2540	7964	2550	10·0741348	2564	5653	2581	10·1051082	2601	8
53	7846	2539	10·0590514	2550	3912	2564	8234	2581	2683	2602	7
54	10·0440385	2540	3064	2551	6476	2564	10·0900815	2582	6285	2601	6
55	2925	2540	5615	2550	9040	2564	3397	2581	8886	2603	5
56	5465	2540	8165	2551	10·0751604	2565	5978	2582	10·1061489	2602	4
57	8005	2540	10·0600716	2551	4169	2565	8560	2582	4091	2603	3
58	10·0450545	2540	3267	2551	6734	2565	10·0911142	2583	6694	2604	2
59	3085	2541	5818	2551	9299	2566	3725	2583	9298	2604	1
60	5626		8369		10·0761865		6308		10·1071902		0
′	42°	*diff.*	41°	*diff.*	40°	*diff.*	39°	*diff.*	38°	*diff.*	′

LOG. COTAN.

′	52°	*dif.*	53°	*dif.*	54°	*dif.*	55°	*dif.*	56°	*dif.*	′
0	9·896 5321	987	9·902 3486	952	9·907 9576	918	9·913 3645	885	9·918 5742	852	60
1	6308	986	4438	951	9·908 0494	917	4530	883	6594	851	59
2	7294	986	5389	950	1411	916	5413	883	7445	851	58
3	8280	985	6339	950	2327	916	6296	883	8296	850	57
4	9265	984	7289	950	3243	916	7179	882	9146	850	56
5	9·897 0249	984	8239	949	4159	914	8061	882	9996	849	55
6	1233	983	9188	948	5073	915	8943	881	9·919 0845	849	54
7	2216	983	9·903 0136	948	5988	913	9824	880	1694	848	53
8	3199	982	1084	947	6901	913	9·914 0704	880	2542	848	52
9	4181	981	2031	946	7814	913	1584	880	3390	847	51
10	5162	981	2977	946	8727	912	2464	878	4237	846	50
11	6143	980	3923	945	9639	911	3342	879	5083	846	49
12	7123	980	4868	945	9·909 0550	911	4221	878	5929	846	48
13	8103	979	5813	944	1461	910	5099	877	6775	844	47
14	9082	978	6757	944	2371	910	5976	876	7619	845	46
15	9·898 0060	978	7701	943	3281	909	6852	877	8464	844	45
16	1038	977	8644	943	4190	909	7729	875	9308	843	44
17	2015	977	9587	942	5099	908	8604	875	9·920 0151	843	43
18	2992	976	9·904 0529	941	6007	908	9479	875	0994	842	42
19	3968	976	1470	941	6915	906	9·915 0354	874	1836	842	41
20	4944	975	2411	940	7821	907	1228	873	2678	841	40
21	5919	974	3351	940	8728	906	2101	873	3519	841	39
22	6893	974	4291	939	9634	905	2974	872	4360	840	38
23	7867	973	5230	938	9·910 0539	905	3846	872	5200	839	37
24	8840	972	6168	938	1444	904	4718	871	6039	839	36
25	9812	972	7106	937	2348	903	5589	871	6878	839	35
26	9·899 0784	972	8043	937	3251	904	6460	870	7717	838	34
27	1756	971	8980	936	4155	902	7330	870	8555	838	33
28	2727	970	9916	936	5057	902	8200	869	9393	836	32
29	3697	970	9·905 0852	935	5959	901	9069	868	9·921 0229	837	31
30	4667	969	1787	935	6860	901	9937	868	1066	836	30
31	5636	968	2722	934	7761	900	9·916 0805	868	1902	835	29
32	6604	968	3656	933	8661	900	1673	866	2737	835	28
33	7572	967	4589	933	9561	899	2539	867	3572	834	27
34	8539	967	5522	932	9·911 0460	899	3406	866	4406	834	26
35	9506	966	6454	932	1359	898	4272	865	5240	833	25
36	9·900 0472	966	7386	931	2257	898	5137	865	6073	833	24
37	1438	965	8317	930	3155	896	6002	864	6906	832	23
38	2403	964	9247	930	4051	897	6866	864	7738	832	22
39	3367	964	9·906 0177	930	4948	896	7730	863	8570	831	21
40	4331	963	1107	929	5844	895	8593	862	9401	831	20
41	5294	963	2036	928	6739	895	9455	862	9·922 0232	830	19
42	6257	962	2964	928	7634	894	9·917 0317	862	1062	829	18
43	7219	962	3892	927	8528	894	1179	861	1891	830	17
44	8181	961	4819	926	9422	893	2040	860	2721	828	16
45	9142	960	5745	926	9·912 0315	892	2900	860	3549	828	15
46	9·901 0102	960	6671	926	1207	892	3760	859	4377	828	14
47	1062	959	7597	925	2099	892	4619	859	5205	827	13
48	2021	959	8522	924	2991	891	5478	858	6032	826	12
49	2980	958	9446	924	3882	890	6336	858	6858	826	11
50	3938	957	9·907 0370	923	4772	890	7194	857	7684	825	10
51	4895	957	1293	923	5662	889	8051	857	8509	825	9
52	5852	956	2216	922	6551	889	8908	856	9334	824	8
53	6808	956	3138	921	7440	888	9764	856	9·923 0158	824	7
54	7764	955	4059	921	8328	887	9·918 0620	855	0982	823	6
55	8719	955	4980	921	9215	887	1475	854	1805	823	5
56	9674	954	5901	919	9·913 0102	887	2329	854	2628	822	4
57	9·902 0628	953	6820	920	0989	886	3183	854	3450	822	3
58	1581	953	7740	918	1875	885	4037	853	4272	821	2
59	2534	952	8658	918	2760	885	4890	852	5093	821	1
60	3486		9576		3645		5742		5914		0
′	37°	*dif.*	36°	*dif.*	35°	*dif.*	34°	*dif.*	33°	*dif.*	′

LOG. COSINE.

′	52°	*diff.*	53°	*diff.*	54°	*diff.*	55°	*diff.*	56°	*diff.*	′
0	10·1071902	2604	10·1228856	2629	10·1387390	2656	10.1547732	2689	10·1710126	2725	60
1	4506	2604	10·1231485	2629	10·1390046	2658	10·1550421	2690	2851	2726	59
2	7110	2605	4114	2629	2704	2658	3111	2690	5577	2727	58
3	9715	2606	6743	2630	5362	2658	5801	2691	8304	2727	57
4	10·1082321	2605	9373	2631	8020	2659	8492	2691	10·1721031	2728	56
5	4926	3606	10·1242004	2631	10·1400679	2660	10·1561183	2692	3759	2728	55
6	7532	2607	4635	2631	3339	2659	3875	2693	6487	2730	54
7	10·1090139	2607	7266	2632	5998	2661	6568	2693	9217	2730	53
8	2746	2607	9898	2632	8659	2661	9261	2693	10·1731947	2730	52
9	5353	2607	10·1252530	2632	10·1411320	2661	10·1571954	2695	4677	2731	51
10	7960	2608	5162	2634	3981	2662	4649	2694	7408	2732	50
11	10·1100568	2609	7796	2633	6643	2663	7343	2696	10·1740140	2733	49
12	3177	2609	10·1260429	2634	9306	2663	10·1580039	2696	2873	2733	48
13	5786	2609	3063	2635	10·1421969	2663	2735	2696	5606	2734	47
14	8395	2609	5698	2634	4632	2664	5431	2698	8340	2734	46
15	10·1111004	2610	8332	2636	7296	2665	8129	2697	10·1751074	2735	45
16	3614	2611	10·1270968	2636	9961	2665	10·1590826	2699	3809	2736	44
17	6225	2610	3604	2636	10·1432626	2666	3525	2699	6545	2736	43
18	8835	2611	6240	2637	5292	2666	6224	2699	9281	2738	42
19	10·1121446	2612	8877	2637	7958	2666	8923	2700	10·1762019	2737	41
20	4058	2612	10·1281514	2638	10·1440624	2668	10·1601623	2701	4756	2739	40
21	6670	2612	4152	2938	3292	2667	4324	2701	7495	2739	39
22	9282	2613	6790	2638	5959	2669	7025	2702	10·1770234	2740	38
23	10·1131895	2613	9428	2639	8628	2668	9727	2702	2974	2740	37
24	4508	2614	10·1292067	2640	10·1451296	2670	10·1612429	2704	5714	2741	36
25	7122	2614	4707	2640	3966	2669	5133	2703	8455	2742	35
26	9736	2614	7347	2640	6635	2671	7836	2704	10·1781197	2743	34
27	10·1142350	2615	9987	2641	9306	2671	10·1620540	2705	3940	2743	33
28	4965	2615	10·1302628	2641	10·1461977	2671	3245	2706	6683	2743	32
29	7580	2615	5269	2642	4648	2672	5951	2706	9426	2745	31
30	10·1150195	2616	7911	2643	7320	2672	8657	2707	10·1792171	2745	30
31	2811	2617	10·1310554	2642	9992	2673	10·1631364	2707	4916	2746	29
32	5428	2616	3196	2644	10·1472665	2674	4071	2708	7662	2746	28
33	8044	2618	5840	2643	5339	2674	6779	2708	10·1800408	2748	27
34	10·1160662	2617	8483	2644	8013	2675	9487	2709	3156	2748	26
35	3279	2618	10·1321127	2645	10·1480688	2675	10·1642196	2710	5904	2748	25
36	5897	2619	3772	2645	3363	2676	4906	2710	8652	2749	24
37	8516	2618	6417	2646	6039	2676	7616	2711	10·1811401	2750	23
38	10·1171134	2620	9063	2646	8715	2677	10·1650327	2712	4151	2751	22
39	3754	2619	10·1331709	2647	10·1491392	2677	3039	2712	6902	2751	21
40	6373	2620	4356	2647	4069	2678	5751	2713	9653	2752	20
41	8993	2621	7003	2647	6747	2678	8464	2713	10·1822405	2753	19
42	10·1181614	2621	9650	2648	9425	2679	10·1661177	2714	5158	2753	18
43	4235	2621	10·1342298	2649	10·1502104	2680	3891	2715	7911	2754	17
44	6856	2622	4947	2649	4784	2680	6606	2715	10·1830665	2755	16
45	9478	2622	7596	2649	7464	2681	9321	2716	3420	2756	15
46	10·1192100	2623	10·1350245	2650	10·1510145	2681	10·1672037	2717	6176	2756	14
47	4723	2623	2895	2651	2826	2682	4754	2717	8932	2757	13
48	7346	2623	5546	2651	5508	2682	7471	2718	10·1841689	2757	12
49	9969	2624	8197	2651	8190	2683	10·1680189	2718	4446	2759	11
50	10·1202593	2625	10·1360848	2652	10·1520873	2683	2907	2719	7205	2759	10
51	5218	2624	3500	2652	3556	2684	5626	2720	9964	2759	9
52	7842	2625	6152	2653	6240	2685	8346	2720	10·1852723	2761	8
53	10·1210467	2626	8805	2654	8925	2685	10·1691066	2721	5484	2761	7
54	3093	2626	10·1371459	2654	10·1531610	2685	3787	2721	8245	2762	6
55	5719	2627	4113	2654	4295	2687	6508	2723	10·1861007	2762	5
56	8346	2627	6767	2655	6982	2686	9231	2722	3769	2763	4
57	10·1220973	2627	9422	2655	9668	2688	10·1701953	2724	6532	2764	3
58	3600	2628	10·1382077	2656	10·1542356	2688	4677	2724	9296	2765	2
59	6228	2628	4733	2657	5044	2688	7401	2725	10·1872061	2765	1
60	8856		7390		7732		10·1710126		4826		0
′	37°	*diff.*	36°	*diff.*	35°	*diff.*	34°	*diff.*	33°	*diff.*	′

LOG. COTAN.

′	57°	*diff.*	58°	*diff.*	59°	*diff.*	60°	*diff.*	61°	*diff.*	′
0	9·923 5914	820	9·928 4205	789	9·933 0656	759	9·937 5306	729	9·941 8193	700	60
1	6734	820	4994	789	1415	758	6035	729	8893	699	59
2	7554	819	5783	788	2173	758	6764	728	9592	699	58
3	8373	818	6571	787	2931	757	7492	728	9·942 0291	699	57
4	9191	819	7358	787	3688	757	8220	727	0990	698	56
5	9·924 0010	817	8145	787	4445	756	8947	727	1688	698	55
6	0827	817	8932	786	5201	756	9674	726	2386	697	54
7	1644	817	9718	786	5957	756	9·938 0400	726	3083	696	53
8	2461	816	9·929 0504	785	6713	754	1126	725	3779	697	52
9	3277	815	1289	784	7467	755	1851	725	4476	695	51
10	4092	815	2073	784	8222	754	2576	724	5171	695	50
11	4907	814	2857	784	8976	753	3300	724	5866	695	49
12	5721	814	3641	783	9729	753	4024	723	6561	694	48
13	6535	814	4424	783	9·934 0482	752	4747	723	7255	694	47
14	7349	812	5207	782	1234	752	5470	722	7949	694	46
15	8161	813	5989	781	1986	751	6192	722	8643	692	45
16	8974	812	6770	781	2737	751	6914	721	9335	693	44
17	9786	811	7551	781	3488	750	7635	721	9·943 0028	692	43
18	9·925 0597	811	8332	780	4238	750	8356	720	0720	691	42
19	1408	810	9112	779	4988	750	9076	720	1411	691	41
20	2218	810	9891	779	5738	748	9796	719	2102	690	40
21	3028	809	9·930 0670	778	6486	749	9·939 0515	719	2792	690	39
22	3837	809	1448	778	7235	748	1234	719	3482	690	38
23	4646	808	2226	778	7983	747	1953	718	4172	689	37
24	5454	807	3004	777	8730	747	2671	717	4861	688	36
25	6261	808	3781	776	9477	746	3388	717	5549	689	35
26	7069	806	4557	776	9·935 0223	746	4105	716	6238	687	34
27	7875	806	5333	776	0969	746	4821	716	6925	687	33
28	8681	806	6109	774	1715	744	5537	716	7612	687	32
29	9487	805	6883	775	2459	745	6253	715	8299	686	31
30	9·926 0292	804	7658	774	3204	744	6968	714	8985	686	30
31	1096	805	8432	773	3948	743	7682	714	9671	685	29
32	1901	803	9205	773	4691	743	8396	714	9·944 0356	685	28
33	2704	803	9978	772	5434	743	9110	713	1041	684	27
34	3507	803	9·931 0750	772	6177	741	9823	712	1725	684	26
35	4310	802	1522	772	6918	742	9·940 0535	713	2409	683	25
36	5112	801	2294	771	7660	741	1248	711	3092	683	24
37	5913	801	3065	770	8401	740	1959	711	3775	682	23
38	6714	800	3835	770	9141	740	2670	711	4457	682	22
39	7514	800	4605	769	9881	740	3381	710	5139	682	21
40	8314	800	5374	769	9·936 0621	739	4091	710	5821	680	20
41	9114	799	6143	768	1360	738	4801	709	6501	681	19
42	9913	798	6911	768	2098	738	5510	709	7182	680	18
43	9·927 0711	798	7679	768	2836	738	6219	708	7862	679	17
44	1509	797	8447	766	3574	737	6927	707	8541	679	16
45	2306	797	9213	767	4311	736	7634	708	9220	679	15
46	3103	796	9980	766	5047	736	8342	706	9899	678	14
47	3899	796	9·932 0746	765	5783	736	9048	707	9·945 0577	678	13
48	4695	795	1511	765	6519	735	9755	706	1255	677	12
49	5490	795	2276	764	7254	734	9·941 0461	705	1932	677	11
50	6285	794	3040	764	7988	734	1166	705	2609	676	10
51	7079	794	3804	763	8722	734	1871	704	3285	675	9
52	7873	793	4567	763	9456	733	2575	704	3960	676	8
53	8666	793	5330	762	9·937 0189	732	3279	703	4636	674	7
54	9459	792	6092	762	0921	732	3982	703	5310	675	6
55	9·928 0251	792	6854	762	1653	732	4685	703	5985	674	5
56	1043	791	7616	760	2385	731	5388	702	6659	673	4
57	1834	791	8376	761	3116	731	6090	701	7332	673	3
58	2625	790	9137	760	3847	730	6791	701	8005	672	2
59	3415	790	9897	759	4577	729	7492	701	8677	672	1
60	4205		9·933 0656		5306		8193		9349		0
′	32°	*diff.*	31°	*diff.*	30°	*diff.*	29°	*diff.*	28°	*diff.*	′

LOG. COSINE.

′	57°	*diff.*	58°	*diff.*	59°	*diff.*	60°	*diff.*	61°	*diff.*	′
0	10·1874826	2766	10·2042108	2811	10·2212263	2862	10·2385606	2918	10·2562480	2980	60
1	7592	2767	4919	2813	5125	2863	8524	2919	5460	2981	59
2	10·1880359	2768	7732	2813	7988	2863	10·2391443	2920	8441	2982	58
3	3127	2768	10·2050545	2814	10·2220851	2865	4363	2921	10·2571423	2983	57
4	5895	2769	3359	2814	3716	2866	7284	2922	4406	2985	56
5	8664	2770	6173	2816	6582	2866	10·2400206	2923	7391	2985	55
6	10·1891434	2770	8989	2816	9448	2867	3129	2924	10·2580376	2986	54
7	4204	2771	10·2061805	2817	10·2232315	2869	6053	2925	3362	2988	53
8	6975	2772	4622	2818	5184	2869	8978	2926	6350	2988	52
9	9747	2773	7440	2819	8053	2870	10·2411904	2926	9338	2990	51
10	10·1902520	2773	10·2070259	2820	10·2240923	2871	4830	2928	10·2592328	2991	50
11	5293	2774	3079	2820	3794	2872	7758	2929	5319	2992	49
12	8067	2775	5899	2821	6666	2872	10·2420687	2930	8311	2993	48
13	10·1910842	2775	8720	2822	9538	2874	3617	2931	10·2601304	2994	47
14	3617	2777	10·2081542	2823	10·2252412	2875	6548	2932	4298	2995	46
15	6394	2777	4365	2824	5287	2875	9480	2933	7293	2997	45
16	9171	2777	7189	2824	8162	2877	10·2432413	2934	10·2610290	2997	44
17	10·1921948	2779	10·2090013	2826	10·2261039	2877	5347	2935	3287	2999	43
18	4727	2779	2839	2826	3916	2878	8282	2935	6286	2999	42
19	7506	2780	5665	2827	6794	2879	10·2441217	2937	9285	3001	41
20	10·1930286	2781	8492	2827	9673	2880	4154	2938	10·2622286	3002	40
21	3067	2781	10·2101319	2829	10·2272553	2881	7092	2939	5288	3003	39
22	5848	2782	4148	2829	5434	2882	10·2450031	2940	8291	3004	38
23	8630	2783	6977	2831	8316	2883	2971	2941	10·2631295	3006	37
24	10·1941413	2784	9808	2831	10·2281199	2884	5912	2942	4301	3006	36
25	4197	2784	10·2112639	2832	4083	2884	8854	2943	7307	3008	35
26	6981	2786	5471	2833	6967	2886	10·2461797	2944	10·2640315	3008	34
27	9767	2786	8304	2833	9853	2846	4741	2945	3323	3010	33
28	10·1952553	2786	10·2121137	2835	10·2292739	2888	7686	2946	6333	3011	32
29	5339	2788	3972	2835	5627	2888	10·2470632	2948	9344	3012	31
30	8127	2788	6807	2836	8515	2889	3580	2948	10·2652356	3013	30
31	10·1960915	2789	9643	2837	10·2301404	2891	6528	2949	5369	3015	29
32	3704	2790	10·2132480	2838	4295	2891	9477	2950	8384	3015	28
33	6494	2790	5318	2838	7186	2892	10·2482427	2951	10·2661399	3017	27
34	9284	2791	8156	2840	10·2310078	2893	5378	2953	4416	3018	26
35	10·1972075	2792	10·2140996	2840	2971	2894	8331	2953	7434	3019	25
36	4867	2793	3836	2841	5865	2895	10·2491284	2954	10·2670453	3020	24
37	7660	2794	6677	2842	8760	2896	4238	2956	3473	3021	23
38	10·1980454	2794	9519	2843	10·2321656	2896	7194	2956	6494	3022	22
39	3248	2795	10·2152362	2844	4552	2898	10·2500150	2958	9516	3024	21
40	6043	2796	5206	2845	7450	2899	3108	2958	10·2682540	3024	20
41	8839	2796	8051	2845	10·2330349	2900	6066	2960	5564	3026	19
42	10·1991635	2798	10·2160896	2846	3249	2900	9026	2961	8590	3027	18
43	4433	2798	3742	2848	6149	2902	10·2511987	2961	10·2691617	3029	17
44	7231	2799	6590	2848	9051	2902	4948	2963	4646	3029	16
45	10·2000030	2800	9438	2849	10·2341953	2904	7911	2964	7675	3030	15
46	2830	2800	10·2172287	2849	4857	2904	10·2520875	2965	10·2700705	3032	14
47	5630	2901	5136	2851	7761	2905	3840	2966	3737	3033	13
48	8431	2802	7987	2851	10·2350666	2907	6806	2967	6770	3034	12
49	10·2011233	2803	10·2180838	2853	3573	2907	9773	2968	9804	3035	11
50	4036	2804	3691	2853	6480	2908	10·2532741	2969	10·2712839	3037	10
51	6840	2804	6544	2854	9388	2910	5710	2970	5876	3037	9
52	9644	2805	9398	2855	10·2362298	2910	8680	2971	8913	3039	8
53	10·2022449	2806	10·2192253	2856	5208	2911	10·2541651	2973	10·2721952	3040	7
54	5255	2807	5109	2857	8119	2912	4624	2973	4992	3041	6
55	8062	2808	7966	2857	10·2371031	2913	7597	2975	8033	3042	5
56	10·2030870	2808	10·2200823	2859	3944	2914	10·2550572	2975	10·2731075	3044	4
57	3678	2809	3682	2859	6858	2915	3547	2977	4119	3044	3
58	6487	2810	6541	2860	9773	2916	6524	2977	7163	3046	2
59	9297	2811	9401	2862	10·2382689	2917	9501	2979	10·2740209	3047	1
60	10·2042108		10·2212263		5606		10·2562480		3256		0
′	32°	*diff.*	31°	*diff.*	30°	*diff.*	29°	*diff.*	28°	*diff.*	′

LOG. COTAN.

′	62°	*dif.*	63°	*dif.*	64°	*dif.*	65°	*dif.*	66°	*dif.*	′
0	9·945 9349	672	9·949 8809	643	9·953 6602	616	9·957 2757	589	9·960 7302	562	60
1	9·946 0021	671	9452	643	7218	615	3346	588	7864	562	59
2	0692	670	9·950 0095	643	7833	615	3934	588	8426	561	58
3	1362	670	0738	642	8448	615	4522	588	8987	561	57
4	2032	670	1380	642	9063	614	5110	587	9548	560	56
5	2702	669	2022	641	9677	614	5697	587	9·961 0108	560	55
6	3371	669	2663	640	9·954 0291	613	6284	586	0668	560	54
7	4040	668	3303	641	0904	613	6870	586	1228	559	53
8	4708	668	3944	639	1517	612	7456	585	1787	559	52
9	5376	667	4583	640	2129	612	8041	585	2346	558	51
10	6043	667	5223	638	2741	611	8626	584	2904	558	50
11	6710	666	5861	639	3352	611	9210	584	3462	558	49
12	7376	666	6500	638	3963	611	9794	584	4020	556	48
13	8042	665	7138	637	4574	610	9·958 0378	583	4576	557	47
14	8707	665	7775	637	5184	609	0961	582	5133	556	46
15	9372	664	8412	637	5793	609	1543	582	5689	556	45
16	9·947 0036	664	9049	636	6402	609	2125	582	6245	555	44
17	0700	664	9685	635	7011	608	2707	581	6800	555	43
18	1364	663	9·951 0320	636	7619	608	3268	581	7355	554	42
19	2027	662	0956	634	8227	607	3869	581	7909	554	41
20	2689	663	1590	634	8834	607	4450	580	8463	553	40
21	3352	661	2224	634	9441	606	5030	579	9016	553	39
22	4013	661	2858	634	9·955 0047	606	5609	579	9569	553	38
23	4674	661	3492	632	0653	606	6188	579	9·962 0122	552	37
24	5335	660	4124	633	1259	605	6767	578	0674	552	36
25	5995	660	4757	632	1864	605	7345	578	1226	551	35
26	6655	659	5389	631	2469	604	7923	577	1777	551	34
27	7314	659	6020	631	3073	603	8500	577	2328	550	33
28	7973	658	6651	631	3676	604	9077	576	2878	550	32
29	8631	658	7282	630	4280	602	9653	576	3428	550	31
30	9289	658	7912	629	4882	603	9·959 0229	576	3978	549	30
31	9947	657	8541	630	5485	602	0805	575	4527	549	29
32	9·948 0604	656	9171	628	6087	601	1380	574	5076	548	28
33	1260	656	9799	629	6688	601	1954	574	5624	548	27
34	1916	656	9·952 0428	627	7289	601	2528	574	6172	547	26
35	2572	655	1055	628	7890	600	3102	573	6719	547	25
36	3227	654	1683	627	8490	599	3675	573	7266	546	24
37	3881	654	2310	626	9089	600	4248	573	7812	546	23
38	4535	654	2936	626	9689	598	4821	572	8358	546	22
39	5189	653	3562	626	9·956 0287	599	5393	571	8904	545	21
40	5842	653	4188	625	0886	597	5964	571	9449	545	20
41	6495	652	4813	624	1483	598	6535	571	9994	544	19
42	7147	652	5437	624	2081	597	7106	570	9·963 0538	544	18
43	7799	651	6061	624	2678	596	7676	570	1082	543	17
44	8450	651	6685	623	3274	596	8246	569	1625	543	16
45	9101	651	7308	623	3870	596	8815	569	2168	543	15
46	9752	650	7931	622	4466	595	9384	568	2711	542	14
47	9·949 0402	649	8553	622	5061	595	9952	568	3253	542	13
48	1051	649	9175	622	5656	594	9·960 0520	568	3795	541	12
49	1700	649	9797	621	6250	594	1088	567	4336	541	11
50	2349	648	9·953 0418	620	6844	593	1655	567	4877	540	10
51	2997	648	1038	620	7437	593	2222	566	5417	540	9
52	3645	647	1658	620	8030	593	2788	566	5957	539	8
53	4292	646	2278	619	8623	592	3354	565	6496	540	7
54	4938	647	2897	618	9215	591	3919	565	7036	538	6
55	5585	645	3515	619	9806	591	4484	564	7574	538	5
56	6230	646	4134	617	9·957 0397	591	5048	564	8112	538	4
57	6876	645	4751	618	0988	590	5612	564	8650	537	3
58	7521	644	5369	616	1578	590	6176	563	9187	537	2
59	8165	644	5985	617	2168	589	6739	563	9724	537	1
60	8809		6602		2757		7302		9·964 0261		0
′	27°	*dif.*	26°	*dif.*	25°	*dif.*	24°	*dif.*	23°	*dif.*	′

LOG. COSINE.

′	62°	*diff.*	63°	*diff.*	64°	*diff.*	65°	*diff.*	66°	*diff.*	′
0	10·2743256	3049	10·2928341	3124	10·3118182	3207	10·3313275	3299	10·3514169	3400	60
1	46305	3049	31465	3125	21389	3209	16574	3300	17569	3403	59
2	49354	3051	34590	3126	24598	3210	19874	3303	20972	3404	58
3	52405	3052	37716	3128	27808	3211	23177	3304	24376	3407	57
4	55457	3053	40844	3129	31019	3213	26481	3305	27783	3407	56
5	58510	3054	43973	3130	34232	3215	29786	3307	31190	3410	55
6	61564	3055	47103	3132	37447	3215	33093	3309	34600	3412	54
7	64619	3057	50235	3133	40662	3218	36402	3310	38012	3413	53
8	67676	3058	53368	3135	43880	3219	39712	3313	41425	3415	52
9	70734	3059	56503	3135	47099	3220	43025	3313	44840	3417	51
10	73793	3060	59638	3137	50319	3222	46338	3316	48257	3419	50
11	76853	3062	62775	3139	53541	3223	49654	3316	51676	3421	49
12	79915	3063	65914	3140	56764	3225	52970	3319	55097	3422	48
13	82978	3064	69054	3141	59989	3226	56259	3320	58519	3424	47
14	86042	3065	72195	3142	63215	3228	59609	3322	61943	3426	46
15	89107	3066	75337	3144	66443	3229	62931	3324	65369	3428	45
16	92173	3068	78481	3145	69672	3230	66255	3325	68797	3430	44
17	95241	3069	81626	3147	72902	3233	69580	3327	72227	3431	43
18	98310	3070	84773	3147	76135	3233	72907	3328	75658	3434	42
19	10·2801380	3071	87920	3150	79368	3236	76235	3331	79092	3435	41
20	04451	3073	91070	3150	82604	3236	79566	3331	82527	3437	40
21	07524	3074	94220	3152	85840	3239	82897	3334	85964	3439	39
22	10598	3075	97372	3154	89079	3239	86231	3335	89403	3441	38
23	13673	3076	10·3000526	3154	92318	3242	89566	3337	92844	3442	37
24	16749	3078	03680	3156	95560	3242	92903	3339	96286	3445	36
25	19827	3079	06836	3158	98802	3245	96242	3340	99731	3446	35
26	22906	3080	09994	3159	10·3202047	3245	99582	3342	10·3603177	3448	34
27	25986	3081	13153	3160	05292	3248	10·3402924	3343	06625	3450	33
28	29067	3082	16313	3161	08540	3249	06267	3346	10075	3452	32
29	32149	3084	19474	3163	11789	3250	09613	3346	13527	3454	31
30	35233	3085	22637	3165	15039	3252	12959	3349	16981	3456	30
31	38318	3087	25802	3166	18291	3253	16308	3351	20437	3457	29
32	41405	3087	28968	3167	21544	3255	19659	3352	23894	3460	28
33	44492	3089	32135	3168	24799	3257	23011	3353	27354	3461	27
34	47581	3090	35303	3170	28056	3258	26364	3356	30815	3463	26
35	50671	3092	38473	3172	31314	3260	29720	3357	34278	3465	25
36	53763	3092	41645	3172	34574	3261	33077	3359	37743	3467	24
37	56855	3094	44817	3174	37835	3262	36436	3360	41210	3469	23
38	59949	3095	47991	3176	41097	3265	39796	3363	44679	3471	22
39	63044	3097	51167	3177	44362	3266	43159	3364	48150	3472	21
40	66141	3098	54344	3178	47628	3267	46523	3365	51622	3475	20
41	69239	3099	57522	3180	50895	3269	49888	3368	55097	3477	19
42	72338	3100	60702	3181	54164	3270	53256	3369	58574	3478	18
43	75438	3101	63883	3183	57434	3272	56625	3371	62052	3480	17
44	78539	3103	67066	3184	60706	3274	59996	3373	65532	3483	16
45	81642	3104	70250	3185	63980	3275	63369	3374	69015	3484	15
46	84746	3106	73435	3187	67255	3277	66743	3376	72499	3486	14
47	87852	3107	76622	3189	70532	3278	70119	3378	75985	3468	13
48	90959	3108	79811	3189	73810	3280	73497	3380	79473	3490	12
49	94067	3109	83000	3191	77090	3282	76877	3381	82963	3492	11
50	97176	3111	86191	3193	80372	3283	80258	3383	86455	3493	10
51	10·2900287	3112	89384	3194	83655	3285	83641	3385	89948	3496	9
52	03399	3113	92578	3196	86940	3286	87026	3387	93444	3498	8
53	06512	3114	95774	3196	90226	3288	90413	3388	96942	3500	7
54	09626	3116	98970	3199	93514	3289	93801	3390	10·3700442	3501	6
55	12742	3117	10·3102169	3200	96803	3291	97191	3392	03943	3504	5
56	15859	3119	05369	3201	10·3300094	3293	10·3500583	3394	07447	3505	4
57	18978	3120	08570	3203	03387	3294	03977	3395	10952	3508	3
58	22098	3121	11773	3204	06681	3296	07372	3398	14460	3509	2
59	25219	3122	14977	3205	09977	3298	10770	3399	17969	3512	1
60	28341		18182		13275		14169		21481		0
′	27°	*diff.*	26°	*diff.*	25°	*diff.*	24°	*diff.*	23°	*diff.*	′

LOG. COTAN.

LOG. SINE.

′	67°	dif.	68°	dif.	69°	dif.	70°	dif.	71°	dif.	′
0	9·964 0261		9·967 1659		9·970 1517		9·972 9858		9·975 6701		60
1	0797	536	2169	510	2002	485	9·973 0318	460	7135	434	59
2	1332	535	2679	510	2486	484	0777	459	7570	435	58
3	1868	536	3188	509	2970	484	1236	459	8004	434	57
4	2402	534	3697	509	3454	484	1694	458	8437	413	56
5	2937	535	4205	508	3937	483	2152	458	8870	433	55
6	3470	533	4713	508	4419	482	2610	458	9303	433	54
7	4004	534	5221	508	4902	483	3067	457	9736	433	53
8	4537	533	5728	507	5383	481	3523	456	9·976 0167	431	52
9	5069	532	6235	507	5865	482	3980	457	0599	432	51
10	5602	533	6741	506	6346	481	4435	455	1030	431	50
11	6133	531	7247	506	6826	480	4891	456	1461	431	49
12	6665	532	7753	506	7306	480	5346	455	1891	430	48
13	7195	530	8258	505	7786	480	5801	455	2321	430	47
14	7726	531	8763	505	8265	479	6255	454	2750	429	46
15	8256	530	9267	504	8744	479	6709	454	3179	429	45
16	8785	529	9771	504	9223	479	7162	453	3608	429	44
17	9314	529	9·968 0274	503	9701	478	7615	453	4036	428	43
18	9843	529	0777	503	9·971 0178	477	8067	452	4464	428	42
19	9·965 0371	528	1279	502	0655	477	8519	452	4891	427	41
20	0899	528	1781	502	1132	477	8971	452	5318	427	40
21	1426	527	2283	502	1608	476	9422	451	5745	427	39
22	1953	527	2784	501	2084	476	9873	451	6171	426	38
23	2480	527	3285	501	2560	476	9·974 0324	451	6597	426	37
24	3006	526	3786	501	3035	475	0774	450	7022	425	36
25	3532	526	4286	500	3509	474	1224	450	7447	425	35
26	4057	525	4785	499	3984	475	1673	449	7872	425	34
27	4582	525	5284	499	4457	473	2122	449	8296	424	33
28	5106	524	5783	499	4931	474	2570	448	8720	424	32
29	5630	524	6281	498	5404	473	3018	448	9143	423	31
30	6153	523	6779	498	5876	472	3466	448	9566	423	30
31	6677	524	7276	497	6348	472	3913	447	9988	422	29
32	7199	522	7773	497	6820	472	4359	446	9·977 0410	422	28
33	7721	522	8270	497	7291	471	4806	447	0832	422	27
34	8243	522	8766	496	7762	471	5252	446	1253	421	26
35	8764	521	9262	496	8233	471	5697	445	1674	421	25
36	9285	521	9757	495	8703	470	6142	445	2095	421	24
37	9806	521	9·969 0252	495	9172	469	6587	445	2515	420	23
38	9·966 0326	520	0746	494	9642	470	7031	444	2934	419	22
39	0846	520	1241	495	9·972 0110	468	7475	444	3354	420	21
40	1365	519	1734	493	0579	469	7918	443	3772	418	20
41	1884	519	2227	493	1047	468	8361	443	4191	419	19
42	2402	518	2720	493	1514	467	8804	443	4609	418	18
43	2920	518	3212	492	1981	467	9246	442	5026	417	17
44	3437	517	3704	492	2448	467	9688	442	5444	418	16
45	3954	517	4196	492	2914	466	9·975 0129	441	5860	416	15
46	4471	517	4687	491	3380	466	0570	441	6277	417	14
47	4987	516	5177	490	3845	465	1011	441	6693	416	13
48	5503	516	5668	491	4310	465	1451	440	7108	415	12
49	6018	515	6158	490	4775	465	1891	440	7523	415	11
50	6533	515	6647	489	5239	464	2330	439	7938	415	10
51	7048	515	7136	489	5703	464	2769	439	8353	415	9
52	7562	514	7624	488	6166	463	3208	439	8766	413	8
53	8075	513	8112	488	6629	463	3646	438	9180	414	7
54	8588	513	8600	488	7092	463	4083	437	9593	413	6
55	9101	513	9087	487	7554	462	4521	438	9·978 0006	413	5
56	9614	513	9574	497	8016	462	4957	436	0418	412	4
57	9·967 0125	511	9·970 0061	487	8477	461	5394	437	0830	412	3
58	0637	512	0547	486	8938	461	5830	436	1241	411	2
59	1148	511	1032	485	9398	460	6265	435	1653	412	1
60	1659	511	1517	485	9858	460	6701	436	2063	410	0
′	22°	dif.	21°	dif.	20°	dif.	19°	dif.	18°	dif.	′

LOG. COSINE.

′	67°	*diff.*	68°	*diff.*	69°	*diff.*	70°	*diff.*	71°	*diff.*	′
0	10·3721481	3513	10·3935904	3639	10·4158226	3777	10·4389341	3932	10·4630281	4106	60
1	24994	3515	39543	3640	62003	3780	93273	3935	34387	4108	59
2	28509	3518	43183	3643	65783	3782	97208	3938	38495	4112	58
3	32027	3519	46826	3645	69565	3784	10 4401146	3940	42607	4115	57
4	35546	3522	50471	3647	73349	3787	05086	3943	46722	4117	56
5	39068	3523	54118	3649	77136	3790	09029	3946	50839	4121	55
6	42591	3525	57767	3652	80926	3792	12975	3948	54960	4124	54
7	46116	3528	61419	3654	84718	3794	16923	3952	59084	4127	53
8	49644	3529	65073	3656	88512	3797	20875	3954	63211	4130	52
9	53173	3531	68729	3658	92309	3799	24829	3957	67341	4133	51
10	56704	3533	72387	3660	96108	3802	28786	3959	71474	4137	50
11	60237	3536	76047	3663	99910	3804	32745	3963	75611	4139	49
12	63773	3537	79710	3665	10·4203714	3807	36708	3965	79750	4143	48
13	67310	3540	83375	3667	07521	3810	40673	3968	83893	4146	47
14	70850	3541	87042	3669	11331	3811	44641	3971	88039	4148	46
15	74391	3543	90711	3672	15142	3815	48612	3973	92187	4152	45
16	77934	3546	94383	3674	18957	3817	52585	3977	96339	4156	44
17	81480	3547	98057	3676	22774	3819	56562	3979	10 4700495	4158	43
18	85027	3550	10·4001733	3679	26593	3822	60541	3982	04653	4161	42
19	88577	3551	05412	3680	30415	3824	64523	3985	08814	4165	41
20	92128	3554	09092	3683	34239	3827	68508	3988	12979	4168	40
21	95682	3556	12775	3685	38066	3830	72496	3990	17147	4171	39
22	99238	3557	16460	3688	41896	3832	76486	3993	21318	4174	38
23	10·3802795	3560	20148	3690	45728	3834	80479	3997	25492	4177	37
24	06355	3562	23838	3692	49562	3837	84476	3999	29669	4181	36
25	09917	3564	27530	3694	53399	3840	88475	4002	33850	4184	35
26	13481	3566	31224	3697	57239	3842	92477	4004	38034	4187	34
27	17047	3568	34921	3699	61081	3845	96481	4008	42221	4190	33
28	20615	3570	38620	3701	64926	3847	10·4500489	4011	46411	4194	32
29	24185	3572	42321	3704	68773	3850	04500	4013	50605	4196	31
30	27757	3574	46025	3706	72623	3853	08513	4016	54801	4200	30
31	31331	3576	49731	3708	76476	3855	12529	4019	59001	4204	29
32	34907	3579	53439	3710	80331	3858	16548	4022	63205	4206	28
33	38486	3580	57149	3713	84189	3860	20570	4025	67411	4210	27
34	42066	3583	60862	3715	88049	3863	24595	4028	71621	4213	26
35	45649	3585	64577	3718	91912	3865	28623	4031	75834	4216	25
36	49234	3586	68295	3720	95777	3868	32654	4034	80050	4220	24
37	52820	3589	72015	3722	99645	3871	36688	4036	84270	4222	23
38	56409	3591	75737	3724	10·4303516	3873	40724	4040	88492	4226	22
39	60000	3593	79461	3727	07389	3876	44764	4043	92718	4230	21
40	63593	3595	83188	3730	11265	3879	48807	4045	96948	4233	20
41	67188	3598	86918	3731	15144	3881	52852	4048	10·4801181	4236	19
42	70786	3599	90649	3734	19025	3884	56900	4052	05417	4239	18
43	74385	3602	94383	3736	22909	3886	60952	4054	09656	4243	17
44	77987	3604	98119	3739	26795	3889	65006	4057	13899	4246	16
45	81591	3605	10·4101858	3741	30684	3892	69063	4060	18145	4249	15
46	85196	3608	05599	3744	34576	3894	73123	4064	22394	4253	14
47	88804	3610	09343	3745	38470	3897	77187	4066	26647	4256	13
48	92414	3613	13088	3749	42367	3900	81253	4069	30903	4259	12
49	96027	3614	16837	3750	46267	3902	85322	4072	35162	4263	11
50	99641	3617	20587	3753	50169	3906	89394	4075	39425	4266	10
51	10·3903258	3618	24340	3756	54075	3907	93469	4078	43691	4270	9
52	06876	3621	28096	3757	57982	3911	97547	4082	47961	4273	8
53	10497	3623	31853	3761	61893	3913	10 4601629	4084	52234	4276	7
54	14120	3626	35614	3762	65806	3916	05713	4087	56510	4280	6
55	17746	3627	39376	3765	69722	3918	09800	4090	60790	4283	5
56	21373	3630	43141	3768	73640	3921	13890	4093	65073	4286	4
57	25003	3631	46909	3770	77561	3924	17983	4097	69359	4290	3
58	28634	3634	50679	3772	81485	3927	22080	4099	73649	4294	2
59	32268	3636	54451	3775	85412	3929	26179	4102	77943	4297	1
60	35904		58226		89341		30281		82240		0
′	22°	*diff.*	21°	*diff.*	20°	*diff.*	19°	*diff.*	18°	*diff.*	′

LOG. COTAN.

′	72°	*dif.*	73°	*dif.*	74°	*dif.*	75°	*dif.*	76°	*dif.*	′
0	9·978 2063	411	9·980 5963	386	9·982 8416	362	9·984 9438	338	9·986 9041	315	60
1	2474	409	6349	386	8778	362	9776	338	9356	314	59
2	2883	410	6735	385	9140	361	9·985 0114	338	9670	314	58
3	3293	409	7120	385	9501	361	0452	337	9984	314	57
4	3702	409	7505	384	9862	361	0789	336	9·987 0298	313	56
5	4111	408	7889	384	9·983 0223	360	1125	337	0611	313	55
6	4519	408	8273	384	0583	359	1462	336	0924	312	54
7	4927	407	8657	383	0942	360	1798	335	1236	313	53
8	5334	407	9040	383	1302	359	2133	335	1549	311	52
9	5741	407	9423	382	1661	358	2468	335	1860	311	51
10	6148	406	9805	382	2019	358	2803	335	2171	311	50
11	6554	406	9·981 0187	382	2377	358	3138	333	2482	311	49
12	6960	405	0569	381	2735	357	3471	334	2793	310	48
13	7365	405	0950	381	3092	357	3805	333	3103	310	47
14	7770	405	1331	380	3449	356	4138	333	3413	309	46
15	8175	404	1711	390	3805	356	4471	332	3722	309	45
16	8579	404	2091	380	4161	356	4803	332	4031	308	44
17	8983	403	2471	379	4517	355	5135	332	4339	309	43
18	9386	403	2850	379	4872	355	5467	331	4648	307	42
19	9789	403	3229	379	5227	355	5798	331	4955	308	41
20	9·979 0192	402	3608	378	5582	354	6129	331	5263	307	40
21	0594	402	3986	377	5936	354	6460	330	5570	306	39
22	0996	401	4363	377	6290	353	6790	329	5876	307	38
23	1397	401	4740	377	6643	353	7119	330	6183	305	37
24	1798	400	5117	377	6996	352	7449	328	6488	306	36
25	2198	401	5494	376	7348	353	7777	329	6794	305	35
26	2599	399	5870	375	7701	351	8106	328	7099	305	34
27	2998	400	6245	375	8052	352	8434	328	7404	304	33
28	3398	398	6620	375	8404	351	8762	327	7708	304	32
29	3796	399	6995	375	8755	350	9089	327	8012	303	31
30	4195	398	7370	374	9105	350	9416	326	8315	303	30
31	4593	398	7744	373	9455	350	9742	327	8618	303	29
32	4991	397	8117	373	9805	349	9·986 0069	325	8921	302	28
33	5388	397	8490	373	9·984 0154	349	0394	326	9223	302	27
34	5785	397	8863	373	0503	349	0720	325	9525	302	26
35	6182	396	9236	372	0852	348	1045	324	9827	301	25
36	6578	395	9608	371	1200	348	1369	324	9·988 0128	301	24
37	6973	396	9979	372	1548	347	1693	324	0429	300	23
38	7369	395	9·982 0351	370	1895	347	2017	323	0729	300	22
39	7764	394	0721	371	2242	347	2340	323	1029	300	21
40	8158	394	1092	370	2589	346	2663	323	1329	299	20
41	8552	394	1462	369	2935	346	2986	322	1628	299	19
42	8946	393	1831	370	3281	345	3308	322	1927	298	18
43	9339	393	2201	368	3626	345	3630	322	2225	298	17
44	9732	392	2569	369	3971	345	3952	321	2523	298	16
45	9·980 0124	392	2938	368	4316	344	4273	320	2821	297	15
46	0516	392	3306	368	4660	344	4593	320	3118	297	14
47	0908	391	3674	367	5004	343	4913	320	3415	297	13
48	1299	391	4041	367	5347	343	5233	320	3712	296	12
49	1690	391	4408	366	5690	343	5553	319	4008	295	11
50	2081	390	4774	366	6033	342	5872	319	4303	296	10
51	2471	339	5140	366	6375	342	6191	318	4599	295	9
52	2860	390	5506	365	6717	342	6509	318	4894	294	8
53	3250	389	5871	365	7059	341	6827	317	5188	294	7
54	3639	388	6236	364	7400	340	7144	317	5482	294	6
55	4027	388	6600	364	7740	341	7461	317	5776	294	5
56	4415	388	6964	364	8081	339	7778	316	6070	293	4
57	4803	387	7328	363	8420	340	8094	316	6363	292	3
58	5190	387	7691	363	8760	339	8410	316	6655	292	2
59	5577	386	8054	362	9099	339	8726	315	6947	292	1
60	5963		8416		9438		9041		7239		0
′	17°	*dif.*	16°	*dif.*	15°	*dif.*	14°	*dif.*	13°	*dif.*	′

LOG. COSINE.

′	72°	*diff.*	73°	*diff.*	74°	*diff.*	75°	*diff.*	76°	*diff.*	′
0	10·4882240	4300	10·5146610	4520	10·5425036	4770	10·5719475	5056	10·6032289	5385	60
1	86540	4304	51130	4524	29806	4774	24531	5061	37674	5391	59
2	90844	4307	55654	4528	34580	4779	29592	5066	43065	5397	58
3	95151	4310	60182	4532	39359	4784	34658	5071	48462	5402	57
4	99461	4315	64714	4536	44143	4788	39729	5077	53864	5409	56
5	10·4903776	4317	69250	4540	48931	4793	44806	5081	59273	5414	55
6	08093	4321	73790	4544	53724	4797	49887	5087	64687	5420	54
7	12414	4325	78334	4548	58521	4801	54974	5091	70107	5427	53
8	16739	4328	82882	4552	63322	4806	60065	5097	75534	5432	52
9	21067	4331	87434	4555	68128	4811	65162	5103	80966	5439	51
10	25398	4335	91989	4560	72939	4815	70265	5107	86405	5444	50
11	29733	4339	96549	4564	77754	4819	75372	5113	91849	5451	49
12	34072	4342	10·5201113	4568	82573	4825	80485	5117	97300	5456	48
13	38414	4346	05681	4571	87398	4828	85602	5123	10·6102756	5463	47
14	42760	4349	10252	4576	92226	4834	90725	5129	08219	5469	46
15	47109	4353	14828	4580	97060	4838	95854	5133	13688	5475	45
16	51462	4356	19408	4583	10·5501898	4842	10·5800987	5139	19163	5481	44
17	55818	4360	23991	4588	06740	4847	06126	5145	24644	5487	43
18	60178	4363	28579	4592	11587	4852	11271	5149	30131	5493	42
19	64541	4367	33171	4596	16439	4857	16420	5155	35624	5500	41
20	68908	4371	37767	4600	21296	4861	21575	5160	41124	5506	40
21	73279	4374	42367	4604	26157	4865	26735	5166	46630	5512	39
22	77653	4378	46971	4608	31022	4871	31901	5171	52142	5518	38
23	82031	4381	51579	4613	35893	4875	37072	5176	57660	5524	37
24	86412	4385	56192	4616	40768	4880	42248	5182	63184	5531	36
25	90797	4389	60808	4620	45648	4884	47430	5187	68715	5537	35
26	95186	4392	65428	4625	50532	4889	52617	5192	74252	5543	34
27	99578	4396	70053	4629	55421	4894	57809	5196	79795	5550	33
28	10·5003974	4400	74682	4633	60315	4899	63007	5204	85345	5555	32
29	08374	4403	79315	4637	65214	4903	68211	5208	90900	5563	31
30	12777	4407	83952	4641	70117	4908	73419	5215	96463	5568	30
31	17184	4410	88593	4645	75025	4913	78634	5220	10·6202031	5575	29
32	21594	4415	93238	4650	79938	4917	83854	5225	07606	5581	28
33	26009	4417	97888	4653	84855	4923	89079	5231	13187	5588	27
34	30426	4422	0·5302541	4658	89778	4927	94310	5236	18775	5594	26
35	34848	4425	07199	4662	94705	4932	99546	5242	24369	5601	25
36	39273	4429	11861	4666	99637	4937	10·5904788	5247	29970	5607	24
37	43702	4433	16527	4671	10·5604574	4941	10035	5253	35577	5613	23
38	48135	4436	21198	4675	09515	4947	15288	5259	41190	5620	22
39	52571	4441	25873	4679	14462	4951	20547	5264	46810	5627	21
40	57012	4443	30552	4683	19413	4956	25811	5270	52437	5633	20
41	61455	4448	35235	4687	24369	4961	31081	5275	58070	5639	19
42	65903	4451	39922	4692	29330	4966	36356	5281	63709	5646	18
43	70354	4456	44614	4696	34296	4971	41637	5287	69355	5653	17
44	74810	4459	49310	4700	39267	4976	46924	5292	75008	5659	16
45	79269	4462	54010	4705	44243	4981	52216	5298	80667	5666	15
46	83731	4467	58715	4709	49224	4985	57514	5304	86333	5673	14
47	88198	4470	63424	4713	54209	4991	62818	5309	92006	5679	13
48	92668	4474	68137	4718	59200	4995	68127	5315	97685	5686	12
49	97142	4478	72855	4722	64195	5001	73442	5321	10·6303371	5692	11
50	10·5101620	4482	77577	4726	69196	5005	78763	5327	09063	5699	10
51	06102	4485	82303	4730	74201	5010	84090	5332	14762	5706	9
52	10587	4489	87033	4735	79211	5016	89422	5338	20468	5713	8
53	15076	4494	91768	4740	84227	5020	94760	5344	26181	5719	7
54	19570	4497	96508	4743	89247	5026	10·6000104	5349	31900	5726	6
55	24067	4500	10·5401251	4748	94273	5030	05453	5356	37626	5733	5
56	28567	4505	05999	4753	99303	5036	10809	5361	43359	5740	4
57	33072	4509	10752	4757	10·5704339	5040	16170	5367	49099	5746	3
58	37581	4512	15509	4761	09379	5046	21537	5374	54845	5754	2
59	42093	4517	20270	4766	14425	5050	26911	5378	60599	5760	1
60	46610		25036		19475		32289		66359		0
′	17°	*diff.*	16°	*diff.*	15°	*diff.*	14°	*diff.*	13°	*diff.*	′

LOG. COTAN.

′	77°	*diff.*	78°	*diff.*	79°	*diff.*	80°	*diff.*	81°	*diff.*	′
0	9·9887 239	292	9·9904 044	268	9·9919 466	245	9·9933 515	222	9·9946 199	200	60
1	531	291	312	268	711	245	737	222	399	200	59
2	822	291	580	268	956	245	959	222	599	199	58
3	9·9888 113	290	848	267	9·9920 201	244	9·9934 181	222	798	199	57
4	403	290	9·9905 115	267	445	244	403	221	997	198	56
5	693	289	382	266	689	243	624	220	9·9947 195	198	55
6	982	289	648	266	932	243	844	221	393	198	54
7	9·9889 271	289	914	266	9·9921 175	243	9·9935 065	220	591	197	53
8	560	289	9·9906 180	265	418	242	285	219	788	197	52
9	849	288	445	265	660	242	504	219	985	196	51
10	9·9890 137	287	710	264	902	242	723	219	9·9948 181	196	50
11	424	287	974	265	9·9922 144	241	942	218	377	196	49
12	711	287	9·9907 239	263	385	241	9·9936 160	218	573	196	48
13	998	287	502	264	626	240	378	218	769	195	47
14	9·9891 285	286	766	263	866	240	596	217	964	194	46
15	571	285	9·9908 029	262	9·9923 106	240	813	217	9·9949 158	194	45
16	856	286	291	262	346	239	9·9937 030	217	352	194	44
17	9·9892 142	285	553	262	585	239	247	216	546	194	43
18	427	284	815	262	824	239	463	216	740	193	42
19	711	284	9·9909 077	261	9·9924 063	238	679	215	933	193	41
20	995	284	338	260	301	238	894	215	9·9950 126	192	40
21	9·9893 279	283	598	261	539	237	9·9938 109	215	318	192	39
22	562	283	859	260	776	237	324	214	510	192	38
23	845	283	9·9910 119	259	9·9925 013	237	538	214	702	191	37
24	9·9894 128	282	378	259	250	236	752	213	893	191	36
25	410	282	637	259	486	236	965	213	9·9951 084	190	35
26	692	281	896	258	722	235	9·9939 178	213	274	190	34
27	973	281	9·9911 154	258	957	235	391	212	464	190	33
28	9·9895 254	281	412	258	9·9926 192	235	603	212	654	190	32
29	535	280	670	257	427	234	815	212	844	189	31
30	815	280	927	257	661	234	9·9940 027	211	9·9952 033	188	30
31	9·9896 095	279	9·9912 184	256	895	234	238	211	221	188	29
32	374	280	440	256	9·9927 129	233	449	210	409	188	28
33	654	278	696	256	362	233	659	211	597	188	27
34	932	279	952	255	595	232	870	209	785	187	26
35	9·9897 211	278	9·9913 207	255	827	232	9·9941 079	210	972	187	25
36	489	277	462	255	9·9928 059	232	289	209	9·9953 159	186	24
37	766	277	717	254	291	231	498	208	345	186	23
38	9·9898 043	277	971	254	522	231	706	208	531	186	22
39	320	277	9·9914 225	253	753	231	914	208	717	185	21
40	597	276	478	253	984	230	9·9942 122	208	902	185	20
41	873	275	731	253	9·9929 214	230	330	207	9·9954 087	184	19
42	9·9899 148	275	984	252	444	229	537	206	271	184	18
43	423	275	9·9915 236	252	673	229	743	207	455	164	17
44	698	275	488	251	902	229	950	206	639	183	16
45	973	274	739	251	9·9930 131	228	9·9943 156	205	822	183	15
46	9·9900 247	274	990	251	359	228	361	205	9·9955 005	183	14
47	521	273	9·9916 241	251	587	227	566	205	188	182	13
48	794	273	492	249	814	227	771	204	370	182	12
49	9·9301 067	272	741	250	9·9931 041	227	975	205	552	182	11
50	339	273	991	249	268	226	9·9944 180	203	734	181	10
51	612	271	9·9917 240	249	494	226	383	204	915	180	9
52	883	272	489	248	720	226	587	202	9·9956 095	181	8
53	9·9902 155	271	737	249	946	225	789	203	276	180	7
54	426	271	986	247	9·9932 171	225	992	202	456	179	6
55	697	270	9·9918 233	247	396	225	9·9945 194	202	635	180	5
56	967	270	480	247	621	224	396	201	815	178	4
57	9·9903 237	269	727	247	845	223	597	201	993	179	3
58	506	269	974	246	9·9933 068	224	798	201	9·9957 172	178	2
59	775	269	9·9919 220	246	292	223	999	200	350	178	1
60	9·9904 044		466		515		9·9946 199		528		0
′	12°	*diff.*	11°	*diff.*	10°	*diff.*	9°	*diff.*	8°	*diff.*	′

LOG. COSINE.

′	77°	*diff.*	78°	*diff.*	79°	*diff.*	80°	*diff.*	81°	*diff.*	′
0	10·6366359	5767	10·6725255	6216	10·7113477	6750	10·7536812	7394	10·8002875	8184	60
1	72126	5774	31471	6224	20227	6759	44206	7405	11059	8198	59
2	77900	5781	37695	6232	26986	6769	51611	7417	19257	8213	58
3	83681	5788	43927	6241	33755	6779	59028	7429	27470	8228	57
4	89469	5795	50168	6248	40534	6789	66457	7440	35698	8243	56
5	95264	5801	56416	6257	47323	6799	73897	7453	43941	8257	55
6	10·6401065	5809	62673	6266	54122	6808	81350	7465	52198	8273	54
7	06874	5816	68939	6273	60930	6819	88815	7477	60471	8288	53
8	12690	5823	75212	6282	67749	6828	96292	7490	68759	8302	52
9	18513	5829	81494	6290	74577	6838	10·7603782	7501	77061	8318	51
10	24342	5837	87784	6298	81415	6849	11283	7514	85379	8334	50
11	30179	5844	94082	6307	88264	6858	18797	7525	93713	8348	49
12	36023	5851	10·6800389	6316	95122	6869	26322	7539	10·8102061	8364	48
13	41874	5859	06705	6323	10·7201991	6878	33861	7550	10425	8379	47
14	47733	5865	18028	6332	08869	6889	41411	7563	18804	8394	46
15	53598	5872	19360	6341	15758	6899	48974	7575	27198	8410	45
16	59470	5880	25701	6349	22657	6909	56549	7588	35608	8426	44
17	65350	5887	32050	6358	29566	6920	64137	7601	44034	8441	43
18	71237	5894	38408	6366	36486	6930	71738	7612	52475	8457	42
19	77131	5901	44774	6375	43416	6940	79350	7626	60932	8473	41
20	83032	5909	51149	6383	50356	6950	86976	7638	69405	8489	40
21	88941	5916	57532	6392	57306	6961	94614	7651	77894	8504	39
22	94857	5923	63924	6401	64267	6971	10·7702265	7664	86398	8520	38
23	10·6500780	5930	70325	6409	71238	6982	09929	7676	94918	8536	37
24	06710	5938	76734	6418	78220	6992	17605	7689	10·8203454	8553	36
25	12648	5945	83152	6427	85212	7002	25294	7702	12007	8568	35
26	18593	5953	89579	6436	92214	7014	32996	7715	20575	8585	34
27	24546	5960	96015	6444	99228	7023	40711	7728	29160	8601	33
28	30506	5967	10·6902459	6453	10·7306251	7035	48439	7742	37761	8617	32
29	36473	5975	08912	6462	13286	7045	56181	7754	46378	8634	31
30	42448	5982	15374	6471	20331	7056	63935	7767	55012	8650	30
31	48430	5990	21845	6480	27387	7066	71702	7780	63662	8666	29
32	54420	5997	28325	6488	34453	7077	79482	7794	72328	8683	28
33	60417	6005	34813	6498	41530	7088	87276	7807	81011	8700	27
34	66422	6012	41311	6506	48618	7099	95083	7820	89711	8717	26
35	72434	6020	47817	6516	55717	7110	10·7802903	7833	98428	8733	25
36	78454	6027	54333	6524	62827	7120	10736	7847	10·8307161	8750	24
37	84481	6035	60857	6534	69947	7132	18583	7861	15911	8767	23
38	90516	6043	67391	6543	77079	7142	26444	7873	24678	8784	22
39	96559	6050	73934	6552	84221	7154	34317	7888	33462	8801	21
40	10·6602609	6058	80486	6560	91375	7164	42205	7901	42263	8818	20
41	08667	6066	87046	6571	98539	7176	50106	7914	51081	8836	19
42	14733	6073	93617	6579	10·7405715	7186	58020	7929	59917	8852	18
43	20806	6081	10·7000196	6588	12901	7198	65949	7942	68769	8870	17
44	26887	6089	06784	6598	20099	7209	73891	7956	77639	8888	16
45	32976	6097	13382	6607	27308	7220	81847	7969	86527	8904	15
46	39073	6104	19989	6616	34528	7232	89816	7984	95431	8923	14
47	45177	6112	26605	6626	41760	7243	97800	7997	10·8404354	8940	13
48	51289	6120	33231	6635	49003	7254	10·7905797	8012	13294	8958	12
49	57409	6128	39866	6645	56257	7266	13809	8026	22252	8975	11
50	63537	6136	46511	6653	63523	7277	21835	8039	31227	8993	10
51	69673	6144	53164	6664	70800	7288	29874	8054	40220	9011	9
52	75817	6152	59828	6672	78088	7300	37928	8068	49231	9030	8
53	81969	6159	66500	6683	85388	7311	45996	8082	58261	9047	7
54	88128	6168	73183	6691	92699	7323	54078	8097	67308	9065	6
55	94296	6176	79874	6702	10·7500022	7335	62175	8111	76373	9084	5
56	10·6700472	6183	86576	6711	07357	7346	70286	8126	85457	9102	4
57	06655	6192	93287	6720	14703	7358	78412	8139	94559	9120	3
58	12847	6200	10·7100007	6730	22061	7370	86551	8155	10·8503679	9139	2
59	19047	6208	06737	6740	29431	7381	94706	8169	12818	9157	1
60	25255		13477		36812		10·8002875		21975		0
′	12°	*diff.*	11°	*diff.*	10°	*diff.*	9°	*diff.*	8°	*diff.*	′

LOG. COTAN.

″	82°	*diff.*	83°	*diff.*	84°	*diff.*	85°	*diff.*	86°	*diff.*	′
0	9·9957 528	177	9·9967 507	155	9·9976 143	133	9·9983 442	111	9·9989 408	88	60
1	705	177	662	155	276	132	553	110	496	88	59
2	882	177	817	154	408	132	663	109	584	87	58
3	9·9958 059	176	971	154	540	132	772	109	671	87	57
4	235	176	9·9968 125	153	672	131	881	109	758	87	56
5	411	175	278	153	803	130	990	109	845	86	55
6	586	175	431	153	933	131	9·9984 099	108	931	86	54
7	761	175	584	152	9·9977 064	130	207	108	9·9990 017	86	53
8	936	175	736	152	194	129	315	107	103	85	52
9	9·9959 111	173	888	152	323	130	422	107	188	85	51
10	284	174	9·9969 040	151	453	129	529	107	273	84	50
11	458	173	191	151	582	128	636	106	357	84	49
12	631	173	342	150	710	128	742	106	441	84	48
13	804	173	492	150	838	128	848	105	525	83	47
14	977	172	642	150	966	127	953	105	608	83	46
15	9·9960 149	172	792	149	9·9978 093	127	9·9985 058	105	691	83	45
16	321	171	941	149	220	127	163	105	774	82	44
17	492	171	9·9970 090	149	347	126	268	104	856	82	43
18	663	171	239	148	473	126	372	103	938	82	42
19	834	170	387	148	599	126	475	104	9·9991 020	81	41
20	9·9961 004	170	535	147	725	125	579	103	101	81	40
21	174	169	682	147	850	125	682	102	182	80	39
22	343	169	829	147	975	124	784	102	262	80	38
23	512	169	976	146	9·9979 099	124	886	102	342	80	37
24	681	168	9·9971 122	146	223	124	988	102	422	79	36
25	849	168	268	146	347	123	9·9986 090	101	501	79	35
26	9·9962 017	168	414	145	470	123	191	101	580	79	34
27	185	167	559	145	593	123	292	100	659	78	33
28	352	167	704	145	716	122	392	100	737	78	32
29	519	167	849	144	838	122	492	99	815	77	31
30	686	166	993	144	960	121	591	100	892	77	30
31	852	166	9·9972 137	143	9·9980 081	121	691	99	969	77	29
32	9·9963 018	165	280	143	202	121	790	98	9·9992 046	76	28
33	183	165	423	143	323	120	888	98	122	76	27
34	348	165	566	142	443	120	986	98	198	76	26
35	513	164	708	142	563	120	9·9987 084	97	274	75	25
36	677	164	850	141	683	119	181	97	349	75	24
37	841	163	991	141	802	119	278	97	424	74	23
38	9·9964 004	163	9·9973 132	141	921	119	375	96	498	74	22
39	167	163	273	141	9·9981 040	118	471	96	572	74	21
40	330	163	414	140	158	117	567	96	646	74	20
41	493	162	554	139	275	118	663	95	720	73	19
42	655	161	693	140	393	117	758	95	793	72	18
43	816	161	833	138	510	116	853	94	865	73	17
44	977	161	971	139	626	117	947	94	938	71	16
45	9·9965 138	161	9·9974 110	138	743	116	9·9988 041	94	9·9993 009	72	15
46	299	160	248	138	859	115	135	93	081	71	14
47	459	160	386	137	974	115	228	93	152	71	13
48	619	159	523	137	9·9982 089	115	321	93	223	70	12
49	778	159	660	137	204	114	414	92	293	71	11
50	937	159	797	136	318	115	506	92	364	69	10
51	9·9966 096	158	933	136	433	113	598	91	433	70	9
52	254	158	9·9975 069	136	546	114	689	91	503	69	8
53	412	158	205	135	660	112	780	91	572	68	7
54	570	157	340	135	772	113	871	91	640	68	6
55	727	157	475	134	885	112	962	90	708	68	5
56	884	156	609	134	997	112	9·9989 052	89	776	68	4
57	9·9967 040	156	743	134	9·9983 109	111	141	89	844	67	3
58	196	156	877	134	220	112	230	89	911	67	2
59	352	155	9·9976 011	132	332	110	319	89	978	66	1
60	507		143		442		408		9 9994 044		0
″	7°	*diff.*	6°	*diff.*	5°	*diff.*	4°	*diff.*	3°	*diff.*	′

LOG. COSINE.

′	82°	*diff.*	83°	*diff.*	84°	*diff.*	85°	*diff.*	′
0	10·85 21975	9176	10·91 08562	10457	10·97 83798	12169	11·05 80482	14574	60
1	31151	9194	19019	10480	95967	12202	95056	14623	59
2	40345	9213	29499	10505	10·98 08169	12237	11·06 09679	14671	58
3	49558	9232	40004	10530	20406	12269	24350	14721	57
4	58790	9251	50534	10555	32675	12304	39071	14769	56
5	68041	9270	61089	10580	44979	12339	53840	14820	55
6	77311	9289	71669	10605	57318	12372	68660	14869	54
7	86600	9308	82274	10630	69690	12407	83529	14919	53
8	95908	9328	92904	10655	82097	12442	98448	14971	52
9	10·86 05236	9347	10·92 03559	10681	94539	12477	11·07 13419	15021	51
10	14583	9366	14240	10707	10·99 07016	12513	28440	15073	50
11	23949	9386	24947	10732	19529	12547	43513	15124	49
12	33335	9405	35679	10758	32076	12584	58637	15177	48
13	42740	9425	46437	10784	44660	12619	73814	15229	47
14	52165	9444	57221	10810	57279	12655	89043	15282	46
15	61609	9465	68031	10836	69934	12691	11·08 04325	15335	45
16	71074	9484	78867	10863	82625	12728	19660	15388	44
17	80558	9505	89730	10889	95353	12764	35048	15443	43
18	90063	9524	10·93 00619	10916	11·00 08117	12802	50491	15497	42
19	99587	9545	11535	10943	20919	12838	65988	15552	41
20	10·87 09132	9565	22478	10969	33757	12876	81540	15607	40
21	18697	9585	33447	10997	46633	12913	97147	15663	39
22	28282	9606	44444	11023	595[illegible]6	12951	11·09 12810	15718	38
23	37888	9626	55467	11051	72497	12989	28528	15775	37
24	47514	9647	66518	11079	85486	13027	44303	15831	36
25	57161	9668	77597	11106	98513	13066	60134	15889	35
26	66829	9689	88703	11133	11·01 11579	13104	76023	15947	34
27	76518	9709	99836	11162	24683	13144	91970	16004	33
28	86227	9730	10·94 10998	11189	37827	13182	11·10 07974	16063	32
29	95957	9752	22187	11218	51009	13222	24037	16121	31
30	10·88 05709	9773	33405	11246	64231	13262	40158	16182	30
31	15482	9794	44651	11275	77493	13301	56340	16240	29
32	25276	9815	55926	11303	90794	13341	72580	16301	28
33	35091	9837	67229	11332	11·02 04135	13382	88881	16362	27
34	44928	9859	78561	11361	17517	13423	11·11 05243	16423	26
35	54787	9880	89922	11389	30940	13463	21666	16484	25
36	64667	9902	10·95 01311	11419	44403	13505	38150	16547	24
37	74569	9923	12730	11449	57908	13545	54697	16609	23
38	84492	9946	24179	11478	71453	13588	71306	16672	22
39	94438	9968	35657	11507	85041	13629	87978	16736	21
40	10·89 04406	9990	47164	11537	98670	13672	11·12 04714	16799	20
41	14396	10013	58701	11568	11·03 12342	13714	21513	16864	19
42	24409	10034	70269	11597	26056	13756	38377	16929	18
43	34443	10057	81866	11628	39812	13800	55306	16995	17
44	44500	10080	93494	11658	53612	13843	72301	17061	16
45	54580	10103	10·96 05152	11689	67455	13886	89362	17127	15
46	64683	10125	16841	11720	81341	13931	11·13 06489	17194	14
47	74808	10148	28561	11751	95272	13974	23683	17262	13
48	84956	10172	40312	11782	11·04 09246	14019	40945	17330	12
49	95128	10194	52094	11813	23265	14063	58275	17398	11
50	10·90 05322	10218	63907	11844	37328	14108	75673	17468	10
51	15540	10241	75751	11876	51436	14154	93141	17538	9
52	25781	10264	87627	11909	65590	14199	11·14 10679	17608	8
53	36045	10288	99536	11940	79789	14244	28287	17679	7
54	46333	10312	10·97 11476	11972	94033	14291	45966	17751	6
55	56645	10335	23448	12004	11·05 08324	14338	63717	17822	5
56	66980	10360	35452	12038	22662	14384	81539	17895	4
57	77340	10383	47490	12069	37046	14431	99434	17969	3
58	87723	10408	59559	12103	51477	14479	11·15 17403	18043	2
59	98131	10431	71662	12136	65956	14526	35446	18117	1
60	10·91 08562		83798		80482		53563		0
′	7°	*diff.*	6°	*diff.*	5°	*diff.*	4°	*diff.*	′

LOG. COTAN.

LOG. SINE.

′	87°	diff.	′
0	9·9994 044	66	60
1	110	66	59
2	176	65	58
3	241	65	57
4	306	64	56
5	370	65	55
6	435	63	54
7	498	64	53
8	562	63	52
9	625	63	51
10	688	62	50
11	750	62	49
12	812	62	48
13	874	61	47
14	935	61	46
15	996	60	45
16	9·9995 056	60	44
17	116	60	43
18	176	60	42
19	236	59	41
20	295	58	40
21	353	58	39
22	411	58	38
23	469	58	37
24	527	57	36
25	584	57	35
26	641	56	34
27	697	56	33
28	753	56	32
29	809	56	31
30	865	54	30
31	919	55	29
32	974	54	28
33	9·9996 028	54	27
34	082	54	26
35	136	53	25
36	189	53	24
37	242	52	23
38	294	52	22
39	346	52	21
40	398	51	20
41	449	51	19
42	500	50	18
43	550	51	17
44	601	49	16
45	650	50	15
46	700	49	14
47	749	49	13
48	798	48	12
49	846	48	11
50	894	48	10
51	942	47	9
52	989	47	8
53	9.9997 036	46	7
54	082	46	6
55	128	46	5
56	174	46	4
57	220	45	3
58	265	44	2
59	309	45	1
60	354		0
′	2°	diff.	′

LOG. COSINE.

LOG. TAN.

′	86°	diff.	87°	diff.	′
0	11·15 53563	18192	11·28 06042	24239	60
1	71755	18268	30281	24374	59
2	90023	18344	54655	24511	58
3	11·16 08367	18422	79166	24649	57
4	26789	18499	11·29 03815	24790	56
5	45288	18578	28605	24930	55
6	63866	18656	53535	25075	54
7	82522	18737	78610	25218	53
8	11·17 01259	18817	11·30 03828	25366	52
9	20076	18898	29194	25514	51
10	38974	18980	54708	25663	50
11	57954	19062	80371	25816	49
12	77016	19146	11·31 06187	25969	48
13	96162	19230	32156	26125	47
14	11·18 15392	19314	58281	26282	46
15	34706	19400	84563	26441	45
16	54106	19487	11·32 11004	26603	44
17	73593	19573	37607	26765	43
18	93166	19662	64372	26931	42
19	11·19 12828	19750	91303	27099	41
20	32578	19839	11·33 18402	27267	40
21	52417	19930	45669	27440	39
22	72347	20021	73109	27612	38
23	92368	20113	11·34 00721	27789	37
24	11·20 12481	20206	28510	27968	36
25	32687	20299	56478	28147	35
26	52986	20394	84625	28331	34
27	73380	20490	11·35 12956	28516	33
28	93870	20586	41472	28703	32
29	11·21 14456	20683	70175	28894	31
30	35139	20782	99069	29086	30
31	55921	20880	11·36 28155	29282	29
32	76801	20981	57437	29480	28
33	97782	21082	86917	29681	27
34	11·22 18864	21184	11·37 16598	29884	26
35	40048	21287	46482	30091	25
36	61335	21391	76573	30300	24
37	82726	21497	11·38 06873	30511	23
38	11·23 04223	21602	37384	30727	22
39	25825	21710	68111	30946	21
40	47535	21818	99057	31166	20
41	69353	21928	11·39 30223	31391	19
42	91281	22038	61614	31619	18
43	11·24 13319	22150	93233	31850	17
44	35469	22262	11·40 25083	32085	16
45	57731	22377	57168	32323	15
46	80108	22492	89491	32564	14
47	11·25 02600	22608	11·41 22055	32809	13
48	25208	22725	54864	33059	12
49	47933	22845	87923	33311	11
50	70778	22964	11·42 21234	33569	10
51	93742	23086	54803	33829	9
52	11·26 16828	23208	88632	34093	8
53	40036	23333	11·43 22725	34363	7
54	63369	23457	57088	34636	6
55	86826	23585	91724	34914	5
56	11·27 10411	23712	11·44 26638	35196	4
57	34123	23842	61834	35483	3
58	57965	23972	97317	35774	2
59	81937	24105	11·45 33091	36071	1
60	11·28 06042		69162		0
′	3°	diff.	2°	diff.	′

LOG. COTAN.

NOTE. *The trigonometrical lines near the extremities of the quadrant are given to every second at the beginning of the table, where we must search for the trigonometrical line belonging to any arc below 2 degrees or above 88 degrees.*

19,000,000,0000

LOG. SINE 88°.

″	0′	1′	2′	3′	4′	5′	6′	7′	″
0	9·9997354	·9997398	·9997441	·9997484	·9997527	·9997570	·9997612	9·9997653	60
1	54	9 98	9 42	9 85	9 28	9 70	9 12	54	59
2	55	99	43	86	29	71	13	55	58
3	56	·9997400	43	86	29	72	14	55	57
4	57	00	44	87	30	72	14	56	56
5	57	01	45	88	31	73	15	57	55
6	58	02	45	89	31	74	16	58	54
7	59	03	46	89	32	74	17	58	53
8	59	03	47	90	33	75	17	59	52
9	9·9997360	·9997404	·9997448	·9997491	·9997534	·9997576	·9997618	9·9997660	51
10	61	05	48	91	34	77	19	60	50
11	62	06	49	92	35	77	19	61	49
12	62	06	49	93	36	78	20	62	48
13	63	07	50	94	36	79	21	62	47
14	64	08	51	94	37	79	21	63	46
15	65	08	52	95	38	80	22	64	45
16	65	09	53	96	38	81	23	64	44
17	66	10	53	96	39	82	24	65	43
18	67	11	54	97	40	82	24	66	42
19	9·9997368	·9997411	·9997455	·9997498	·9997541	·9997583	·9997625	9·9997667	41
20	68	12	56	99	41	84	26	67	40
21	69	13	56	99	42	84	26	68	39
22	70	14	57	·9997500	43	85	27	69	38
23	70	14	58	01	43	86	28	69	37
24	71	15	58	01	44	86	28	70	36
25	72	16	59	02	45	87	29	71	35
26	73	16	60	03	46	88	30	71	34
27	73	17	61	04	46	89	30	72	33
28	74	18	61	04	47	89	31	73	32
29	9·9997375	·9997419	·9997462	·9997505	·9997548	·9997590	·9997632	9·9997673	31
30	76	19	63	06	48	91	33	74	30
31	76	20	63	06	49	91	33	75	29
32	77	21	64	07	50	92	34	75	28
33	78	22	65	08	51	93	35	76	27
34	79	22	66	09	51	93	35	77	26
35	79	23	66	09	52	94	36	78	25
36	80	24	67	10	53	95	37	78	24
37	81	24	68	11	53	96	37	79	23
38	81	25	68	11	54	96	38	80	22
39	9·9997382	·9997426	·9997469	·9997512	·9997555	·9997597	·9997639	9·9997680	21
40	83	27	70	13	55	98	40	81	20
41	84	27	71	14	56	98	40	82	19
42	84	28	71	14	57	99	41	82	18
43	85	29	72	15	58	·9997600	42	83	17
44	86	30	73	16	58	00	42	84	16
45	87	30	74	16	59	01	43	84	15
46	87	31	74	17	60	02	44	85	14
47	88	32	75	18	60	03	44	86	13
48	89	32	76	19	61	03	45	87	12
49	9·9997389	·9997433	·9997476	·9997519	·9997562	·9997604	·9997646	9·9997687	11
50	90	34	77	20	63	05	46	88	10
51	91	35	78	21	63	05	47	89	9
52	92	35	79	21	64	06	48	89	8
53	92	36	79	22	65	07	49	90	7
54	93	37	80	23	65	07	49	91	6
55	94	37	81	24	66	08	50	91	5
56	95	38	81	24	67	09	51	92	4
57	95	39	82	25	67	10	51	93	3
58	96	40	83	26	68	10	52	93	2
59	9·9997397	·9997440	·9997484	·9997526	·9997569	·9997611	·9997653	9·9997694	1
60	98	41	84	27	70	12	53	95	0
″	59′	58′	57′	56′	55′	54′	53′	52′	″

LOG. COSINE 1°.

″	0′	1′	2′	3′	4′	5′	6′	7′	″
0	11·4569162	4605534	4642213	4679203	4716510	4754140	4792098	11·4830390	60
1	9766	6143	2827	9822	7135	4770	2733	1031	59
2	11·4570369	6752	3441	4680441	7759	5400	3369	1672	58
3	0973	7361	4055	1061	8384	6030	4005	2313	57
4	1577	7970	4669	1680	9009	6660	4640	2955	56
5	2181	8579	5283	2300	9634	7291	5276	3596	55
6	2786	9188	5898	2919	4720259	7921	5912	4238	54
7	3390	9797	6512	3539	0884	8551	6548	4879	53
8	3994	4610407	7127	4159	1509	9182	7184	5521	52
9	4598	1016	7741	4779	2134	9813	7820	6163	51
10	5203	1626	8356	5399	2759	4760443	8457	6805	50
11	5807	2235	8971	6019	3385	1074	9093	7447	49
12	6412	2845	9586	6639	4010	1705	9729	8089	48
13	7017	3455	4650201	7259	4636	2336	4800366	8731	47
14	7622	4065	0816	7879	5261	2967	1003	9373	46
15	8227	4675	1431	8500	5887	3599	1639	11·4840016	45
16	8832	5285	2046	9120	6513	4230	2276	0658	44
17	9437	5895	2661	9741	7139	4861	2913	1301	43
18	11·4580042	6505	3277	4690362	7765	5493	3550	1943	42
19	0647	7116	3892	0982	8391	6124	4187	2586	41
20	1252	7726	4508	1603	9017	6756	4825	3229	40
21	1858	8336	5124	2224	9644	7388	5462	3872	39
22	2463	8947	5739	2845	4730270	8020	6099	4515	38
23	3069	9558	6355	3466	0897	8651	6737	5158	37
24	3674	4620168	6971	4088	1523	9283	7374	5801	36
25	4280	0779	7587	4709	2150	9916	8012	6445	35
26	4886	1390	8203	5330	2777	4770548	8650	7088	34
27	5492	2001	8819	5952	3403	1180	9288	7732	33
28	6098	2612	9436	6573	4030	1813	9926	8375	32
29	6704	3223	4660052	7195	4657	2445	4810564	9019	31
30	7310	3835	0669	7817	5284	3078	1202	9663	30
31	7916	4446	1285	8438	5912	3710	1840	11·4850307	29
32	8523	5058	1902	9060	6539	4343	2478	0951	28
33	9129	5669	2518	9682	7166	4976	3117	1595	27
34	9736	6281	3135	4700304	7794	5609	3755	2239	26
35	11·4590342	6892	3752	0927	8421	6242	4394	2883	25
36	0949	7504	4369	1549	9049	6875	5033	3528	24
37	1555	8116	4986	2171	9677	7508	5671	4172	23
38	2162	8728	5603	2794	4740305	8142	6310	4817	22
39	2769	9340	6621	3416	0933	8775	6949	5461	21
40	3376	9952	6838	4039	1561	9409	7588	6106	20
41	3983	4630564	7455	4662	2189	4780042	8228	6751	19
42	4591	1177	8073	5284	2817	0676	8867	7396	18
43	5198	1789	8690	5907	3445	1310	9506	8041	17
44	5805	2401	9308	6530	4074	1943	4820146	8686	16
45	6413	3014	9926	7153	4702	2577	0785	9332	15
46	7020	3627	4670544	7777	5331	3211	1425	9977	14
47	7628	4239	1162	8400	5959	3846	2065	11·4860622	13
48	8235	4852	1780	9023	6588	4480	2704	1268	12
49	8843	5465	2398	9647	7217	5114	3344	1913	11
50	9451	6078	3016	4710270	7846	5749	3984	2559	10
51	11·4600059	6691	3634	0894	8475	6383	4625	3205	9
52	0667	7304	4253	1517	9104	7018	5265	3851	8
53	1275	7918	4871	2141	9733	7652	5905	4497	7
54	1883	8531	5490	2765	4750362	8287	6545	5143	6
55	2491	9144	6108	3389	0992	8922	7186	5789	5
56	3100	9758	6727	4013	1621	9557	7827	6436	4
57	3708	4640371	7346	4637	2251	4790192	8467	7082	3
58	4317	0985	7965	5261	2880	0827	9108	7728	2
59	4925	1599	8584	5886	3510	1463	9749	8375	1
60	5534	2213	9203	6510	4140	2098	4830390	9022	0
″	59′	58′	57′	56′	55′	54′	53′	52′	″

LOG. COTAN. 1°.

LOG. SINE 88°.

″	8′	9′	10′	11′	12′	13′	14′	15′	″
0	9·9997695	·9997736	·9997776	·9997817	·9997856	·9997896	·9997935	9·9997974	60
1	95	36	77	17	57	97	36	75	59
2	96	37	78	18	58	97	36	75	58
3	97	38	78	19	58	98	37	76	57
4	97	38	79	19	59	99	38	77	56
5	98	39	80	20	60	99	38	77	55
6	99	40	80	21	60	·9997900	39	78	54
7	9·9997700	40	81	21	61	01	40	78	53
8	00	41	82	22	62	01	40	79	52
9	9·9997701	·9997742	·9997782	·9997823	·9997862	·9997902	·9997941	9·9997980	51
10	02	43	83	23	63	03	42	80	50
11	02	43	84	24	64	03	42	81	49
12	03	44	84	25	64	04	43	82	48
13	04	45	85	25	65	05	44	82	47
14	04	45	86	26	66	05	44	83	46
15	05	46	86	27	66	06	45	84	45
16	06	47	87	27	67	06	46	84	44
17	06	47	88	28	68	07	46	85	43
18	07	48	88	29	68	08	47	86	42
19	9·9997708	·9997749	·9997789	·9997829	·9997869	·9997908	·9997947	9·9997986	41
20	08	49	90	30	70	09	48	87	40
21	09	50	90	31	70	10	49	87	39
22	10	51	91	31	71	10	49	88	38
23	10	51	92	32	72	11	50	89	37
24	11	52	93	33	72	12	51	89	36
25	12	53	93	33	73	12	51	90	35
26	13	53	94	34	74	13	52	91	34
27	13	54	95	35	74	14	53	91	33
28	14	55	95	35	75	14	53	92	32
29	9·9997715	·9997755	·9997796	·9997836	·9997876	·9997915	·9997954	9·9997993	31
30	15	56	97	37	76	16	55	93	30
31	16	57	97	37	77	16	55	94	29
32	17	57	98	38	78	17	56	94	28
33	17	58	99	39	78	18	57	95	27
34	18	59	99	39	79	18	57	96	26
35	19	59	·9997800	40	80	19	58	96	25
36	19	60	01	41	80	20	58	97	24
37	20	61	01	41	81	20	59	98	23
38	21	62	02	42	82	21	60	98	22
39	9·9997721	·9997762	·9997803	·9997843	·9997882	·9997921	·9997960	9·9997999	21
40	22	63	03	43	83	22	61	9·9998000	20
41	23	64	04	44	84	23	62	00	19
42	23	64	05	45	84	23	62	01	18
43	24	65	05	45	85	24	63	02	17
44	25	66	06	46	86	25	64	02	16
45	26	66	07	47	86	25	64	03	15
46	26	67	07	47	87	26	65	03	14
47	27	68	08	48	87	27	66	04	13
48	28	68	09	49	88	27	66	05	12
49	9·9997728	·9997769	·9997809	·9997849	·9997889	·9997928	·9997967	9·9998005	11
50	29	70	10	50	89	29	68	06	10
51	30	70	11	51	90	29	68	07	9
52	30	71	11	51	91	30	69	07	8
53	31	72	12	52	91	31	70	08	7
54	32	72	13	53	92	31	70	09	6
55	32	73	13	53	93	32	71	09	5
56	33	74	14	54	93	33	71	10	4
57	34	74	15	55	94	33	72	10	3
58	34	75	15	55	95	34	73	11	2
59	9·9997735	·9997776	·9997816	·9997856	·9997895	·9997935	·9997973	9·9998012	1
60	36	76	17	56	96	35	74	12	0
″	51′	50′	49′	48′	47′	46′	45′	44′	″

LOG. COSINE 1°.

″	8′	9′	10′	11′	12′	13′	14′	15′	″
0	11·4869022	4907999	4947329	4987018	5027072	5067498	5108304	11·5149495	60
1	9668	11· 8652	11· 7988	11· 7683	11· 7743	11· 8175	11· 8987	11·5150185	59
2	11·4870315	9305	8647	8347	8414	8852	9670	0875	58
3	0962	9958	9305	9012	9085	9529	5110354	1565	57
4	1609	4910610	9964	9677	9756	5070207	1038	2255	56
5	2257	1263	4950623	4990342	5030427	0884	1721	2945	55
6	2904	1916	1282	1007	1098	1562	2405	3636	54
7	3551	2570	1941	1672	1769	2239	3089	4326	53
8	4199	3223	2600	2337	2441	2917	3773	5017	52
9	4846	3876	3260	3003	3112	3595	4457	5708	51
10	5494	4530	3919	3668	3784	4273	5142	6398	50
11	6141	5183	4579	4334	4456	4951	5826	7089	49
12	6789	5837	5238	5000	5127	5629	6511	7780	48
13	7437	6491	5898	5665	5799	6307	7195	8472	47
14	8085	7145	6558	6331	6471	6985	7880	9163	46
15	8733	7799	7218	6997	7144	7664	8565	9854	45
16	9382	8453	7878	7663	7816	8342	9250	11·5160546	44
17	11·4880030	9107	8538	8329	8498	9021	9935	1237	43
18	0678	9761	9198	8996	9161	9700	5120620	1929	42
19	1327	4920416	9858	9662	9833	5080379	1305	2621	41
20	1975	1070	4960519	5000329	5040506	1058	1991	3313	40
21	2624	1725	1179	0995	1179	1737	2676	4005	39
22	3273	2379	1840	1662	1852	2416	3362	4697	38
23	3922	3034	2501	2329	2524	3095	4048	5389	37
24	4571	3689	3162	2996	3198	3774	4733	6081	36
25	5220	4344	3822	3663	3871	4454	5419	6774	35
26	5869	4999	4483	4330	4544	5134	6105	7467	34
27	6518	5654	5145	4997	5217	5813	6791	8159	33
28	7168	6309	5806	5664	5891	6493	7478	8852	32
29	7817	6965	6467	6332	6565	7173	8164	9545	31
30	8467	7620	7129	6999	7238	7853	8851	11·5170238	30
31	9117	8276	7790	7667	7912	8533	9537	0931	29
32	9766	8931	8452	8334	8586	9213	5130224	1624	28
33	11·4890416	9587	9113	9002	9260	9894	0911	2318	27
34	1066	0243	9775	9670	9934	5090574	1597	3011	26
35	1716	4930899	4970437	5010338	5050608	1255	2284	3705	25
36	2366	1555	1099	1006	1283	1935	2972	4398	24
37	3017	2211	1761	1675	1957	2616	3659	5092	23
38	3667	2867	2424	2343	2632	3297	4346	5786	22
39	4317	3523	3086	3011	3306	3978	5034	6480	21
40	4968	4180	3748	3680	3981	4659	5721	7174	20
41	5619	4836	4411	4348	4656	5340	6409	7869	19
42	6269	5493	5073	5017	5331	6022	7097	8563	18
43	6920	6150	5736	5686	6006	6703	7784	9257	17
44	7571	6807	6399	6355	6681	7385	8472	9952	16
45	8222	7464	7062	7024	7357	8066	9161	11·5180647	15
46	8873	8121	7725	7693	8032	8748	9849	1341	14
47	9525	8778	8388	8362	8707	9430	5140537	2036	13
48	11·4900176	9435	9051	9032	9383	5100112	1225	2731	12
49	0827	4940092	9715	9701	5060059	0794	1914	3426	11
50	1479	0750	4980378	5020371	0734	1476	2603	4122	10
51	2130	1407	1042	1041	1410	2158	3291	4817	9
52	2782	2065	1705	1710	2086	2841	3980	5513	8
53	3434	2723	2369	2380	2763	3523	4669	6208	7
54	4086	3380	3033	3050	3439	4206	5358	6904	6
55	4738	4038	3697	3720	4115	4888	6047	7600	5
56	5390	4696	4361	4390	4792	5571	6737	8296	4
57	6042	5354	5025	5061	5468	6254	7426	8992	3
58	6695	6013	5689	5731	6145	6937	8116	9688	2
59	7347	6671	6354	6402	6821	7620	8805	11·5190384	1
60	7999	7329	7018	7072	7498	8304	9495	1080	0
″	51′	50′	49′	48′	47′	46′	45′	44′	″

LOG. COTAN. 1°.

LOG. SINE 88°.

″	16′	17′	18′	19′	20′	21′	22′	23′	″
0	9·9998012	·9998050	·9998088	·9998125	·9998162	·9998199	·9998235	9·9998271	60
1	13	51	89	26	63	99	36	72	59
2	14	52	89	27	64	·9998200	36	72	58
3	14	52	90	27	64	01	37	73	57
4	15	53	91	28	65	01	38	73	56
5	16	54	91	28	65	02	38	74	55
6	16	54	92	29	66	03	39	75	54
7	17	55	92	30	67	03	39	75	53
8	17	55	93	30	67	04	40	76	52
9	9·9998018	·9998056	·9998094	·9998131	·9998168	·9998204	·9998241	9.9998276	51
10	19	57	94	32	68	05	41	77	50
11	19	57	95	32	69	06	42	77	49
12	20	58	96	33	70	06	42	78	48
13	21	59	96	33	70	07	43	79	47
14	21	59	97	34	71	07	44	79	46
15	22	60	97	35	72	08	44	80	45
16	23	60	98	35	72	09	45	80	44
17	23	61	99	36	73	09	45	81	43
18	2	62	99	37	73	10	46	82	42
19	9·9998024	·9998062	·9998100	·9998137	·9998174	·9998210	·9998246	9·9998282	41
20	25	63	01	38	75	11	47	83	40
21	26	64	01	38	75	12	48	83	39
22	26	64	02	39	76	12	48	84	38
23	27	65	02	40	76	13	49	85	37
24	28	66	03	40	77	13	49	85	36
25	28	66	04	41	78	14	50	86	35
26	29	67	04	41	78	15	51	86	34
27	30	67	05	42	79	15	51	87	33
28	30	68	06	43	79	16	52	88	32
29	9·9998031	·9998069	·9998106	·9998143	·9998180	·9998216	·9998252	9·9998288	31
30	31	69	07	44	81	17	53	89	30
31	32	70	07	45	81	18	54	89	29
32	33	71	08	45	82	18	54	90	28
33	33	71	09	46	82	19	55	91	27
34	34	72	09	46	83	19	55	91	26
35	35	72	10	47	84	20	56	92	25
36	35	73	11	48	84	21	57	92	24
37	36	74	11	48	85	21	57	93	23
38	36	74	12	49	86	22	58	93	22
39	9·9998037	·9998075	·9998112	·9998149	·9998186	·9998222	·9998258	9·9998294	21
40	38	76	13	50	87	23	59	95	20
41	38	76	14	51	87	24	60	95	19
42	39	77	14	51	88	24	60	96	18
43	40	77	15	52	89	25	61	96	17
44	40	78	15	53	89	25	61	97	16
45	41	79	16	53	90	26	62	98	15
46	42	79	17	54	90	27	63	98	14
47	42	80	17	54	91	27	63	99	13
48	43	81	18	55	92	28	64	9·9998300	12
49	9·9998043	·9998081	·9998119	·9998156	·9998192	·9998229	·9998264	9·9998300	11
50	44	82	19	56	93	29	65	01	10
51	45	82	20	57	93	30	66	01	9
52	45	83	20	57	94	30	66	02	8
53	46	84	21	58	95	31	67	02	7
54	47	84	22	59	95	32	67	03	6
55	47	85	22	59	96	32	68	03	5
56	48	86	23	60	96	33	69	04	4
57	49	86	24	60	97	33	69	05	3
58	49	87	24	61	98	34	70	05	2
59	9·9998050	·9998087	·9998125	·9998162	·9998198	·9998235	·9998270	9·9998306	1
60	50	88	25	62	99	35	71	06	0
″	43′	42′	41′	40′	39′	38′	37′	36′	″

LOG. COSINE 1°.

LOG. TAN. 88°.

″	16′	17′	18′	19′	20′	21′	22′	23′	″
0	11·5191080	5233067	5275462	5318275	5361514	5405186	5449301	11·5493869	60
1	1777	3770	6173	8992	2238	5918	5450040	4615	59
2	2473	4473	6883	9710	2962	6649	0780	5362	58
3	3170	5177	7593	5320427	3687	7381	1519	6109	57
4	3867	5880	8304	1145	4412	8113	2258	6856	56
5	4564	6584	9014	1862	5136	8845	2998	7603	55
6	5261	7288	9725	2580	5861	9578	3738	8351	54
7	5958	7992	5280436	3298	6586	5410310	4477	9098	53
8	6655	8696	1147	4016	7311	1042	5217	9846	52
9	7352	9400	1858	4734	8037	1775	5957	11·5500593	51
10	8050	5240104	2569	5452	8762	2508	6698	1341	50
11	8748	0808	3280	6170	9488	3240	7438	2089	49
12	9445	1513	3991	6889	5370213	3973	8178	2837	48
13	11·5200143	2217	4703	7607	0939	4707	8919	3585	47
14	0841	2922	5414	8326	1665	5440	9660	4334	46
15	1539	3627	6126	9045	2391	6173	5460401	5082	45
16	2237	4332	6838	9764	3117	6906	1141	5831	44
17	2935	5037	7550	5330483	3843	7640	1883	6580	43
18	3634	5742	8262	1202	4569	8374	2624	7328	42
19	4332	6447	8974	1921	5296	9108	3365	8077	41
20	5031	7153	9687	2640	6022	9842	4107	8827	40
21	5729	7858	5290399	3360	6749	5420576	4848	9576	39
22	6428	8564	1112	4079	7476	1310	5590	11·5510325	38
23	7127	9270	1824	4799	8203	2044	6332	1075	37
24	7826	9975	2537	5519	8930	2779	7074	1824	36
25	8525	5250681	3250	6239	9657	3513	7816	2574	35
26	9225	1388	3963	6959	5380385	4248	8558	3324	34
27	9924	2094	4676	7679	1112	4983	9300	4074	33
28	11·5210624	2800	5389	8400	1840	5718	5470043	4824	32
29	1323	3506	6103	9120	2567	6453	0785	5574	31
30	2023	4213	6816	9841	3295	7188	1528	6325	30
31	2723	4920	7530	5340561	4023	7923	2271	7075	29
32	3423	5626	8244	1282	4751	8659	3014	7826	28
33	4123	6333	8957	2003	5479	9394	3757	8577	27
34	4823	7040	9671	2724	6208	5430130	4500	9328	26
35	5523	7747	5300385	3445	6936	0866	5243	11·5520079	25
36	6224	8455	1100	4167	7664	1602	5987	0830	24
37	6924	9162	1814	4888	8393	2338	6731	1581	23
38	7625	9869	2528	5610	9122	3074	7474	2333	22
39	8325	5260577	3243	6331	9851	3810	8218	3084	21
40	9026	1285	3957	7053	5390580	4547	8962	3836	20
41	9727	1992	4672	7775	1309	5283	9706	4588	19
42	11·5220428	2700	5387	8497	2038	6020	5480451	5340	18
43	1129	3408	6102	9219	2768	6757	1195	6092	17
44	1831	4116	6817	9941	3497	7494	1939	6844	16
45	2532	4825	7532	5350664	4227	8231	2684	7596	15
46	3234	5533	8248	1386	4957	8968	3429	8349	14
47	3935	6242	8963	2109	5686	9705	4174	9102	13
48	4637	6950	9679	2832	6416	5440442	4919	9854	12
49	5339	7659	5310395	3554	7147	1180	5664	11·5530607	11
50	6041	8368	1110	4277	7877	1918	6409	1360	10
51	6743	9077	1826	5000	8607	2656	7154	2113	9
52	7445	9786	2542	5724	9338	3393	7900	2867	8
53	8147	5270495	3259	6447	5400068	4132	8646	3620	7
54	8850	1204	3975	7170	0799	4870	9391	4373	6
55	9552	1914	4691	7894	1530	5608	5490137	5127	5
56	11·5230255	2623	5408	8618	2261	6346	0883	5881	4
57	0958	3333	6125	9341	2992	7085	1629	6635	3
58	1661	4043	6841	5360065	3723	7824	2376	7389	2
59	2364	4752	7558	0789	4455	8562	3122	8143	1
60	3067	5462	8275	1514	5186	9301	3869	8897	0
″	43′	42′	41′	40′	39′	38′	37′	36′	″

LOG. COTAN. 1°.

″	24′	25′	26′	27′	28′	29′	30′	31′	″
0	9·9998306	·9998342	·9998376	·9998411	·9998445	·9998478	·9998512	9·9998544	60
1	07	42	77	11	45	79	12	45	59
2	08	43	77	12	46	79	13	46	58
3	08	43	78	12	46	80	13	46	57
4	09	44	79	13	47	80	14	47	56
5	09	44	79	13	47	81	14	47	55
6	10	45	80	14	48	82	15	48	54
7	11	46	80	15	49	82	15	48	53
8	11	46	81	15	49	83	16	49	52
9	12	47	81	16	50	83	16	49	51
10	9 9998312	·9998347	·9998382	·9998416	·9998450	·9998484	·9998517	9·9998550	50
11	13	48	83	17	51	84	18	50	49
12	13	49	83	17	51	85	18	51	48
13	14	49	84	18	52	85	19	52	47
14	15	50	84	19	53	86	19	52	46
15	15	50	85	19	53	87	20	53	45
16	16	51	85	20	54	87	20	53	44
17	16	51	86	20	54	88	21	54	43
18	17	52	87	21	55	88	21	54	42
19	18	53	87	21	55	89	22	55	41
20	9·9998318	·9998353	·9998388	·9998422	·9998456	·9998489	·9998523	9·9998555	40
21	19	54	88	23	56	90	23	56	39
22	19	54	89	23	57	91	24	56	38
23	20	55	89	24	58	91	24	57	37
24	21	55	90	24	58	92	25	57	36
25	21	56	91	25	59	92	25	58	35
26	22	57	91	25	59	93	26	59	34
27	22	57	92	26	60	93	26	59	33
28	23	58	92	27	60	94	27	60	32
29	23	58	93	27	61	94	27	60	31
30	9·9998324	·9998359	·9998393	·9998428	·9998461	·9998495	·9998528	9·9998561	30
31	25	60	94	28	62	95	29	61	29
32	25	60	95	29	63	96	29	62	28
33	26	61	95	29	63	97	30	62	27
34	26	61	96	30	64	97	30	63	26
35	27	62	96	31	64	98	31	63	25
36	28	62	97	31	65	98	31	64	24
37	28	63	97	32	65	99	32	65	23
38	29	64	98	32	66	99	32	65	22
39	29	64	99	33	67	·9998500	33	66	21
40	9·9998330	·9998365	·9998399	·9998433	·9998467	·9998500	·9998533	9·9998566	20
41	30	65	·9998400	34	68	01	34	67	19
42	31	66	00	34	68	02	35	67	18
43	32	66	01	35	69	02	35	68	17
44	32	67	01	36	69	03	36	68	16
45	33	68	02	36	70	03	36	69	15
46	33	68	03	37	70	04	37	69	14
47	34	69	03	37	71	04	37	70	13
48	35	69	04	38	72	05	38	70	12
49	35	70	04	38	72	05	38	71	11
50	9·9998336	·9998370	·9998405	·9998439	·9998473	·9998506	·9998539	9·9998572	10
51	36	71	05	40	73	07	40	72	9
52	37	72	06	40	74	07	40	73	8
53	37	72	07	41	74	08	41	73	7
54	38	73	07	41	75	08	41	74	6
55	39	73	08	42	75	09	42	74	5
56	39	74	08	42	76	09	42	75	4
57	40	75	09	43	77	10	43	75	3
58	40	75	09	44	77	10	43	76	2
59	41	76	10	44	78	11	44	76	1
60	42	76	11	45	78	12	44	77	0
″	35′	34′	33′	32′	31′	30′	29′	28′	″

LOG. COSINE 1°.

″	24′	25′	26′	27′	28′	29′	30′	31′	″
0	11·5538897	5584397	5630378	5676850	5723824	5771310	5819321	11·5867868	60
1	9652	11· 5159	11· 1148	11· 7628	11· 4611	11· 2106	5820126	8682	59
2	11·5540406	5922	1919	9407	5398	2902	11· 0931	9496	58
3	1161	6685	2690	9186	6186	3698	1736	11·5870310	57
4	1916	7447	3460	9966	6973	4495	2541	1124	56
5	2671	8210	4232	5680745	7761	5291	3346	1938	55
6	3426	8973	5003	1524	8549	6088	4152	2753	54
7	4181	• 9737	5774	2304	9337	6884	4957	3568	53
8	4937	5590500	6545	3084	5730125	7681	5763	4382	52
9	5692	1263	7317	3864	0914	8478	6569	5197	51
10	6448	2027	8089	4644	1702	9275	7375	6012	50
11	7203	2791	8861	5424	2491	5780073	8181	6828	49
12	7959	3554	9633	6204	3280	0870	8988	7643	48
13	8715	4318	5640405	6984	4068	1668	9794	8459	47
14	9471	5082	1177	7765	4858	2466	5830601	9274	46
15	11·5550228	5847	1949	8546	5647	3264	1407	11·5880090	45
16	0984	6611	2722	9327	6436	4062	2214	0906	44
17	1741	7376	3494	5690108	7226	4860	3021	1722	43
18	2497	8140	4267	0889	8015	5658	3829	2539	42
19	3254	8905	5040	1670	8805	6457	4636	3355	41
20	4011	9670	5813	2451	9595	7255	5444	4172	40
21	4768	5600435	6587	3233	5740385	8054	6251	4989	39
22	5525	1200	7360	4015	1175	8853	7059	5806	38
23	6283	1966	8133	4796	1966	9652	7867	6623	37
24	7040	2731	8907	5578	2756	5790451	8675	7440	36
25	7798	3497	9681	6361	3547	1251	9483	8257	35
26	8556	4262	5650455	7143	4338	2050	5840292	9075	34
27	9313	5028	1229	7925	5128	2850	1100	9893	33
28	11·5560071	5794	2003	8708	5920	3650	1909	11·5890710	32
29	0829	6560	2777	9490	6711	4450	2718	1528	31
30	1588	7327	3552	5700273	7502	5250	3527	2347	30
31	2346	8093	4326	1056	8294	6050	4336	3165	29
32	3105	8860	5101	1839	9085	6850	5146	3983	28
33	3863	9626	5876	2623	9877	7651	5955	4802	27
34	4622	5610393	6651	3406	5750669	8451	6765	5621	26
35	5381	1160	7426	4189	1461	9252	7575	6440	25
36	6140	1927	8201	4973	2253	5800053	8384	7259	24
37	6899	2694	8977	5757	3046	0854	9195	8078	23
38	7659	3462	9752	6541	3838	1656	5850005	8897	22
39	8418	4229	5660528	7325	4631	2457	0815	9717	21
40	9178	4997	1304	8109	5424	3259	1626	11·5900537	20
41	9937	5765	2080	8894	6217	4060	2436	1357	19
42	11·5570697	6532	2856	9678	7010	4862	3247	2177	18
43	1457	7300	3632	5710463	7803	5664	4058	2997	17
44	2217	8069	4409	1248	8596	6467	4869	3817	16
45	2977	8837	5185	2032	9390	7269	5681	4638	15
46	3738	9605	5962	2818	5760184	8071	6492	5458	14
47	4498	5620374	6739	3603	0977	8874	7304	6279	13
48	5259	1143	7516	4388	1771	9677	8115	7100	12
49	6020	1911	8293	5174	2566	5810490	8927	7921	11
50	6781	2680	9070	5959	3360	1283	9739	8742	10
51	7542	3450	9847	6745	4154	2086	5860552	9564	9
52	8303	4219	5670625	7531	4949	2889	1364	11·5910385	8
53	9064	4988	1402	8317	5743	3693	2177	1207	7
54	9826	5758	2180	9103	6538	4496	2989	2029	6
55	11·5580587	6527	2958	9890	7333	5300	3802	2851	5
56	1349	7297	3736	5720676	8128	6104	4615	3673	4
57	2111	8067	4514	1463	8924	6908	5428	4495	3
58	2873	8837	5293	2250	9719	7712	6241	5318	2
59	3635	9607	6071	3037	5770515	8517	7055	6141	1
60	4397	5630378	6850	3824	1310	9321	7868	6963	0
″	35′	34′	33′	32′	31′	30′	29′	28′	″

LOG. COTAN. 1°.

LOG. SINE 88°.

″	32′	33′	34′	35′	36′	37′	38′	39′	″
0	9·9998577	·9998609	·9998641	·9998672	·9998703	·9998734	·9998764	9·9998794	60
1	77	9 10	9 41	9 73	9 04	9 35	9 65	95	59
2	78	10	42	73	04	35	65	95	58
3	79	11	42	74	05	36	66	96	57
4	79	11	43	74	05	36	66	96	56
5	80	12	44	75	06	37	67	97	55
6	80	12	44	75	07	37	67	97	54
7	81	13	45	76	07	38	68	98	53
8	81	13	45	76	08	38	68	98	52
9	9·9998582	·9998614	·9998646	·9998677	·9998708	·9998739	·9998769	9·9998799	51
10	82	14	46	78	09	39	69	99	50
11	83	15	47	78	09	40	70	9·9998800	49
12	83	16	47	79	10	40	70	00	48
13	84	16	48	79	10	41	71	01	47
14	84	17	48	80	11	41	71	01	46
15	85	17	49	80	11	42	72	02	45
16	86	18	49	81	12	42	72	02	44
17	86	18	50	81	12	43	73	03	43
18	87	19	50	82	13	43	73	03	42
19	9·9998587	·9998619	·9998651	·9998682	·9998713	·9998744	·9998774	9·9998804	41
20	88	20	51	83	14	44	74	05	40
21	88	20	52	83	14	45	75	05	39
22	89	21	52	84	15	45	75	05	38
23	89	21	53	84	15	46	76	06	37
24	90	22	54	85	16	46	76	06	36
25	90	22	54	85	16	47	77	07	35
26	91	23	55	86	17	47	77	07	34
27	91	23	55	86	17	48	78	08	33
28	92	24	56	87	18	48	78	08	32
29	9·9998593	·9998625	·9998656	·9998687	·9998718	·9998749	·9998779	9·9998809	31
30	93	25	57	88	19	49	79	09	30
31	94	26	57	88	19	50	80	10	29
32	94	26	58	89	20	50	80	10	28
33	95	27	58	89	20	51	81	11	27
34	95	27	59	90	21	51	81	11	26
35	96	28	59	90	21	52	82	12	25
36	96	28	60	91	22	52	82	12	24
37	97	29	60	92	22	53	83	13	23
38	97	29	61	92	23	53	83	13	22
39	9·9998598	·9998630	·9998661	·9998693	·9998723	·9998754	·9998784	9·9998814	21
40	98	30	62	93	24	54	84	14	20
41	99	31	62	94	24	55	85	15	19
42	9·9998600	31	63	94	25	55	85	15	18
43	00	32	63	95	25	56	86	16	17
44	01	32	64	95	26	56	86	16	16
45	01	33	65	96	26	57	87	17	15
46	02	34	65	96	27	57	87	17	14
47	02	34	66	97	27	58	88	18	13
48	03	35	66	97	28	58	88	18	12
49	9·9998603	·9998635	·9998667	·9998698	·9998728	·9998759	·9998789	9·9998819	11
50	04	36	67	98	29	59	89	19	10
51	04	36	68	99	30	60	90	20	9
52	05	37	68	99	30	60	90	20	8
53	05	37	69	·9998700	31	61	91	21	7
54	06	38	69	00	31	61	91	21	6
55	06	38	70	01	32	62	92	21	5
56	07	39	70	01	32	62	92	22	4
57	08	39	71	02	33	63	93	22	3
58	08	40	71	02	33	63	93	23	2
59	9·9998669	·9998640	·9998672	·9998703	·9998734	·9998764	·9998794	9·9998823	1
60	09	41	72	03	34	64	94	24	0
″	27′	26′	25′	24′	23′	22′	21′	20′	″

LOG. COSINE 1°.

″	32′	33′	34′	35′	36′	37′	38′	39′	″
0	11·5916963	5966619	6016848	6067664	6119082	6171114	6223777	11·6277085	60
1	7786	7451	7690	8516	9944	1986	4660	7979	59
2	8609	8284	8533	9369	6120806	2859	5543	8873	58
3	9433	9117	9375	6070221	1668	3732	6426	9768	57
4	11·5920256	9950	6020218	1073	2531	4605	7310	11·6280662	56
5	1080	5970783	1060	1926	3394	5478	8194	1557	55
6	1903	1616	1903	2779	4257	6352	9078	2452	54
7	2727	2449	2746	3632	5120	7225	9962	3347	53
8	3551	3283	3589	4485	5983	8099	6230847	4242	52
9	4376	4116	4433	5338	6847	8973	1731	5138	51
10	5200	4950	5276	6192	7710	9847	2616	6033	50
11	6025	5784	6120	7045	8574	6180721	3501	6929	49
12	6849	6619	6964	7899	9438	1596	4386	7825	48
13	7674	7453	7806	8753	6130302	2470	5271	8722	47
14	8499	8287	8652	9607	1167	3345	6157	9618	46
15	9324	9122	9497	6080462	2031	4220	7042	11·6290515	45
16	11·5930150	9957	6030341	1316	2896	5095	7928	1411	44
17	0975	5980792	1186	2171	3761	5970	8814	2308	43
18	1801	1627	2031	3026	4626	6846	9701	3206	42
19	2626	2462	2876	3881	5491	7722	6240587	4103	41
20	3452	3298	3721	4736	6357	8597	1473	5001	40
21	4278	4133	4566	5591	7222	9473	2360	5898	39
22	5105	4969	5412	6447	8088	6190350	3247	6796	38
23	5931	5805	6258	7303	8954	1226	4134	7694	37
24	6758	6641	7103	8158	9820	2102	5021	8593	36
25	7584	7477	7950	9014	6140686	2979	5909	9491	35
26	8411	8314	8796	9871	1552	3856	6797	11·6300390	34
27	9238	9150	9642	6090727	2419	4733	7684	1289	33
28	11·5940065	9987	6040489	1583	3286	5610	8572	2188	32
29	0893	5990824	1335	2440	4153	6488	9461	3087	31
30	1720	1661	2182	3297	5020	7366	6250349	3986	30
31	2548	2498	3029	4154	5887	8243	1238	4886	29
32	3376	3336	3876	5011	6755	9121	2126	5785	28
33	4204	4173	4724	5869	7622	9999	3015	6685	27
34	5032	5011	5571	6726	8490	6200878	3904	7586	26
35	5860	5849	6419	7584	9358	1756	4794	8486	25
36	6689	6687	7267	8442	6150226	2635	5683	9386	24
37	7517	7525	8115	9300	1095	3514	6573	11·6310287	23
38	8346	8363	8963	6100158	1963	4393	7462	1188	22
39	9175	9202	9811	1016	2832	5272	8353	2089	21
40	11·5950004	6000041	6050660	1875	3701	6151	9243	2990	20
41	0833	0879	1508	2734	4570	7031	6260133	3892	19
42	1663	1718	2357	3592	5439	7911	1024	4793	18
43	2492	2558	3206	4452	6308	8791	1914	5695	17
44	3322	3397	4055	5311	7178	9671	2805	6597	16
45	4152	4236	4905	6170	8047	6210551	3696	7499	15
46	4982	5076	5754	7030	8917	1431	4588	8402	14
47	5812	5916	6604	7889	9787	2312	5479	9304	13
48	6642	6756	7454	8749	6160658	3193	6371	11·6320207	12
49	7473	7596	8304	9609	1528	4074	7263	1110	11
50	8304	8436	9154	6110470	2399	4955	8155	2013	10
51	9134	9277	6060004	1330	3269	5836	9047	2917	9
52	9965	6010117	0855	2191	4140	6718	9939	3820	8
53	11·5960797	0958	1705	3051	5011	7600	6270832	4724	7
54	1628	1799	2556	3912	5883	8481	1725	5628	6
55	2459	2640	3407	4773	6754	9364	2617	6532	5
56	3291	3481	4258	5635	7626	6220246	3511	7436	4
57	4123	4323	5109	6496	8497	1128	4404	8340	3
58	4955	5165	5961	7358	9369	2011	5297	9245	2
59	5787	6006	6813	8220	6170242	2894	6191	11·6330150	1
60	6619	6848	7664	9082	1114	3777	7085	1055	0
″	27′	26′	25′	24′	23′	22′	21′	20′	″

LOG. COTAN. 1°.

"	40′	41′	42′	43′	44′	45′	46′	47′	"
0	9·9998824	·9998853	·9998882	·9998911	·9998939	·9998966	·9998994	9·9999021	60
1	24	54	83	11	39	67	94	21	59
2	25	54	83	11	40	67	95	22	58
3	25	55	83	12	40	68	95	22	57
4	26	55	84	12	40	68	96	23	56
5	26	56	84	13	41	69	96	23	55
6	27	56	85	13	41	69	96	23	54
7	27	57	85	14	42	70	97	24	53
8	28	57	86	14	42	70	97	24	52
9	9·9998828	·9998858	·9998886	·9998915	·9998943	·9998971	·9998998	9·9999025	51
10	29	58	87	15	43	71	98	25	50
11	29	58	87	16	44	71	99	26	49
12	30	59	88	16	44	72	99	26	48
13	30	59	88	17	45	72	·9999000	27	47
14	31	60	89	17	45	73	00	27	46
15	31	60	89	18	46	73	01	27	45
16	32	61	90	18	46	74	01	28	44
17	32	61	90	19	47	74	01	28	43
18	33	62	91	19	47	75	02	29	42
19	9·9998833	·9998862	·9998891	·9998919	·9998947	·9998975	·9999002	9·9999029	41
20	34	63	92	20	48	76	03	30	40
21	34	63	92	20	48	76	03	30	39
22	35	64	93	21	49	76	04	31	38
23	35	64	93	21	49	77	04	31	37
24	36	65	93	22	50	77	05	31	36
25	36	65	94	22	50	78	05	32	35
26	37	66	94	23	51	78	06	32	34
27	37	66	95	23	51	79	06	33	33
28	38	67	95	24	52	79	06	33	32
29	9·9998838	·9998867	·9998896	·9998924	·9998952	·9998980	·9999007	9·9999034	31
30	39	68	96	25	53	80	07	34	30
31	39	68	97	25	53	81	08	35	29
32	40	69	97	26	53	81	08	35	28
33	40	69	98	26	54	81	09	35	27
34	41	70	98	26	54	82	09	36	26
35	41	70	99	27	55	82	10	36	25
36	42	71	99	27	55	83	10	37	24
37	42	71	·9998900	28	56	83	10	37	23
38	42	71	00	28	56	84	11	38	22
39	9·9998843	·9998872	·9998901	·9998929	·9998957	·9998984	·9999011	9·9999038	21
40	43	72	01	29	57	85	12	39	20
41	44	73	02	30	58	85	12	39	19
42	44	73	02	30	58	86	13	39	18
43	45	74	02	31	59	86	13	40	17
44	45	74	03	31	59	86	14	40	16
45	46	75	03	32	59	87	14	41	15
46	46	75	04	32	60	87	14	41	14
47	47	76	04	33	60	88	15	42	13
48	47	76	05	33	61	88	15	42	12
49	9·9998848	·9998877	·9998905	·9998933	·9998961	·9998989	·9999016	9·9999043	11
50	48	77	06	34	62	89	16	43	10
51	49	78	06	34	62	90	17	43	9
52	49	78	07	35	63	90	17	44	8
53	50	79	07	35	63	91	18	44	7
54	50	79	08	36	64	91	18	45	6
55	51	80	08	36	64	91	19	45	5
56	51	80	09	37	65	92	19	46	4
57	52	81	09	37	65	92	19	46	3
58	52	81	10	38	65	93	20	47	2
59	9·9998853	·9998882	·9998910	·9998938	·9998966	·9998993	·9999020	9·9999047	1
60	53	82	11	39	66	94	21	47	0
"	19′	18′	17′	16′	15′	14′	13′	12′	"

LOG. COSINE 1°.

″	40′	41′	42′	43′	44′	45′	46′	47′	″
0	11·6331055	6385703	6441047	6497105	6553895	6611437	6669751	11·6728857	60
1	1960	11· 6620	11· 1976	11· 8046	11· 4848	11· 2403	6670729	9849	59
2	2865	7537	2904	8986	5801	3368	11· 1708	11·6730842	58
3	3771	8454	3833	9927	6754	4334	2687	1834	57
4	4677	9371	4762	6500868	7708	5300	3666	2827	56
5	5583	6390289	5691	1809	8661	6267	4646	3819	55
6	6489	1206	6621	2751	9615	7233	5625	4812	54
7	7395	2124	7550	3693	6560569	8200	6605	5806	53
8	8302	3042	8480	4635	1524	9167	7585	6799	52
9	9208	3960	9410	5577	2478	6620134	8566	7793	51
10	11·6340115	4879	6450340	6519	3433	1102	9546	8787	50
11	1022	5797	1271	7461	4388	2070	6680527	9781	49
12	1930	6716	2201	8404	5343	3037	1508	11·6740776	48
13	2837	7635	3132	9347	6298	4006	2489	1770	47
14	3745	8554	4063	6510290	7254	4974	3471	2765	46
15	4653	9473	4994	1233	8209	5942	4452	3760	45
16	5561	6400393	5926	2177	9165	6911	5434	4756	44
17	6469	1313	6857	3121	6570122	7880	6416	5751	43
18	7377	2233	7789	4064	1078	8849	7399	6747	42
19	8286	3153	8721	5009	2035	9819	8381	7743	41
20	9195	4073	9653	5953	2991	6630788	9364	8740	40
21	11·6350104	4994	6460586	6897	3948	1758	6690347	9736	39
22	1013	5914	1518	7842	4906	2728	1330	11·6750733	38
23	1922	6835	2451	8787	5863	3698	2313	1730	37
24	2832	7757	3384	9732	6821	4669	3297	2727	36
25	3742	8678	4317	3520678	7779	5639	4281	3724	35
26	4652	9599	5250	1623	8737	6610	5265	4722	34
27	5562	6410521	6184	2569	9695	7581	6249	5720	33
28	6472	1443	7118	3515	6580653	8553	7234	6718	32
29	7383	2365	8052	4461	1612	9524	8219	7716	31
30	8293	3287	8986	5408	2571	6640496	9204	8715	30
31	9204	4210	9920	6354	3530	1468	6700189	9714	29
32	11·6360115	5132	6470855	7301	4489	2440	1174	11·6760713	28
33	1026	6055	1790	8248	5449	3413	2160	1712	27
34	1938	6978	2725	9195	6409	4385	3146	2711	26
35	2850	7902	3660	6530143	7369	5358	4132	3711	25
36	3761	8825	4595	1090	8329	6331	5118	4711	24
37	4673	9749	5531	2038	9289	7305	6105	5711	23
38	5586	6420673	6467	2986	6590250	8278	7092	6712	22
39	6498	1597	7403	3934	1211	9252	8079	7712	21
40	7411	2521	8339	4883	2172	6650226	9066	8713	20
41	8324	3445	9275	5831	3133	1200	6710053	9714	19
42	9237	4370	6480212	6780	4094	2174	1041	11·6770715	18
43	11·6370150	5295	1149	7729	5056	3149	2029	1717	17
44	1063	6220	2086	8679	6018	4124	3017	2719	16
45	1977	7145	3023	9628	6980	5099	4005	3721	15
46	2890	8071	3960	3540578	7942	6074	4994	4723	14
47	3804	8996	4898	1528	8905	7050	5983	5726	13
48	4719	9922	5836	2478	9868	8025	6972	6728	12
49	5633	6430848	6774	3428	6600831	9001	7961	7731	11
50	6547	1774	7712	4379	1794	9977	8950	8734	10
51	7462	2701	8650	5329	2757	6660954	9940	9738	9
52	8377	3627	9589	6280	3721	1930	6720930	11·6780741	8
53	9292	4554	6490528	7231	4684	2907	1920	1745	7
54	11·6380207	5481	1467	8183	5649	3884	2910	2749	6
55	1123	6408	2406	9134	6613	4861	3901	3754	5
56	2039	7336	3345	6550086	7577	5839	4892	4758	4
57	2955	8263	4285	1038	8542	6816	5883	5763	3
58	3871	9191	5225	1990	9507	7794	6874	6768	2
59	4787	6440119	6165	2943	6610472	8772	7866	7773	1
60	5703	1047	7105	3895	1437	9751	8857	8779	0
″	19′	18′	17′	16′	15′	14′	13′	12′	″

LOG. COTAN. 1°.

LOG. SINE 88°.

″	48′	49′	50′	51′	52′	53′	54′	55′	″
0	9·9999047	·9999074	·9999100	·9999125	·9999150	·9999175	·9999200	9·9999224	60
1	48	74	00	26	51	76	00	24	59
2	49	75	00	26	51	76	00	24	58
3	49	75	01	26	52	76	01	25	57
4	49	75	01	27	52	77	01	25	56
5	50	76	02	27	52	77	02	26	55
6	50	76	02	28	53	77	02	26	54
7	51	77	03	28	53	78	02	26	53
8	51	77	03	29	54	78	03	27	52
9	9·9999051	·9999078	·9999103	·9999129	·9999154	·9999179	·9999203	9·9999227	51
10	52	78	04	29	54	79	04	28	50
11	52	78	04	30	55	80	04	28	49
12	53	79	05	30	55	80	04	28	48
13	53	79	05	31	56	80	05	29	47
14	54	80	06	31	56	81	05	29	46
15	54	80	06	31	57	81	06	30	45
16	54	81	06	32	57	82	06	30	44
17	55	81	07	32	57	82	06	30	43
18	55	82	07	33	58	83	07	31	42
19	9·9999056	·9999082	·9999108	·9999133	·9999158	·9999183	·9999207	9·9999231	41
20	56	82	08	34	59	83	08	32	40
21	57	83	09	34	59	84	08	32	39
22	57	83	09	34	59	84	08	32	38
23	58	84	09	35	60	85	09	33	37
24	58	84	10	35	60	85	09	33	36
25	58	85	10	36	61	85	10	34	35
26	59	85	11	36	61	86	10	34	34
27	59	85	11	37	62	86	10	35	33
28	60	86	12	37	62	87	11	35	32
29	9·9999060	·9999086	·9999112	·9999137	·9999162	·9999187	·9999211	9·9999235	31
30	61	87	12	38	63	87	12	36	30
31	61	87	13	38	63	88	12	36	29
32	61	88	13	39	64	88	12	36	28
33	62	88	14	39	64	89	13	37	27
34	62	88	14	39	64	89	13	37	26
35	63	89	15	40	65	89	14	38	25
36	63	89	15	40	65	90	14	38	24
37	64	90	15	41	66	90	14	38	23
38	64	90	16	41	66	91	15	39	22
39	9·9999065	·9999091	·9999116	·9999142	·9999166	·9999191	·9999215	9·9999239	21
40	65	91	17	42	67	91	16	39	20
41	65	91	17	42	67	92	16	40	19
42	66	92	18	43	68	92	16	40	18
43	66	92	18	43	68	93	17	41	17
44	67	93	18	44	69	93	17	41	16
45	67	93	19	44	69	93	18	41	15
46	68	94	19	44	69	94	18	42	14
47	68	94	20	45	70	94	18	42	13
48	68	94	20	45	70	95	19	43	12
49	9·9999069	·9999095	·9999120	·9999146	·9999171	·9999195	·9999219	9·9999243	11
50	69	95	21	46	71	96	20	43	10
51	70	96	21	47	71	96	20	44	9
52	70	96	22	47	72	96	20	44	8
53	71	97	22	47	72	97	21	45	7
54	71	97	23	48	73	97	21	45	6
55	72	97	23	48	73	98	22	45	5
56	72	98	23	49	73	98	22	46	4
57	72	98	24	49	74	98	22	46	3
58	73	99	24	49	74	99	23	47	2
59	9·9999073	·9999099	·9999125	·9999150	·9999175	·9999199	·9999223	9·9999247	1
60	74	·9999100	25	50	75	·9999200	24	47	0
″	11′	10′	9′	8′	7′	6′	5′	4′	″

LOG. COSINE 1°.

LOG. TAN. 88°.

″	48′	49′	50′	51′	52′	53′	54′	55′	″
0	11·6788779	6849538	6911158	6973665	7037083	7101441	7166766	11·7233088	60
1	9785	6850558	11· 2193	11· 4714	11· 8148	11· 2522	11· 7863	4202	59
2	11·6790790	11· 1578	3227	5764	9213	3603	8960	5316	58
3	1797	2598	4262	6814	7040279	4684	7170058	6430	57
4	2803	3619	5297	7864	1344	5765	1156	7545	56
5	3810	4640	6333	8914	2410	6847	2254	8660	55
6	4817	5661	7369	9965	3476	7929	3353	9776	54
7	5824	6682	8404	6981016	4543	9012	4451	11·7240892	53
8	6831	7704	9441	2067	5609	7110094	5550	2008	52
9	7839	8725	6920477	3119	6676	1177	6650	3124	51
10	8846	9747	1514	4170	7744	2260	7749	4240	50
11	9855	6860770	2551	5222	8811	3344	8849	5357	49
12	11·6800863	1792	3588	6275	9879	4428	9949	6474	48
13	1871	2815	4625	7327	7050947	5512	7181050	7592	47
14	2880	3838	5663	8380	2015	6596	2151	8709	46
15	3889	4861	6701	9433	3084	7680	3252	9827	45
16	4898	5885	7739	6990486	4152	8765	4353	11·7250946	44
17	5908	6908	8777	1540	5221	9850	5455	2064	43
18	6917	7932	9816	2593	6291	7120935	6556	3183	42
19	7927	8957	6930855	3647	7360	2021	7658	4302	41
20	8938	9981	1894	4702	8430	3107	8761	5422	40
21	9948	6871006	2933	5756	9500	4193	9864	6542	39
22	11·6810959	2031	3973	6811	7060571	5280	7190966	7662	38
23	1969	3056	5013	7866	1641	6366	2070	8782	37
24	2981	4081	6053	8921	2712	7453	3173	9902	36
25	3992	5107	7093	9977	3783	8540	4277	11·7261023	35
26	5003	6133	8134	7001033	4855	9628	5381	2144	34
27	6015	7159	9175	2089	5926	7130716	6485	3266	33
28	7027	8185	6940216	3145	6998	1804	7590	4388	32
29	8040	9212	1257	4201	8070	2892	8695	5510	31
30	9052	6880239	2299	5258	9143	3981	9800	6632	30
31	11·6820065	1266	3341	6315	7070216	5069	7200906	7755	29
32	1078	2293	4383	7373	1289	6159	2012	8878	28
33	2091	3321	5425	8430	2362	7248	3118	11·7270001	27
34	3105	4349	6468	9488	3435	8338	4224	1124	26
35	4118	5377	7511	7010546	4509	9428	5330	2248	25
36	5132	6405	8554	1605	5583	7140518	6437	3372	24
37	6146	7434	9597	2663	6658	1608	7545	4496	23
38	7161	8462	6950641	3722	7732	2699	8652	5621	22
39	8175	9492	1685	4781	8807	3790	9760	6746	21
40	9190	6890521	2729	5841	9882	4882	7210868	7871	20
41	11·6830205	1550	3774	6900	7080958	5973	1976	8997	19
42	1221	2580	4818	7960	2033	7065	3085	11·7280123	18
43	2236	3610	5863	9020	3109	8157	4194	1249	17
44	3252	4640	6908	7020081	4185	9250	5303	2375	16
45	4268	5671	7954	1142	5262	7150342	6412	3502	15
46	5285	6702	8999	2203	6339	1435	7522	4629	14
47	6301	7733	6960045	3264	7416	2529	8632	5756	13
48	7318	8764	1091	4325	8493	3622	9742	6884	12
49	8335	9795	2138	5387	9570	4716	7220853	8011	11
50	9352	6900827	3184	6449	7090648	5810	1964	9140	10
51	11·6840370	1859	4231	7511	1726	6904	3075	11·7290268	9
52	1387	2891	5278	8574	2805	7999	4186	1397	8
53	2405	3924	6326	9637	3883	9094	5298	2526	7
54	3424	4957	7373	7030700	4962	7160189	6410	3655	6
55	4442	5990	8421	1763	6041	1284	7522	4785	5
56	5461	7023	9469	2826	7121	2380	8635	5915	4
57	6480	8056	6970518	3890	8200	3476	9747	7045	3
58	7499	9090	1567	4954	9280	4572	7230861	8175	2
59	8518	6910124	2615	6019	7100360	5669	1974	9306	1
60	9538	1158	3665	7083	1441	6766	3088	11·7300437	0
″	11′	10′	9′	8′	7′	6′	5′	4′	″

LOG. COTAN. 1°.

	LOG. SINE 88°.				LOG. SINE 89°.				
″	56′	57′	58′	59′	0′	1′	2′	3′	″
0	9·9999247	·9999271	·9999294	·9999316	·9999338	·9999360	·9999382	9·9999403	60
1	48	9 71	9 94	9 17	9 39	9 61	9 82	03	59
2	48	71	94	17	39	61	83	04	58
3	49	72	95	17	40	61	83	04	57
4	49	72	95	18	40	62	83	04	56
5	49	73	96	18	40	62	84	05	55
6	50	73	96	19	41	63	84	05	54
7	50	73	96	19	41	63	84	05	53
8	50	74	97	19	41	63	85	06	52
9	9·9999251	·9999274	·9999297	·9999320	·9999342	·9999364	·9999385	9·9999406	51
10	51	75	97	20	42	64	85	06	50
11	52	75	98	20	43	64	86	07	49
12	52	75	98	21	43	65	86	07	48
13	52	76	99	21	43	65	86	08	47
14	53	76	99	21	44	65	87	08	46
15	53	76	99	22	44	66	87	08	45
16	54	77	·9999300	22	44	66	88	09	44
17	54	77	00	23	45	66	88	09	43
18	54	78	00	23	45	67	88	09	42
19	9·9999255	·9999278	·9999301	·9999323	·9999345	·9999367	·9999389	9·9999410	41
20	55	78	01	24	46	68	89	10	40
21	56	79	02	24	46	68	89	10	39
22	56	79	02	24	47	68	90	11	38
23	56	80	02	25	47	69	90	11	37
24	57	80	03	25	47	69	90	11	36
25	57	80	03	26	48	69	91	12	35
26	58	81	04	26	48	70	91	12	34
27	58	81	04	26	48	70	91	12	33
28	58	81	04	27	49	70	92	13	32
29	9·9999259	·9999282	·9999305	·9999327	·9999349	·9999371	·9999392	9·9999413	31
30	59	82	05	27	49	71	92	13	30
31	59	83	05	28	50	72	93	14	29
32	60	83	06	28	50	72	93	14	28
33	60	83	06	29	51	72	94	14	27
34	61	84	07	29	51	73	94	15	26
35	61	84	07	29	51	73	94	15	25
36	61	85	07	30	52	73	95	16	24
37	62	85	08	30	52	74	95	16	23
38	62	85	08	30	52	74	95	16	22
39	9·9999263	·9999286	·9999308	·9999331	·9999353	·9999374	·9999396	9·9999417	21
40	63	86	09	31	53	75	96	17	20
41	63	86	09	31	53	75	96	17	19
42	64	87	10	32	54	75	97	18	18
43	64	87	10	32	54	76	97	18	17
44	65	88	10	33	55	76	97	18	16
45	65	88	11	33	55	77	98	19	15
46	65	88	11	33	55	77	98	19	14
47	66	89	11	34	56	77	98	19	13
48	66	89	12	34	56	78	99	20	12
49	9·9999266	·9999289	·9999312	·9999334	·9999356	·9999378	·9999399	9·9999420	11
50	67	90	13	35	57	78	·9999400	20	10
51	67	90	13	35	57	79	00	21	9
52	68	91	13	36	57	79	00	21	8
53	68	91	14	36	58	79	01	21	7
54	68	91	14	36	58	80	01	22	6
55	69	92	14	37	59	80	01	22	5
56	69	92	15	37	59	80	02	22	4
57	70	93	15	37	59	81	02	23	3
58	70	93	16	38	60	81	02	23	2
59	9·9999270	·9999293	·9999316	·9999338	·9999360	·9999382	·9999403	9·9999423	1
60	71	94	16	38	60	82	03	24	0
″	3′	2′	1′	0′	59′	58′	57′	56′	″
	LOG. COSINE 1°.				LOG. COSINE 0°.				

″	56′	57′	58′	59′	0′	1′	2′	3′	″
0	11·7300437	7368847	7438351	7508985	7580785	7653792	7728047	11·7803592	60
1	1569	9996	9519	7510172	11· 1992	11· 5020	9295	4863	59
2	2700	7371146	7440667	11· 1359	3199	6247	7730544	6133	58
3	3832	11· 2296	11· 1856	2547	4407	7475	11· 1793	7404	57
4	4965	3446	3024	3735	5614	8703	3043	8676	56
5	6097	4597	4194	4923	6823	9932	4292	9947	55
6	7230	5748	5363	6112	8031	7661161	5543	11·7811220	54
7	8363	6899	6533	7301	9240	2390	6793	2492	53
8	9497	8050	7703	8490	7590449	3620	8044	3765	52
9	11·7310630	9202	8873	9679	1658	4850	9295	5038	51
10	1764	7380354	7450044	7520869	2868	6080	7740547	6312	50
11	2899	1507	1215	2060	4078	7311	1799	7586	49
12	4033	2659	2386	3250	5289	8542	3051	8860	48
13	5168	3812	3558	4441	6500	9773	4304	11·7820135	47
14	6304	4966	4730	5632	7711	7671005	5557	1410	46
15	7439	6119	5902	6824	8922	2237	6810	2686	45
16	8575	7273	7075	8016	7600134	3470	8064	3962	44
17	9711	8427	8248	9208	1346	4703	9318	5238	43
18	11·7320847	9582	9421	7530401	2559	5936	7750573	6514	42
19	1984	7390737	7460594	1593	3772	7169	1828	7791	41
20	3121	1892	1768	2787	4985	8403	3083	9069	40
21	4258	3047	2942	3980	6198	9637	4339	11·7830347	39
22	5396	4203	4117	5174	7412	7680872	5595	1625	38
23	6534	5359	5292	6368	8627	2107	6851	2903	37
24	7672	6515	6467	7563	9841	3342	8108	4182	36
25	8811	7672	7642	8758	7611056	4578	9365	5461	35
26	9949	8829	8818	9953	2271	5814	7760622	6741	34
27	11·7331089	9986	9994	7541148	3487	7050	1880	8021	33
28	2228	7401144	7471171	2344	4703	8287	3138	9301	32
29	3368	2302	2347	3540	5919	9524	4396	11·7840582	31
30	4508	3460	3524	4737	7135	7690761	5655	1863	30
31	5648	4619	4702	5934	8352	1999	6915	3145	29
32	6788	5777	5879	7131	9570	3237	8174	4427	28
33	7929	6937	7057	8328	7620787	4475	9434	5709	27
34	9071	8096	8236	9526	2005	5714	7770695	6992	26
35	11·7340212	9256	9414	7550724	3224	6953	1955	8275	25
36	1354	7410416	7480593	1923	4442	8193	3216	9558	24
37	2496	1576	1773	3121	5661	9432	4478	11·7850842	23
38	3638	2737	2952	4320	6880	7700673	5740	2126	22
39	4781	3898	4132	5520	8100	1913	7002	3411	21
40	5924	5059	5312	6720	9320	3154	8264	4696	20
41	7067	6221	6493	7920	7630540	4395	9527	5981	19
42	8211	7383	7674	9120	1761	5637	7780790	7267	18
43	9355	8545	8855	7560321	2982	6879	2054	8553	17
44	11·7350499	9708	7490036	1522	4204	8121	3318	9839	16
45	1643	7420871	1218	2724	5425	9364	4582	11·7861126	15
46	2788	2034	2400	3925	6647	7710607	5847	2413	14
47	3933	3197	3583	5128	7870	1850	7112	3701	13
48	5079	4361	4766	6330	9092	3094	8378	4939	12
49	6224	5525	5949	7533	7640316	4338	9644	6277	11
50	7370	6690	7132	8736	1539	5583	7790910	7566	10
51	8517	7854	8316	9939	2763	6827	2176	8855	9
52	9663	9019	9500	7571143	3987	8073	3443	11·7870145	8
53	11·7360810	7430185	7500685	2347	5211	9318	4711	1434	7
54	1957	1350	1869	3552	6436	7720564	5978	2725	6
55	3105	2516	3054	4756	7661	1810	7246	4015	5
56	4253	3683	4240	5962	8887	3057	8515	5306	4
57	5401	4849	5426	7167	7650113	4304	9784	6598	3
58	6549	6016	6612	8373	1339	5551	7801053	7890	2
59	7698	7183	7798	9579	2565	6799	2322	9182	1
60	8847	8351	8985	7580785	3792	8047	3592	11·7880474	0
″	3′	2′	1′	0′	59′	58′	57′	56′	″

LOG. COTAN. 1°. LOG. COTAN. 0°.

LOG. SINE 89°.

″	4′	5′	6′	7′	8′	9′	10′	11′	″
0	9·9999424	·9999444	·9999464	·9999484	·9999503	·9999522	·9999541	9·9999559	60
1	24	44	65	84	03	22	41	59	59
2	24	45	65	84	04	23	41	59	58
3	25	45	65	85	04	23	42	60	57
4	25	46	66	85	04	23	42	60	56
5	25	46	66	85	05	24	42	60	55
6	26	46	66	86	05	24	42	61	54
7	26	47	67	86	05	24	43	61	53
8	27	47	67	86	06	25	43	61	52
9	9·9999427	·9999447	·9999467	·9999487	·9999506	·9999525	·9999543	9·9999562	51
10	27	48	67	87	06	25	44	62	50
11	28	48	68	87	07	26	44	62	49
12	28	48	68	88	07	26	44	62	48
13	28	49	68	88	07	26	45	63	47
14	29	49	69	88	08	26	45	63	46
15	29	49	69	89	08	27	45	63	45
16	29	50	69	89	08	27	46	64	44
17	30	50	70	89	09	27	46	64	43
18	30	50	70	90	09	28	46	64	42
19	9·9999430	·9999451	·9999470	·9999490	·9999509	·9999528	·9999546	9·9999565	41
20	31	51	71	90	09	28	47	65	40
21	31	51	71	91	10	29	47	65	39
22	31	52	71	91	10	29	47	65	38
23	32	52	72	91	10	29	48	66	37
24	32	52	72	92	11	30	48	66	36
25	32	53	72	92	11	30	48	66	35
26	33	53	73	92	11	30	49	67	34
27	33	53	73	93	12	30	49	67	33
28	33	54	73	93	12	31	49	67	32
29	9·9999434	·9999454	·9999474	·9999493	·9999512	·9999531	·9999549	9·9999567	31
30	34	54	74	94	13	31	50	68	30
31	34	55	74	94	13	32	50	68	29
32	35	55	75	94	13	32	50	68	28
33	35	55	75	95	14	32	51	69	27
34	35	56	75	95	14	33	51	69	26
35	36	56	76	95	14	33	51	69	25
36	36	56	76	95	15	33	52	70	24
37	36	57	76	96	15	34	52	70	23
38	37	57	77	96	15	34	52	79	22
39	9·9999437	·9999457	·9999477	·9999496	·9999515	·9999534	·9999552	9·9999570	21
40	37	58	77	97	16	34	53	71	20
41	38	58	78	97	16	35	53	71	19
42	38	58	78	97	16	35	53	71	18
43	38	59	78	98	17	35	54	72	17
44	39	59	79	98	17	36	54	72	16
45	39	59	79	98	17	36	54	72	15
46	39	60	79	99	18	36	55	73	14
47	40	60	80	99	18	37	55	73	13
48	40	60	80	99	18	37	55	73	12
49	9·9999440	·9999461	·9999480	·9999500	·9999519	·9999537	·9999556	9·9999573	11
50	41	61	81	00	19	38	56	74	10
51	41	61	81	00	19	38	56	74	9
52	41	62	81	01	20	38	56	74	8
53	42	62	82	01	20	38	57	75	7
54	42	62	82	01	20	39	57	75	6
55	42	63	82	02	21	39	57	75	5
56	43	63	83	02	21	39	58	75	4
57	43	63	83	02	21	40	58	76	3
58	43	64	83	03	21	40	58	76	2
59	9·9999444	·9999464	·9999484	·9999503	·9999522	·9999540	·9999559	9·9999576	1
60	44	64	84	03	22	41	59	77	0
″	55′	54′	53′	52′	51′	50′	49′	48′	″

LOG. COSINE 0°.

LOG. TAN. 89°.

″	4′	5′	6′	7′	8′	9′	10′	11′	″
0	11·7880474	7958741	8038444	8119636	8202374	8286718	8372733	11·8460484	60
1	1767	7960058	9785	8121001	11· 3767	11· 8138	11· 4181	1962	59
2	3061	11· 1375	8041126	11· 2369	5159	9558	5629	3440	58
3	4354	2692	11· 2468	3736	6553	8290979	7078	4918	57
4	5649	4010	3810	5103	7946	2400	8528	6397	56
5	6943	5328	5152	6471	9341	3822	9978	7877	55
6	8238	6646	6495	7839	8210735	5244	8381428	9357	54
7	9533	7965	7839	9208	2130	6666	2879	11·8470838	53
8	11·7890829	9284	9182	8130577	3526	8089	4331	2319	52
9	2125	7970604	8050527	1947	4922	9513	5783	3801	51
0	3421	1924	1871	3317	6318	8300936	7235	5283	50
1	4718	3244	3216	4687	7715	2361	8688	6766	49
2	6015	4565	4561	6058	9113	3786	8390142	8249	48
3	7313	5887	5907	7429	8220510	5211	1596	9733	47
4	8611	7208	7254	8801	1909	6637	3050	11·8481217	46
5	9909	8530	8600	8140173	3307	8063	4505	2701	45
6	11·7901208	9853	9947	1546	4706	9490	5960	4187	44
7	2507	7981176	8061295	2919	6106	8310917	7416	5672	43
8	3807	2499	2643	4292	7506	2344	8872	3159	42
9	5107	3823	3991	5666	8906	3772	8400329	8645	41
0	6407	5147	5340	7041	8230307	5201	1787	11·8490133	40
1	7708	6471	6689	8415	1709	6630	3244	1620	39
2	9009	7796	8039	9791	3111	8060	4703	3109	38
3	11·7910310	9121	9389	8151166	4513	9490	6161	4598	37
4	1612	7990447	8070739	2542	5916	8320920	7621	6087	36
5	2914	1773	2090	3919	7319	2351	9080	7577	35
6	4217	3100	3441	5296	8722	3782	8410541	9067	34
7	5520	4427	4793	6673	8240127	5214	2001	11·8500558	33
8	6824	5754	6145	8051	1531	6647	3463	2049	32
9	8127	7082	7497	9429	2936	8079	4924	3541	31
0	9432	8410	8850	8160808	4342	9513	6387	5033	30
1	11·7920736	9738	8080204	2187	5748	8330946	7849	6526	29
2	2041	8001067	1558	3567	7154	2381	9313	8020	28
3	3347	2397	2912	4947	8561	3815	8420776	9513	27
4	4652	3727	4266	6327	9968	5251	2241	11·8511008	26
5	5959	5057	5621	7708	8251376	6686	3705	2503	25
6	7265	6387	6977	9090	2784	8122	5170	3998	24
7	8572	7718	8333	8170471	4193	9559	6636	5494	23
8	9880	9050	9689	1854	5602	8340996	8102	6991	22
9	11·7931187	8010381	8091046	3236	7011	2434	9569	8488	21
0	2495	1714	2403	4619	8421	3872	8431036	9985	20
1	3804	3046	3761	6003	9832	5310	2504	11·8521483	19
2	5113	4379	5119	7387	8261243	6749	3972	2982	18
3	6422	5713	6477	8771	2654	8188	5441	4481	17
4	7732	7047	7836	8180156	4066	9628	6910	5980	16
5	9042	8381	9195	1541	5478	8351069	8379	7480	15
6	11·7940353	9716	8100555	2927	6891	2510	9849	8981	14
7	1663	8021051	1915	4313	8304	3951	8441320	11·8530482	13
8	2975	2386	3276	5700	9718	5393	2791	1984	12
9	4286	3722	4637	7087	8271132	6835	4263	3486	11
0	5599	5058	5998	8475	2547	8278	5735	4989	10
1	6911	6395	7360	9863	3962	9721	7208	6492	9
2	8224	7732	8722	8191251	5377	8361165	8681	7996	8
3	9537	9070	8110085	2640	6793	2609	8450154	9500	7
4	11·7950851	8030408	1448	4029	8210	4054	1629	11·8541005	6
5	2165	1746	2812	5419	9627	5499	3103	2510	5
6	3479	3085	4176	6809	8281044	6945	4578	4016	4
7	4794	4424	5540	8200	2462	8391	6054	5522	3
8	6110	5764	6905	9591	3880	9838	7530	7029	2
9	7425	7104	8270	8200982	5299	8371285	9007	8536	1
0	8741	8444	9636	2374	6718	2733	8460484	11·8550044	0
	55′	54′	53′	52′	51′	50′	49′	48′	″

LOG. COTAN. 0°.

″	12′	13′	14′	15′	16′	17′	18′	19′	″
0	9·9999577	·9999591	·9999611	·9999628	·9999644	·9999660	·9999676	9·9999691	60
1	77	9 94	9 11	9 28	9 45	9 61	9 76	91	59
2	77	95	12	28	45	61	76	92	58
3	78	95	12	29	45	61	77	92	57
4	78	95	12	29	45	61	77	92	56
5	78	96	13	29	46	62	77	92	55
6	78	96	13	30	46	62	77	93	54
7	79	96	13	30	46	62	78	93	53
8	79	96	13	30	46	62	78	93	52
9	9·9999579	·9999597	·9999614	·9999630	·9999647	·9999663	·9999678	9·9999693	51
10	80	97	14	31	47	63	78	94	50
11	80	97	14	31	47	63	79	94	49
12	80	98	15	31	47	63	79	94	48
13	80	98	15	31	48	64	79	94	47
14	81	98	15	32	48	64	79	95	46
15	81	98	15	32	48	64	80	95	45
16	81	99	16	32	49	64	80	95	44
17	82	99	16	33	49	65	80	95	43
18	82	99	16	33	49	65	80	96	42
19	9·9999582	·9999600	·9999617	·9999633	·9999649	·9999665	·9999681	9·9999696	41
20	83	00	17	33	50	66	81	96	40
21	83	00	17	34	50	66	81	96	39
22	83	00	17	34	50	66	82	97	38
23	83	01	18	34	50	66	82	97	37
24	84	01	18	34	51	67	82	97	36
25	84	01	18	35	51	67	82	97	35
26	84	02	18	35	51	67	83	98	34
27	85	02	19	35	52	67	83	98	33
28	85	02	19	36	52	68	83	98	32
29	9·9999585	·9999602	·9999619	·9999636	·9999652	·9999668	·9999683	9·9999698	31
30	85	03	20	36	52	68	84	99	30
31	86	03	20	36	53	68	84	99	29
32	86	03	20	37	53	69	84	99	28
33	86	04	20	37	53	69	84	99	27
34	87	04	21	37	53	69	85	9·9999700	26
35	87	04	21	37	54	69	85	00	25
36	87	04	21	38	54	70	85	90	24
37	87	05	22	38	54	70	85	00	23
38	88	05	22	38	54	70	86	01	22
39	9·9999588	·9999605	·9999622	·9999639	·9999655	·9999670	·9999686	9·9999701	21
40	88	06	22	39	55	71	86	04	20
41	89	06	23	39	55	71	86	01	19
42	89	06	23	39	55	71	87	02	18
43	89	06	23	40	56	71	87	02	17
44	89	07	23	40	56	72	87	02	16
45	90	07	24	40	56	72	87	02	15
46	90	07	24	40	57	72	88	03	14
47	90	08	24	41	57	73	88	03	13
48	91	08	25	41	57	73	88	03	12
49	9·9999591	·9999608	·9999625	·9999641	·9999657	·9999673	·9999688	9·9999703	11
50	91	08	25	42	58	73	89	04	10
51	92	09	25	42	58	74	89	04	9
52	92	09	26	42	58	74	89	04	8
53	92	09	26	42	58	74	89	04	7
54	92	09	26	43	59	74	90	05	6
55	93	10	27	43	59	75	90	05	5
56	93	10	27	43	59	75	90	05	4
57	93	10	27	44	59	75	90	05	3
58	94	11	27	44	60	75	91	06	2
59	9·9999594	·9999611	·9999628	·9999644	·9999660	·9999676	·9999691	9·9999706	1
60	94	11	28	44	60	76	91	06	0
″	47′	46′	45′	44′	43′	42′	41′	40′	″

LOG. COSINE 0°.

″	12′	13′	14′	15′	16′	17′	18′	19′	″
0	11·8550044	8641490	8734901	8830366	8927975	9027828	9130030	11·9234694	60
1	1553	11· 3030	11· 6475	11· 1975	9620	9512	11· 1754	6460	59
2	3062	4571	8050	3584	8931267	9031196	3478	8227	58
3	4571	6113	9625	5194	11· 2913	11· 2881	5204	9994	57
4	6081	7655	8741201	6805	4561	4567	6930	11·9241762	56
5	7592	9196	2777	8416	6209	6253	8656	3531	55
6	9103	8650741	4354	8840028	7858	7940	9140384	5301	54
7	11·8560614	2285	5931	1641	9507	9628	2112	7071	53
8	2126	3829	7509	3254	8941157	9041317	3840	8842	52
9	3639	5374	9088	4868	2807	3006	5570	11·9250614	51
10	5152	6919	8750667	6482	4458	4695	7300	2386	50
11	6666	8465	2247	8097	6110	6386	9031	4159	49
12	8180	8660012	3827	9713	7763	8077	9150762	5933	48
13	9695	1559	5408	8851329	9416	9768	2494	7703	47
14	11·8571210	3107	6989	2946	8951069	9051461	4227	9483	46
15	2726	4655	8571	4563	2724	3154	5961	11·9261259	45
16	4242	6204	8760154	6181	4379	4847	7695	3035	44
17	5759	7753	1737	7800	6034	6542	9430	4814	43
18	7276	9303	3321	9419	7691	8237	9161165	6592	42
19	8794	8670853	4905	8861039	9347	9932	2902	8371	41
20	11·8580313	2404	6490	2659	8961005	9061629	4639	11·9270150	40
21	1832	3956	8076	4280	2663	3326	6376	1931	39
22	3351	5508	9662	5902	4322	5023	8115	3712	38
23	4871	7060	8771248	7524	5981	6722	9854	5494	37
24	6392	8614	2836	9147	7641	8421	9171594	7277	36
25	7913	8680167	4423	8870770	9302	9070120	3334	9060	35
26	9434	1722	6012	2394	8970963	1820	5075	11·9280844	34
27	11·8590957	3277	7601	4019	2625	3521	6817	2629	33
28	2479	4832	9190	5644	4287	5223	8560	4414	32
29	4003	6388	8780781	7270	5951	6925	9180303	6201	31
30	5526	7944	2371	8896	7614	8628	2047	7988	30
31	7051	9502	3963	8880523	9279	9080332	3792	9775	29
32	8575	8691059	5554	2151	8980944	2036	5537	11·9291564	28
33	11·8600101	2617	7147	3779	2610	3741	7283	3353	27
34	1627	4176	8740	5408	4276	5447	9030	5143	26
35	3153	5735	8790334	7038	5943	7153	9190777	6934	25
36	4680	7295	1928	8668	7611	8860	2525	8725	24
37	6208	8856	3523	8890298	9279	9090568	4274	11·9300517	23
38	7736	8700417	5118	1930	8990948	2276	6024	2310	22
39	9264	1978	6714	3562	2618	3985	7774	4104	21
40	11·8610793	3540	8311	5194	4288	5695	9525	5896	20
41	2323	5103	9908	6827	5959	7405	9201277	7694	19
42	3853	6666	8801505	8461	7630	9116	3029	9489	18
43	5384	8230	3104	8900096	9302	9100828	4782	11·9311286	17
44	6915	9794	4703	1731	9000975	2540	6536	3083	16
45	8447	8711359	6302	3366	2649	4253	8291	4882	15
46	9980	2925	7902	5002	4323	5967	9210046	6680	14
47	11·8621512	4491	9503	6639	5997	7681	1802	8480	13
48	3046	6058	8811104	8277	7673	9396	3559	11·9320290	12
49	4580	7625	2706	9915	9349	9111112	5316	2081	11
50	6114	9193	4309	8911554	9011025	2828	7074	3883	10
51	7650	8720761	5912	3193	2703	4545	8833	5686	9
52	9185	2330	7515	4833	4381	6263	9220593	7489	8
53	11·8630721	3899	9119	6474	6059	7982	2353	9293	7
54	2258	5469	8820724	8115	7739	9701	4114	11·9331098	6
55	3795	7040	2330	9757	9418	9121421	5875	2904	5
56	5333	8611	3936	8921399	9021099	3141	7638	4710	4
57	6871	8730183	5542	3042	2780	4862	9401	6517	3
58	8410	1755	7149	4686	4462	6584	9231165	8325	2
59	9950	3328	8757	6330	6145	8307	2929	11·9340134	1
60	11·8641490	4901	8830366	7975	7828	9130030	4694	1943	0
″	47′	46′	45′	44′	43′	42′	41′	40′	″

LOG. COTAN. 0°.

″	20′	21′	22′	23′	24′	25′	26′	27′	″
0	9·9999706	·9999721	·9999735	·9999748	·9999762	·9999775	·9999788	9·9999800	60
1	06	21	35	49	62	75	88	00	59
2	06	21	35	49	62	75	88	00	58
3	07	21	35	49	63	76	88	01	57
4	07	21	36	49	63	76	88	01	56
5	07	22	36	50	63	76	89	01	55
6	07	22	36	50	63	76	89	01	54
7	08	22	36	50	63	76	89	01	53
8	08	22	37	50	64	77	89	02	52
9	08	23	37	50	64	77	89	02	51
10	9·9999708	·9999723	·9999737	·9999751	·9999764	·9999777	·9999790	9·9999802	50
11	09	23	37	51	64	77	90	02	49
12	09	23	37	51	65	77	90	02	48
13	09	24	38	51	65	78	90	03	47
14	09	24	38	52	65	78	90	03	46
15	10	24	38	52	65	78	91	03	45
16	10	24	38	52	65	78	91	03	44
17	10	25	39	52	66	79	91	03	43
18	10	25	39	53	66	79	91	04	42
19	11	25	39	53	66	79	92	04	41
20	9·9999711	·9999725	·9999739	·9999753	·9999766	·9999779	·9999792	9·9999804	40
21	11	26	40	53	66	79	92	04	39
22	11	26	40	53	67	80	92	04	38
23	12	26	40	54	67	80	92	05	37
24	12	26	40	54	67	80	93	05	36
25	12	26	40	54	67	80	93	05	35
26	12	27	41	54	68	80	93	05	34
27	13	27	41	55	68	81	93	05	33
28	13	27	41	55	68	81	93	06	32
29	13	27	41	55	68	81	94	06	31
30	9·9999713	·9999728	·9999742	·9999755	·9999768	·9999781	·9999794	9·9999806	30
31	14	28	42	55	69	82	94	06	29
32	14	28	42	56	69	82	94	06	28
33	14	28	42	56	69	82	94	07	27
34	14	29	43	56	69	82	95	07	26
35	15	29	43	56	70	82	95	07	25
36	15	29	43	57	70	83	95	07	24
37	15	29	43	57	70	83	95	07	23
38	15	30	43	57	70	83	95	08	22
39	15	30	44	57	70	83	96	08	21
40	9·9999716	·9999730	·9999744	·9999757	·9999771	·9999783	·9999796	9·9999808	20
41	16	30	44	58	71	84	96	08	19
42	16	30	44	58	71	84	96	08	18
43	16	31	45	58	71	84	96	08	17
44	17	31	45	58	71	84	97	09	16
45	17	31	45	59	72	84	97	09	15
46	17	31	45	59	72	85	97	09	14
47	17	32	45	59	72	85	97	09	13
48	18	32	46	59	72	85	97	09	12
49	18	32	46	59	73	85	98	10	11
50	9·9999718	·9999732	·9999746	·9999760	·9999773	·9999786	·9999798	9·9999810	10
51	18	33	46	60	73	86	98	10	9
52	19	33	47	60	73	86	98	10	8
53	19	33	47	60	73	86	98	10	7
54	19	33	47	61	74	86	99	11	6
55	19	34	47	61	74	87	99	11	5
56	20	34	48	61	74	87	99	11	4
57	20	34	48	61	74	87	99	11	3
58	20	34	48	61	74	87	99	11	2
59	20	34	48	62	75	87	·9999800	12	1
60	21	35	48	62	75	88	00	12	0
″	39′	38′	37′	36′	35′	34′	33′	32′	″

LOG. COSINE 0°.

″	20′	21′	22′	23′	24′	25′	26′	27′	″
0	11·9341943	9451906	9564726	9680554	9799555	11·9921908	12·0047808	12·0177466	60
1	3753	11· 3763	11· 6631	11· 2511	9801566	3977	9938	9660	59
2	5564	5620	8538	4469	11· 3578	6047	12·0052068	12·0181855	58
3	7375	7478	9570445	6427	5592	8117	4200	4052	57
4	9188	9337	2352	8387	7606	11·9930189	6333	6249	56
5	11·9351001	9461197	4261	9690347	9621	2262	8466	8448	55
6	2815	3057	6171	2308	9811636	4335	12·0060601	12·0190647	54
7	4629	4919	8081	4271	3653	6410	2737	2848	53
8	6445	6781	9992	6234	5671	8486	4874	5050	52
9	8261	8644	9581904	8198	7690	11·9940562	7012	7253	51
10	11·9360078	9470507	3817	9700162	9709	2640	9151	9457	50
11	1896	2372	5731	2128	9821730	4718	12·0071291	12·0201662	49
12	3714	4237	7646	4095	3752	6768	3432	3869	48
13	5533	6103	9561	6062	5774	8879	5574	6076	47
14	7353	7970	9591478	8031	7797	11·9950960	7717	8285	46
15	9174	9838	3395	9710000	9822	3043	9862	12·0210494	45
16	11·9370995	9481706	5313	1970	9831847	5126	12·0082007	2705	44
17	2818	3576	7232	3941	3873	7211	4153	4917	43
18	4641	5446	9151	5913	5901	9297	6301	7130	42
19	6464	7317	9601072	7886	7929	11·9961383	8449	9345	41
20	8289	9188	2993	9860	9958	3471	12·0090599	12·0221560	40
21	11·9380114	9491061	4915	9721834	9841988	5559	2749	3776	39
22	1940	2934	6838	3810	4019	7649	4901	5994	38
23	3767	4808	8762	5787	6051	9740	7054	8213	37
24	5595	6683	9610687	7764	8084	11·9971831	9208	12·0230433	36
25	7423	8559	2613	9742	9850117	3924	12·0101363	2654	35
26	9252	9500436	4539	9731721	2152	6017	3519	4876	34
27	11·9391082	2313	6467	3701	4188	8112	5676	7099	33
28	2913	4191	8395	5682	6225	11·9980208	7834	9324	32
29	4745	6070	9620324	7664	8262	2304	9993	12·0241549	31
30	6577	7950	2254	9647	9860301	4402	12·0112153	3776	30
31	8410	9831	4185	9741631	2340	6501	4315	6004	29
32	11·9400244	9511712	6116	3615	4381	8600	6477	8233	28
33	2078	3594	8049	5601	6422	11·9990701	8641	12·0250463	27
34	3913	5477	9982	7587	8465	2803	12·0120805	2694	26
35	5750	7361	9631916	9574	9870508	4906	2971	4927	25
36	7586	9246	3851	9751563	2553	7009	5138	7160	24
37	9424	9521131	5787	3552	4598	9114	7305	9395	23
38	11·9411263	3018	7724	5542	6644	12·0001220	9474	12·0261631	22
39	3102	4905	9662	7533	8692	3327	12·0131644	3868	21
40	4942	6793	9641600	9525	9880740	5435	3815	6106	20
41	6783	8682	3540	9761517	2789	7544	5987	8345	19
42	8624	9530571	5480	3511	4839	9654	8161	12·0270586	18
43	11·9420466	2462	7421	5506	6890	12·0011764	12·0140335	2827	17
44	2310	4353	9363	7501	8942	3876	2510	5070	16
45	4154	6245	9651306	9498	9890995	5989	4687	7314	15
46	5998	8138	3250	9771495	3049	8103	6865	9559	14
47	7844	9540032	5194	3493	5104	12·0020218	9043	12·0281806	13
48	9690	1926	7140	5493	7160	2334	12·0151223	4053	12
49	11·9431537	3822	9086	7493	9217	4452	3404	6302	11
50	3385	5718	9661033	9494	9901275	6570	5586	8551	10
51	5233	7615	2982	9781496	3334	8689	7769	12·0290802	9
52	7083	9513	4931	3499	5394	12·0030809	9953	3055	8
53	8933	9551412	6880	5502	7455	2930	12·0162138	5308	7
54	11·9440784	3311	8831	7507	9517	5053	4325	7562	6
55	2636	5212	9670783	9513	9911580	7176	6512	9818	5
56	4488	7113	2735	9791519	3643	9300	8701	12·0302075	4
57	6342	9015	4689	3527	5708	12·0041426	12.0170890	4333	3
58	8196	9560918	6643	5535	7774	3552	3081	6592	2
59	11·9450051	2821	8598	7545	9841	5680	5273	8852	1
60	1906	4726	9680554	9555	9921908	7808	7466	12·0311114	0
″	39′	38′	37′	36′	35′	34′	33′	32′	″

LOG. COTAN. 0°.

LOG. SINE 89°.

″	28′	29′	30′	31′	32′	33′	34′	35′	″
0	9·9999812	·9999823	·9999835	·9999845	·9999856	·9999866	·9999876	9·9999885	60
1	12	9 24	9 35	9 46	9 56	9 66	9 76	85	59
2	12	24	35	46	56	66	76	85	58
3	12	24	35	46	56	67	76	86	57
4	13	24	35	46	57	67	76	86	56
5	13	24	36	46	57	67	77	86	55
6	13	25	36	47	57	67	77	86	54
7	13	25	36	47	57	67	77	86	53
8	13	25	36	47	57	67	77	86	52
9	9·9999814	·9999825	·9999836	·9999847	·9999857	·9999868	·9999877	9·9999887	51
10	14	25	36	47	58	68	77	87	50
11	14	26	37	47	58	68	78	87	49
12	14	26	37	48	58	68	78	87	48
13	14	26	37	48	58	68	78	87	47
14	15	26	37	48	58	68	78	87	46
15	15	26	37	48	59	69	78	87	45
16	15	26	38	48	59	69	78	88	44
17	15	27	38	48	59	69	78	88	43
18	15	27	38	49	59	69	79	88	42
19	9·9999816	·9999827	·9999838	·9999849	·9999859	·9999869	·9999879	9·9999888	41
20	16	27	38	49	59	69	79	88	40
21	16	27	38	49	60	70	79	88	39
22	16	28	39	49	60	70	79	89	38
23	16	28	39	50	60	70	79	89	37
24	17	28	39	50	60	70	80	89	36
25	17	28	39	50	60	70	80	89	35
26	17	28	39	50	60	70	80	89	34
27	17	29	40	50	61	70	80	89	33
28	17	29	40	50	61	71	80	89	32
29	9·9999817	·9999829	·9999840	·9999851	·9999861	·9999871	·9999880	9·9999890	31
30	18	29	40	51	61	71	81	90	30
31	18	29	40	51	61	71	81	90	29
32	18	29	40	51	61	71	81	90	28
33	18	30	41	51	62	71	81	90	27
34	18	30	41	51	62	72	81	90	26
35	19	30	41	52	62	72	81	90	25
36	19	30	41	52	62	72	81	91	24
37	19	30	41	52	62	72	82	91	23
38	19	31	42	52	62	72	82	91	22
39	9·9999819	·9999831	·9999842	·9999852	·9999863	·9999872	·9999882	9·9999891	21
40	20	31	42	52	63	73	82	91	20
41	20	31	42	53	63	73	82	91	19
42	20	31	42	53	63	73	82	92	18
43	20	31	42	53	63	73	83	92	17
44	20	32	43	53	63	73	83	92	16
45	21	32	43	53	64	73	83	92	15
46	21	32	43	54	64	74	83	92	14
47	21	32	43	54	64	74	83	92	13
48	21	32	43	54	64	74	83	92	12
49	9·9999821	·9999833	·9999844	·9999854	·9999864	·9999874	·9999883	9·9999893	11
50	22	33	44	54	64	74	84	93	10
51	22	33	44	54	65	74	84	93	9
52	22	33	44	55	65	75	84	93	8
53	22	33	44	55	65	75	84	93	7
54	22	34	44	55	65	75	84	93	6
55	22	34	45	55	65	75	84	93	5
56	23	34	45	55	65	75	85	94	4
57	23	34	45	55	66	75	85	94	3
58	23	34	45	56	66	75	85	94	2
59	9·9999823	·9999834	·9999845	·9999856	·9999866	·9999876	·9999885	9·9999894	1
60	23	35	45	56	66	76	85	94	0
″	31′	30′	29′	28′	27′	26′	25′	24′	″

LOG. COSINE 0°.

″	28′	29′	30′	31′	32′	33′	34′	35′	″
0	12·0311114	0449004	0591416	0738656	0891062	1049012	1212923	12·1383262	60
1	3376	0451340	12· 3830	0741153	12· 93648	12· 51694	12· 15707	86159	59
2	5640	12· 3677	6244	12· 3651	96236	54377	18494	89057	58
3	7905	6015	8661	6150	98825	57062	21283	91957	57
4	12·0320171	8354	0601078	8652	0901416	59749	24073	94859	56
5	2439	0460695	3497	0751154	04008	62437	26865	97763	55
6	4707	3037	5917	3658	06601	65127	29659	12·1400669	54
7	6977	5380	8339	6164	09197	67819	32455	03577	53
8	9248	7725	0610762	8670	11793	70513	35252	06487	52
9	12·0331520	0470071	3186	0761179	14392	73208	38051	09399	51
10	3794	2418	5612	3688	16992	75904	40853	12313	50
11	6068	4766	8039	6200	19593	78603	43656	15229	49
12	8344	7115	0620467	8712	22196	81303	46460	18147	48
13	12·0340621	9466	2897	0771226	24801	84005	49267	21066	47
14	2899	0481818	5328	3742	27407	86708	52075	23988	46
15	5178	4172	7761	6259	30015	89413	54885	26912	45
16	7459	6526	0630195	8778	32624	92120	57697	29837	44
17	9740	8882	2630	0781298	35235	94829	60511	32765	43
18	12·0352023	0491240	5066	3819	37847	97539	63327	35695	42
19	4307	3598	7504	6342	40461	1100251	66144	38626	41
20	6592	5958	9943	8866	43077	02964	68963	41560	40
21	8879	8319	0642384	0791392	45694	05680	71785	44495	39
22	12·0361167	0500681	4826	3919	48313	08397	74607	47433	38
23	3456	3045	7270	6448	50933	11115	77432	50372	37
24	5746	5410	9714	8978	53555	13836	80259	53314	36
25	8037	7776	0652161	0801510	56178	16558	83087	56257	35
26	12·0370330	0510144	4608	4043	58803	19282	85918	59203	34
27	2623	2512	7057	6578	61430	22007	88750	62150	33
28	4918	4882	9507	9114	64058	24734	91584	65100	32
29	7214	7254	0661959	0811652	66688	27463	94420	68051	31
30	9512	9626	4412	4191	69319	30194	97257	71004	30
31	12·0381810	0522000	6867	6731	71952	32926	1300097	73960	29
32	4110	4376	9322	9273	74587	35661	02938	76917	28
33	6411	6752	0671780	0821817	77223	38396	05782	79877	27
34	8713	9130	4238	4362	79861	41134	08627	82839	26
35	12·0391017	0531509	6698	6909	82500	43873	11474	85802	25
36	3322	3890	9160	9457	85141	46614	14323	88768	24
37	5627	6272	0681622	0832006	87784	49357	17173	91735	23
38	7935	8655	4087	4557	90428	52101	20026	94705	22
39	12·0400243	0541039	6552	7110	93074	54848	22880	97677	21
40	2553	3425	9019	9664	95721	57596	25737	12·1500650	20
41	4863	5812	0691488	0842219	98370	60345	28595	03626	19
42	7175	8200	3957	4776	1001021	63097	31455	06604	18
43	9489	0550590	6429	7335	03673	65850	34317	09584	17
44	12·0411803	2981	8901	9895	06327	68605	37181	12565	16
45	4119	5373	0701375	0852457	08983	71361	40047	15549	15
46	6436	7767	3851	5020	11640	74120	42915	18535	14
47	8754	0560161	6328	7584	14299	76880	45784	21523	13
48	12·0421074	2558	8806	0860150	16959	79642	48656	24513	12
49	3394	4955	0711286	2718	19621	82406	51529	27506	11
50	5716	7354	3767	5287	22285	85171	54404	30500	10
51	8039	9754	6249	7858	24950	87938	57281	33496	9
52	12·0430364	0572156	8733	0870430	27617	90707	60161	36494	8
53	2690	4559	0721218	3004	30286	93478	63042	39495	7
54	5016	6963	3705	5579	32956	96250	65924	42497	6
55	7345	9368	6193	8156	35628	99025	68809	45502	5
56	9674	0581775	8683	0880734	38301	1201801	71696	48508	4
57	12·0442005	4183	0731174	3314	40977	04578	74585	51517	3
58	4337	6593	3667	5895	43653	07358	77475	54528	2
59	6670	9004	6160	8478	46332	10139	80368	57541	1
60	9004	0591416	8656	0891062	49012	12923	83262	60556	0
″	31′	30′	29′	28′	27′	26′	25′	24′	″

LOG. COTAN. 0°.

″	36′	37′	38′	39′	40′	41′	42′	43′	″
0	9·9999894	·9999903	·9999911	·9999919	·9999927	·9999934	·9999940	9·9999947	60
1	94	9 03	9 11	9 19	9 27	9 34	9 41	47	59
2	94	03	11	19	27	34	41	47	58
3	95	03	11	19	27	34	41	47	57
4	95	03	12	19	27	34	41	47	56
5	95	04	12	20	27	34	41	47	55
6	95	04	12	20	27	34	41	48	54
7	95	04	12	20	27	34	41	48	53
8	95	04	12	20	27	35	41	48	52
9	9·9999895	·9999904	·9999912	·9999920	·9999928	·9999935	·9999941	9·9999948	51
10	96	04	12	20	28	35	42	48	50
11	96	04	13	20	28	35	42	48	49
12	96	04	13	21	28	35	42	48	48
13	96	05	13	21	28	35	42	48	47
14	96	05	13	21	28	35	42	48	46
15	96	05	13	21	28	35	42	48	45
16	97	05	13	21	28	36	42	49	44
17	97	05	13	21	29	36	42	49	43
18	97	05	13	21	29	36	43	49	42
19	9·9999897	·9999905	·9999914	·9999921	·9999929	·9999936	·9999943	9·9999949	41
20	97	06	14	22	29	36	43	49	40
21	97	06	14	22	29	36	43	49	39
22	97	06	14	22	29	36	43	49	38
23	98	06	14	22	29	36	43	49	37
24	98	06	14	22	29	36	43	49	36
25	98	06	14	22	30	37	43	49	35
26	98	06	15	22	30	37	43	50	34
27	98	07	15	22	30	37	43	50	33
28	96	07	15	23	30	37	44	50	32
29	9·9999898	·9999907	·9999915	·9999923	·9999930	·9999937	·9999944	9·9999950	31
30	99	07	15	23	30	37	44	50	30
31	99	07	15	23	30	37	44	50	29
32	99	07	15	23	30	37	44	50	28
33	99	07	15	23	30	37	44	50	27
34	99	08	16	23	31	38	44	50	26
35	99	08	16	23	31	38	44	50	25
36	99	08	16	24	31	38	44	51	24
37	9·9999900	08	16	24	31	38	44	51	23
38	00	08	16	24	31	38	45	51	22
39	9·9999900	·9999908	·9999916	·9999924	·9999931	·9999938	·9999945	9·9999951	21
40	00	08	16	24	31	38	45	51	20
41	00	08	17	24	31	38	45	51	19
42	00	09	17	24	32	38	45	51	18
43	00	09	17	24	32	39	45	51	17
44	01	09	17	25	32	39	45	51	16
45	01	09	17	25	32	39	45	51	15
46	01	09	17	25	32	39	45	52	14
47	01	09	17	25	32	39	46	52	13
48	01	09	17	25	32	39	46	52	12
49	9·9999901	·9999910	·9999918	·9999925	·9999932	·9999939	·9999946	9·9999952	11
50	01	10	18	25	33	39	46	52	10
51	02	10	18	25	33	39	46	52	9
52	02	10	18	26	33	40	46	52	8
53	02	10	18	26	33	40	46	52	7
54	02	10	18	26	33	40	46	52	6
55	02	10	18	26	33	40	46	52	5
56	02	11	18	26	33	40	46	53	4
57	02	11	19	26	33	40	47	53	3
58	03	11	19	26	33	40	47	53	2
59	9·9999903	·9999911	·9999919	·9999926	·9999934	·9999940	·9999947	9·9999953	1
60	03	11	19	27	34	40	47	53	0
″	23′	22′	21′	20′	19′	18′	17′	16′	″

LOG. COSINE 0°.

″	36′	37′	38′	39′	40′	41′	42′	43′	″
0	12·1560556	1745396	1938453	2140492	2352390	2575159	2809974	12·3058214	60
1	63573	12· 48544	12· 41744	12· 43940	12· 56011	12· 78970	12· 13997	62474	59
2	66592	51695	45038	47391	59634	82785	18024	66738	58
3	69613	54847	48335	50845	63261	86603	22055	71007	57
4	72637	58003	51634	54301	66891	90424	26089	75279	56
5	75662	61160	54935	57760	70524	94249	30127	79556	55
6	78690	64320	58239	61222	74160	98077	34169	83837	54
7	81720	67482	61545	64687	77798	2601909	38215	88122	53
8	84751	70646	64854	68155	81440	05743	42264	92411	52
9	87785	73813	68166	71625	85085	09582	46318	96705	51
10	90821	76982	71480	75098	88734	13423	50375	12·3101003	50
11	93860	80153	74797	78574	92385	17269	54435	05305	49
12	96900	83327	78116	82052	96039	21117	58500	09611	48
13	99942	86503	81437	85534	99696	24969	62568	13922	47
14	12·1602987	89681	84762	89018	2403357	28824	66641	18237	46
15	06034	92861	88088	92505	07020	32683	70717	22556	45
16	09082	96044	91418	95995	10687	36545	74797	26880	44
17	12133	99230	94749	99487	14356	40411	78880	31208	43
18	15187	1802417	98084	2202983	18029	44280	82968	35540	42
19	18242	05607	2001421	06481	21705	48152	87059	39876	41
20	21299	08799	04760	09982	25384	52028	91154	44217	40
21	24359	11994	08102	13486	29066	55908	95254	48562	39
22	27421	15191	11447	16993	32751	59791	99357	52912	38
23	30485	18390	14794	20502	36440	63677	2903463	57266	37
24	33551	21592	18144	24015	40131	67567	07574	61624	36
25	36619	24796	21497	27530	43826	71460	11689	65987	35
26	39689	28002	24852	31048	47523	75357	15807	70354	34
27	42762	31211	28209	34569	51224	79258	19930	74725	33
28	45837	34422	31569	38093	54928	83161	24056	79101	32
29	48913	37636	34932	41619	58636	87069	28187	83481	31
30	51993	40852	38298	45149	62346	90980	32321	87866	30
31	55074	44070	41666	48681	66060	94894	36459	92255	29
32	58157	47291	45036	52216	69776	98812	40601	96649	28
33	61243	50514	48410	55754	73496	2702733	44747	12·3201047	27
34	64331	53739	51785	59295	77220	06658	48897	05449	26
35	67421	56967	55164	62839	80946	10587	53051	09856	25
36	70513	60197	58545	66386	84675	14519	57209	14267	24
37	73608	63430	61929	69936	88408	18455	61371	18683	23
38	76704	66665	65315	73468	92144	22394	65537	23104	22
39	79803	69902	68704	77044	95883	26337	69707	27529	21
40	82904	73142	72096	80602	99626	30283	73881	31958	20
41	86008	76385	75490	84164	2503371	34233	78059	36392	19
42	89113	79629	78887	87728	07120	38187	82241	40830	18
43	92221	82876	82287	91295	10872	42144	86427	45273	17
44	95331	86126	85689	94865	14628	46105	90617	49721	16
45	98443	89378	89094	98438	18386	50069	94811	54173	15
46	12·1701557	92632	92502	2302014	22146	54037	99010	58629	14
47	04674	95889	95912	05593	25913	58009	3003212	63091	13
48	07793	99149	99325	09175	29681	61984	07418	67557	12
49	10914	1902410	2102741	12760	33453	65963	11629	72027	11
50	14038	05675	06159	16348	37228	69946	15843	76502	10
51	17163	08941	09580	19939	41006	73932	20062	80982	9
52	20291	12211	13004	23532	44788	77922	24284	85466	8
53	23421	15482	16431	27129	48572	81916	28511	89955	7
54	26554	18756	19860	30729	52360	85913	32742	94448	6
55	29688	22033	23292	34331	56152	89914	36977	98947	5
56	32825	25312	26726	37937	59947	93919	41216	12·3303449	4
57	35964	28593	30164	41546	63745	97927	45459	07957	3
58	39106	31877	33604	45157	67546	2801939	49707	12469	2
59	42250	35164	37046	48772	71351	05955	53958	16986	1
60	45396	38453	40492	52390	75159	09974	58214	21508	0
″	23′	22′	21′	20′	19′	18′	17′	16′	″

LOG. COTAN. 0°.

″	44′	45′	46′	47′	48′	49′	50′	51′	″
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1	53	59	64	69	74	78	82	85	59
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3	53	59	64	69	74	78	82	85	57
4	53	59	64	69	74	78	82	85	56
5	53	59	64	69	74	78	82	85	55
6	54	59	64	69	74	78	82	85	54
7	54	59	65	70	74	78	82	86	53
8	54	59	65	70	74	78	82	86	52
9	9·9999954	·9999959	·9999965	·9999970	·9999974	·9999978	·9999982	9·9999986	51
10	54	60	65	70	74	78	82	86	50
11	54	60	65	70	74	79	82	86	49
12	54	60	65	70	74	79	82	86	48
13	54	60	65	70	74	79	82	86	47
14	54	60	65	70	75	79	82	86	46
15	54	60	65	70	75	79	83	86	45
16	55	60	65	70	75	79	83	86	44
17	55	60	65	70	75	79	83	86	43
18	55	60	66	70	75	79	83	86	42
19	9·9999955	·9999960	·9999966	·9999970	·9999975	·9999979	·9999983	9·9999986	41
20	55	60	66	71	75	79	83	86	40
21	55	61	66	71	75	79	83	86	39
22	55	61	66	71	75	79	83	86	38
23	55	61	66	71	75	79	83	86	37
24	55	61	66	71	75	79	83	86	36
25	55	61	66	71	75	79	83	86	35
26	55	61	66	71	75	79	83	87	34
27	56	61	66	71	75	80	83	87	33
28	56	61	66	71	76	80	83	87	32
29	9·9999956	·9999961	·9999966	·9999971	·9999976	·9999980	·9999983	9·9999987	31
30	56	61	67	71	76	80	83	87	30
31	56	61	67	71	76	80	83	87	29
32	56	62	67	71	76	80	84	87	28
33	56	62	67	72	76	80	84	87	27
34	56	62	67	72	76	80	84	87	26
35	56	62	67	72	76	80	84	87	25
36	56	62	67	72	76	80	84	87	24
37	57	92	67	72	76	80	84	87	23
38	57	62	67	72	76	80	84	87	22
39	9·9999957	·9999962	·9999967	·9999972	·9999976	·9999980	·9999984	9 9999987	21
40	57	62	67	72	76	80	84	87	20
41	57	62	67	72	76	80	84	87	19
42	57	62	67	72	77	81	84	87	18
43	57	63	68	72	77	81	84	87	17
44	57	63	68	72	77	81	84	87	16
45	57	63	68	72	77	81	84	87	15
46	57	63	68	73	77	81	84	88	14
47	57	63	68	73	77	61	84	88	13
48	58	63	68	73	77	81	84	88	12
49	9·9999958	·9999963	·9999968	·9999973	·9999977	·9999981	·9999985	9·9999988	11
50	58	63	68	73	77	81	85	88	10
51	58	63	68	73	77	81	85	88	9
52	58	63	68	73	77	81	85	88	8
53	58	63	68	73	77	81	85	88	7
54	58	63	68	73	77	81	85	88	6
55	58	64	69	73	77	81	85	88	5
56	58	64	69	73	77	81	85	88	4
57	58	64	69	73	78	81	85	88	3
58	58	64	69	73	78	82	85	88	2
59	9·9999959	·9999964	·9999969	·9999973	·9999978	·9999982	9999985	9·9999988	1
60	59	61	69	74	78	82	85	88	0
″	15′	14′	13′	12′	11′	10′	9′	8′	″

LOG. COSINE 0°.

″	44′	45′	46′	47′	48′	49′	50′	51′	″
0	12·3321508	3601799	3901434	4223285	4570909	4948797	5362727	12·5820304	60
1	26034	12· 06627	12· 06608	12· 28856	12· 76945	12· 55382	12· 69970	28354	59
2	30565	11461	11787	34435	82989	61978	77227	36419	58
3	35101	16300	16973	40021	89042	68583	84496	44499	57
4	39641	21144	22165	45614	95104	75198	91777	52594	56
5	44187	25994	27363	51214	4601174	81824	99070	60704	55
6	48787	30849	32567	56821	07252	88459	5406375	68829	54
7	53291	35710	37778	62436	13339	95105	13692	76970	53
8	57851	40576	42995	68058	19434	5001761	21022	85125	52
9	62415	45447	48218	73687	25538	08427	28365	93297	51
10	66985	50324	53447	79324	31651	15103	35719	12·5901483	50
11	71559	55207	58683	84968	37772	21790	43087	09685	49
12	76137	60095	63925	90619	43902	28487	50466	17903	48
13	80721	64988	69173	96278	50040	35194	57859	26136	47
14	85310	69887	74428	4301944	56187	41912	65263	34384	46
15	89903	74792	79689	07617	62343	48640	72681	42649	45
16	94501	79702	84956	13298	68508	55379	80111	50929	44
17	99104	84618	90230	18986	74681	62128	87554	59225	43
18	12·3403712	89539	95510	24682	80863	68887	95010	67537	42
19	08325	94466	4000797	30385	87054	75657	5502479	75865	41
20	12943	99398	06090	36096	93254	82438	09960	84209	40
21	17565	3704336	11389	41814	99463	89229	17454	92569	39
22	22193	09280	16696	47540	4705681	96031	24962	12·6000945	38
23	26826	14229	22008	53273	11907	5102843	32482	09337	37
24	31463	19184	27327	59014	18142	09666	40015	17745	36
25	36105	24145	32653	64762	24387	16500	47562	26170	35
26	40753	29111	37985	70519	30640	23345	55121	34611	34
27	45405	34083	43323	76282	36903	30201	62694	43069	33
28	50063	39061	48669	82054	43174	37067	70280	51543	32
29	54725	44044	54020	87833	49455	43944	77879	60033	31
30	59392	49033	59379	93620	55744	50832	85492	68541	30
31	64065	54028	64744	99414	62043	57731	93118	77065	29
32	68742	59028	70116	4405216	68351	64641	5600757	85605	28
33	73425	64035	75494	11026	74668	71563	08410	94163	27
34	78112	69047	80879	16844	80994	78495	16076	12·6102737	26
35	82805	74065	86270	22670	87330	85438	23756	11329	25
36	87503	79089	91669	28503	93674	92392	31449	19937	24
37	92205	84118	97074	34344	4800028	99358	39157	28563	23
38	96913	89153	4102486	40194	06392	5206334	46877	37206	22
39	12·3501626	94195	07904	46051	12764	13322	54612	45866	21
40	06344	99242	13330	51916	19146	20321	62360	54543	20
41	11067	3804295	18762	57788	25538	27332	70123	63237	19
42	15796	09353	24201	63669	31939	34354	77899	71949	18
43	20529	14418	29647	69558	38349	41387	85689	80679	17
44	25268	19489	35099	75455	44769	48431	93493	89426	16
45	30012	24565	40559	81360	51198	55487	5701311	98191	15
46	34761	29648	46025	87272	57637	62555	09143	12·6206974	14
47	39515	34736	51498	93193	64085	69634	16990	15774	13
48	44275	39831	56978	99122	70543	76724	24850	24592	12
49	49039	44931	62465	4505059	77011	83827	32725	33428	11
50	53809	50037	67959	11005	83488	90940	40614	42282	10
51	58584	55150	73460	16958	89975	98066	48518	51155	9
52	63365	60268	78968	22920	96472	5305203	56436	60045	8
53	68150	65393	84483	28889	4902978	12352	64368	68954	7
54	72941	70523	90005	34867	09494	19513	72315	77881	6
55	77738	75660	95534	40853	16020	26685	80276	86826	5
56	82539	80803	4201070	46848	22556	33870	88252	95790	4
57	87346	85951	06613	52851	29101	41066	96243	12·6304772	3
58	92158	91106	12163	58862	35657	48274	5804248	13773	2
59	96976	96267	17720	64881	42222	55494	12269	22793	1
60	12·3601799	3901434	23285	70909	48797	62727	20304	31831	0
″	15′	14′	13′	12′	11′	10′	9′	8′	″

LOG. COTAN. 0°.

″	52′	53′	54′	55′	56′	57′	58′	59′	″
0	9·9999988	·9999991	·9999993	·9999995	·9999997	·9999998	9·9999999	10·0000000	60
1	88	91	93	95	97	98	99	00	59
2	88	91	93	95	97	98	99	00	58
3	88	91	93	95	97	98	99	00	57
4	88	91	94	95	97	98	99	00	56
5	89	91	94	96	97	98	99	00	55
6	89	91	94	96	97	98	99	00	54
7	89	91	94	96	97	98	99	00	53
8	89	91	94	96	97	98	99	00	52
9	9·9999989	·9999991	·9999994	·9999996	·9999997	·9999999	9·9999999	10·0000000	51
10	89	91	94	96	97	99	99	00	50
11	89	91	94	96	97	99	99	00	49
12	89	92	94	96	97	99	99	00	48
13	89	92	94	96	97	99	99	00	47
14	89	92	94	96	97	99	99	00	46
15	89	92	94	96	97	99	99	00	45
16	89	92	94	96	97	99	99	00	44
17	89	92	94	96	97	99	99	00	43
18	89	92	94	96	97	99	99	00	42
19	9·9999989	·9999992	·9999994	·9999996	·9999998	·9999999	9·9999999	10·0000000	41
20	89	92	94	96	98	99	99	00	40
21	89	92	94	96	98	99	99	00	39
22	89	92	94	96	98	99	10·0000000	00	38
23	89	92	94	96	98	99	00	00	37
24	89	92	94	96	98	99	00	00	36
25	90	92	94	96	98	99	00	00	35
26	90	92	94	96	98	99	00	00	34
27	90	92	94	96	98	99	00	00	33
28	90	92	94	96	98	99	00	00	32
29	9·9999990	·9999992	·9999994	·9999996	·9999998	·9999999	10·0000000	10·0000000	31
30	90	92	94	96	98	99	00	00	30
31	90	92	94	96	98	99	00	00	29
32	90	92	95	96	98	99	00	00	28
33	90	92	95	96	98	99	00	00	27
34	90	92	95	96	98	99	00	00	26
35	90	92	95	96	98	99	00	00	25
36	90	92	95	96	98	99	00	00	24
37	90	93	95	96	98	99	00	00	23
38	90	93	95	96	98	99	00	00	22
39	9·9999990	·9999993	·9999995	·9999997	·9999998	·9999999	10·0000000	10·0000000	21
40	90	93	95	97	98	99	00	00	20
41	90	93	95	97	98	99	00	00	19
42	90	93	95	97	98	99	00	00	18
43	90	93	95	97	98	99	00	00	17
44	90	93	95	97	98	99	00	00	16
45	90	93	95	97	98	99	00	00	15
46	90	93	95	97	98	99	00	00	14
47	90	93	95	97	98	99	00	00	13
48	90	93	95	97	98	99	00	00	12
49	9·9999991	·9999993	·9999995	·9999997	·9999998	·9999999	10·0000000	10·0000000	11
50	91	93	95	97	98	99	00	00	10
51	91	93	95	97	98	99	00	00	9
52	91	93	95	97	98	99	00	00	8
53	91	93	95	97	98	99	00	00	7
54	91	83	95	97	98	99	00	00	6
55	91	93	95	97	98	99	00	00	5
56	91	93	95	97	98	99	00	00	4
57	91	93	95	97	98	99	00	00	3
58	91	93	95	97	98	99	00	00	2
59	9·9999991	·9999993	·9999995	·9999997	·9999998	·9999999	10·0000000	10 0000000	1
60	91	93	95	97	98	99	00	00	0
″	7′	6′	5′	4′	3′	2′	1′	0′	″

LOG. COSINE 0°.

″	52′	53′	54′	55′	56′	57′	58′	59′	″
0	12·6331831	6911752	7581222	8373036	13·9342137	13·0591525	13·2352438	13·5362739	60
1	40888	12 22105	93302	87536	360270	615720	388781	5435731	59
2	49965	32483	7605417	8402086	378480	640050	425430	5509971	58
3	59060	42885	12 17565	12 16684	396766	664518	462392	5585503	57
4	68174	53312	29747	31331	415129	689124	499671	5662371	56
5	77308	63765	41964	46028	433571	713870	537272	5740624	55
6	86460	74242	54214	60775	452091	738758	575202	5820314	54
7	95632	84745	66500	75572	470690	763789	613466	5901493	53
8	12·6404824	95273	78820	90420	489370	788966	652071	5984218	52
9	14035	7005827	91176	8505319	508130	814289	691021	6068550	51
10	23265	16407	7703567	20268	526971	839761	730324	6154551	50
11	32515	27013	15993	35270	545895	865383	769986	6242291	49
12	41785	37644	28455	50324	564901	891158	810013	6331839	48
13	51075	48302	40952	65430	583991	917086	850413	6423273	47
14	60385	58985	53486	80588	603165	943170	891192	6516673	46
15	69714	69696	66056	95800	622424	969411	932358	6612126	45
16	79064	80432	78663	8611065	641769	995812	973918	6709725	44
17	88434	91195	91306	26384	661201	13·1022374	13·3015879	6809567	43
18	97824	7101985	7803986	41758	680720	049100	058249	6911758	42
19	12·6507235	12802	16704	57185	700327	075992	101037	7016413	41
20	16666	23646	29458	72668	720023	103051	144251	7123651	40
21	26117	34517	42250	88207	739809	130279	187899	7233605	39
22	35589	45415	55080	8703801	759685	157680	231990	7346415	38
23	45082	56341	67948	19451	779652	185254	276534	7462234	37
24	54596	67294	80855	35158	799712	213005	321539	7581226	36
25	64130	78275	93799	50922	819865	240934	367015	7703571	35
26	73686	89284	7906783	66743	840112	269043	412973	7829462	34
27	83262	7200321	19805	82622	860454	297336	459422	7959112	33
28	92860	11385	32867	98560	880891	325815	506373	8092752	32
29	12·6602479	22479	45968	8814556	901425	354481	553837	8230634	31
30	12119	33600	59108	30611	922057	383338	601827	8373039	30
31	21781	44750	72289	46726	942787	412388	650351	8520271	29
32	31464	55929	85509	62901	963617	441633	699424	8672671	28
33	41169	67137	98770	79136	984546	471077	749059	8830614	27
34	50896	78373	8012072	95433	13·0005578	500722	799267	8994518	26
35	60644	89639	25414	8911790	026711	530571	850062	9164851	25
36	70415	7300934	38798	28210	047948	560626	901458	9342139	24
37	80207	12259	52223	44691	069290	590890	953470	9526973	23
38	90022	23613	65689	61236	090736	621367	13·4006113	9720025	22
39	99859	34997	79198	77844	112289	652060	059401	9922058	21
40	12·6709718	46410	92748	94516	133950	682970	113351	14·0133951	20
41	09600	57854	8106341	9011251	155719	714103	167980	0356715	19
42	29504	69328	19977	28052	177598	745460	223305	0591526	18
43	39430	80833	33665	44918	199588	777045	279344	0839762	17
44	49380	92368	47377	61849	221689	808862	336115	1103052	16
45	59352	7403934	61142	78847	243904	840913	393639	1383339	15
46	69348	15531	74951	95912	266233	873203	451934	1682971	14
47	79366	27158	88805	9113044	288677	905734	511023	2004818	13
48	89408	38817	8202702	30244	311238	938511	570926	2352439	12
49	99472	50508	16644	47512	333916	971538	631668	2730324	11
50	12·6809560	62229	30631	64849	356714	13·2004817	693271	3144251	10
51	19672	73983	44663	82256	379632	038354	755760	3601826	9
52	29807	85769	58741	99732	402672	072151	819162	4113351	8
53	39966	97586	72864	9217280	425834	106214	883503	4693271	7
54	50149	7509436	87034	34898	449121	140545	948812	5362739	6
55	60356	21318	8301250	52588	472533	175151	13·5015118	6154551	5
56	70587	33233	15512	70351	496072	210034	082451	7123651	4
57	80842	45181	29822	88187	519739	245200	150846	8373039	3
58	91121	57161	44179	9306096	543536	280653	220334	15·0133951	2
59	12·6901424	69175	58583	24079	567464	316397	290953	3144251	1
60	11752	81222	73036	42137	591525	352438	362739	+ ∞	0
″	7′	6′	5′	4′	3′	2′	1′	0′	″

LOG. COTAN. 0°.

NATURAL SIGNS AND TANGENTS

TO EVERY DEGREE AND MINUTE OF THE QUADRANT.

′	0°	1°	2°	3°	4°	5°	6°	7°	′
0	000 0000	017 4524	034 8995	052 3360	069 7565	087 1557	104 5285	121 8693	60
1	2909	7432	035 1902	6264	070 0467	4455	8178	122 1581	59
2	5816	018 0341	4809	9169	3368	7353	105 1070	4468	58
3	8727	3249	7716	053 2074	6270	088 0251	3963	7355	57
4	001 1636	6158	036 0623	4979	9171	3148	6856	123 0241	56
5	4544	9066	3530	7883	071 2073	6046	9748	312[illegible]	55
6	7453	019 1974	6437	054 0788	4974	8943	106 2641	6015	54
7	002 0362	4883	9344	3693	7876	089 1840	5533	8901	53
8	3271	7791	037 2251	6597	072 0777	4738	8425	124 1788	52
9	6180	020 0699	5158	9502	3678	7635	107 1318	4674	51
10	9089	3608	8065	055 2406	6580	090 0532	4210	7560	50
11	003 1998	6516	038 0971	5311	9481	3429	7102	125 0446	49
12	4907	9424	3878	8215	073 2382	6326	9994	3332	48
13	7815	021 2332	6785	066 1119	5283	9223	108 2885	6218	47
14	004 0724	5241	9692	4024	8184	091 2119	5777	9104	46
15	3633	8149	039 2598	6928	074 1085	5016	8669	126 1990	45
16	6542	022 1057	5505	9832	3986	7913	109 1560	4875	44
17	9451	3965	8411	057 2736	6887	092 0809	4452	7761	43
18	005 2360	6873	040 0318	5640	9787	3706	7343	127 0646	42
19	5268	9781	4224	8544	075 2688	6602	110 0234	3531	41
20	8177	023 2690	7131	058 1448	5589	9499	3126	6416	40
21	006 1086	5598	041 0037	4352	8489	093 2395	6017	9302	39
22	3995	8506	2944	7256	076 1390	5291	8908	128 2186	38
23	6904	024 1414	5850	059 0160	4290	8187	111 1799	5071	37
24	9813	4322	8757	3064	7190	094 1083	4689	7956	36
25	007 2721	7230	042 1663	5967	077 0091	3979	7580	129 0841	35
26	5630	025 0138	4569	8871	2991	6875	112 0471	3725	34
27	8539	3046	7475	060 1775	5891	9771	3361	6609	33
28	008 1448	5954	043 0382	4678	8791	095 2666	6252	9494	32
29	4357	8862	3288	7582	078 1691	5562	9142	130 2378	31
30	7265	026 1769	6194	061 0485	4591	8458	113 2032	5262	30
31	009 0174	4677	9100	3389	7491	096 1353	4922	8146	29
32	3083	7585	044 2006	6292	079 0391	4248	7812	131 1030	28
33	5992	027 0493	4912	9196	3290	7144	114 0702	3913	27
34	8900	3401	7818	062 2099	6190	097 0039	3592	6797	26
35	010 1809	6309	045 0724	5002	9090	2934	6482	9681	25
36	4718	9216	3630	7905	080 1989	5829	9372	132 2564	24
37	7627	028 2124	6536	063 0808	4889	8724	115 2261	5447	23
38	011 0535	5032	9442	3711	7788	098 1619	5151	8330	22
39	3444	7940	046 2347	6614	081 0687	4514	8040	133 1213	21
40	6353	029 0847	5253	9517	3587	7408	116 0929	4096	20
41	9261	3755	8159	064 2420	6486	099 0303	3818	6979	19
42	012 2170	6662	047 1065	5323	9385	3197	6707	9862	18
43	5079	9570	3970	8226	082 2284	6092	9596	134 2744	17
44	7987	030 2478	6876	065 1129	5183	8986	117 2485	5627	16
45	013 0896	5385	9781	4031	8082	100 1881	5374	8509	15
46	3805	8293	048 2687	6934	083 0981	4775	8263	135 1392	14
47	6713	031 1200	5592	9836	3880	7669	118 1151	4274	13
48	9622	4108	8498	066 2739	6778	101 0563	4040	7156	12
49	014 2530	7015	049 1403	5641	9677	3457	6928	136 0038	11
50	5439	9922	4308	8544	084 2576	6351	9816	2919	10
51	8348	032 2830	7214	067 1446	5474	9245	119 2704	5801	9
52	015 1256	5737	050 0119	4349	8373	102 2138	5593	8683	8
53	4165	8644	3024	7251	085 1271	5032	8481	137 1564	7
54	7073	033 1552	5929	068 0153	4169	7925	120 1368	4445	6
55	9982	4459	8835	3055	7067	103 0819	4256	7327	5
56	016 2890	7366	051 1740	5957	9966	3712	7144	138 0208	4
57	5799	034 0274	4645	8859	086 2864	6605	121 0031	3089	3
58	8707	3181	7550	069 1761	5762	9499	2919	5970	2
59	017 1616	6088	052 0455	4663	8660	104 2392	5806	8850	1
60	4524	8995	3360	7565	087 1557	5285	8693	139 1731	0
′	89°	88°	87°	86°	85°	84°	83°	82°	′

NAT. COSINE.

′	0°	1°	2°	3°	4°	5°	6°	7°	′
0	000 0000	017 4551	034 9208	052 4078	069 9268	087 4887	105 1042	122 7846	60
1	2909	7460	035 2120	6995	070 2191	7818	3983	123 0798	59
2	5818	018 0370	5033	9912	5115	088 0749	6925	3752	58
3	8727	3280	7945	053 2829	8038	3681	9866	6705	57
4	001 1636	6190	036 0858	5746	071 0961	6612	106 2808	9658	56
5	4544	9100	3771	8663	3885	9544	5750	124 2612	55
6	7453	019 2010	6683	054 1581	6809	089 2476	8692	5566	54
7	002 0362	4920	9596	4498	9733	5408	107 1634	8520	53
8	3271	7830	037 2509	7416	072 2657	8341	4576	125 1474	52
9	6180	020 0740	5422	055 0333	5581	090 1273	7519	4429	51
10	9089	3650	8335	3251	8505	4206	108 0462	7384	50
11	003 1998	6560	038 1248	6169	073 1430	7138	3405	126 0339	49
12	4907	9470	4161	9087	4354	091 0071	6348	3294	48
13	7816	021 2380	7074	056 2005	7279	3004	9291	6249	47
14	004 0725	5291	9988	4923	074 0203	5938	109 2234	9205	46
15	3634	8201	039 2901	7841	3128	8871	5178	127 2161	45
16	6542	022 1111	5814	057 0759	6053	092 1804	8122	5117	44
17	9451	4021	8728	3678	8979	4738	110 1066	8073	43
18	005 2360	6932	040 1641	6596	075 1904	7672	4010	128 1030	42
19	5269	9842	4555	9515	4829	093 0606	6955	3986	41
20	8178	023 2753	7469	058 2434	7755	3540	9899	6943	40
21	006 1087	5663	041 0383	5352	076 0680	6474	111 2844	9900	39
22	3996	8574	3296	8271	3606	9409	5789	129 2858	38
23	6905	024 1484	6210	059 1190	6532	094 2344	8734	5815	37
24	9814	4395	9124	4109	9458	5278	112 1680	8773	36
25	007 2723	7305	042 2038	7029	077 2384	8213	4625	130 1731	35
26	5632	025 0216	4952	9948	5311	095 1148	7571	4690	34
27	8541	3127	7866	060 2867	8237	4084	113 0517	7648	33
28	008 1450	6038	043 0781	5787	078 1164	7019	3463	131 0607	32
29	4360	8948	3695	8706	4090	9955	6410	3566	31
30	7269	026 1859	6609	061 1626	7017	096 2890	9356	6525	30
31	009 0178	4770	9524	4546	9944	5826	114 2303	9484	29
32	3087	7681	044 2438	7466	079 2871	8763	5250	132 2444	28
33	5996	027 0592	5353	062 0386	5796	097 1699	8197	5404	27
34	8905	3503	8268	3306	8726	4635	115 1144	8364	26
35	010 1814	6414	045 1183	6226	080 1653	7572	4092	133 1324	25
36	4724	9325	4097	9147	4581	098 0509	7039	4285	24
37	7633	028 2236	7012	063 2067	7509	3446	9987	7246	23
38	011 0542	5148	9927	4988	081 0437	6383	116 2936	134 0207	22
39	3451	8059	046 2842	7908	3365	9320	5884	3168	21
40	6361	029 0970	5757	064 0829	6293	099 2257	8832	6129	20
41	9270	3882	8673	3750	9221	5194	117 1781	9091	19
42	012 2179	6793	047 1588	6671	082 2150	8133	4730	135 2053	18
43	5088	9705	4503	9592	5078	100 1071	7679	5015	17
44	7998	030 2616	7419	065 2513	8007	4009	118 0628	7978	16
45	013 0907	5528	048 0334	5435	083 0936	6947	3578	136 0940	15
46	3817	8439	3250	8356	3865	9886	6529	3903	14
47	6726	031 1351	6166	066 1278	6794	101 2824	9478	6866	13
48	9635	4263	9082	4199	9723	5763	119 2428	9830	12
49	014 2545	7174	049 1997	7121	084 2653	8702	5378	137 2793	11
50	5454	032 0086	4913	067 0043	5583	102 1641	8329	5757	10
51	8364	2998	7829	2965	8512	4580	120 1279	8721	9
52	015 1273	5910	050 0746	5887	085 1442	7520	4230	138 1685	8
53	4183	8822	3662	8809	4372	103 0460	7182	4650	7
54	7093	033 1734	6578	068 1732	7302	3399	121 0133	7615	6
55	016 0002	4646	9495	4654	086 0233	6340	3085	139 0580	5
56	2912	7558	051 2411	7577	3163	9280	6036	3545	4
57	5821	034 0471	5328	069 0499	6094	104 2220	8988	6510	3
58	8731	3383	8244	3422	9025	5161	122 1941	9476	2
59	017 1641	6295	052 1161	6345	087 1956	8101	4893	140 2442	1
60	4551	9208	4078	9268	4887	105 1042	7846	5408	0
′	89°	88°	87°	86°	85°	84°	83°	82°	′

NAT. COTAN.

′	8°	9°	10°	11°	12°	13°	14°	15°	′
0	139 1731	156 4345	173 6482	190 8090	207 9117	224 9511	241 9219	258 8190	60
1	4612	7218	9346	191 0945	208 1962	225 2345	242 2041	259 1000	59
2	7492	157 0091	174 2211	3801	4807	5179	4863	3810	58
3	140 0372	2963	5075	6656	7652	8013	7685	6619	57
4	3252	5836	7939	9510	209 0497	226 0846	243 0507	9428	56
5	6132	8708	175 0803	192 2365	3341	3680	3329	260 2237	55
6	9012	158 1581	3667	5220	6186	6513	6150	5045	54
7	141 1892	4453	6531	8074	9030	9346	8971	7853	53
8	4772	7325	9395	193 0928	210 1874	227 2179	244 1792	261 0662	52
9	7651	159 0197	176 2258	3782	4718	5012	4613	3469	51
10	142 0531	3069	5121	6636	7561	7844	7433	6277	50
11	3410	5940	7984	9490	211 0405	228 0677	245 0254	9085	49
12	6289	8812	177 0847	194 2344	3248	3509	3074	262 1892	48
13	9168	160 1683	3710	5197	6091	6341	5894	4699	47
14	143 2047	4555	6573	8050	8934	9172	8713	7506	46
15	4926	7426	9435	195 0903	212 1777	229 2004	246 1533	263 0312	45
16	7805	161 0297	178 2298	3756	4619	4835	4352	3118	44
17	144 0684	3167	5160	6609	7462	7666	7171	5925	43
18	3562	6038	8022	9461	213 0304	230 0497	9990	8730	42
19	6440	8909	179 0884	196 2314	3146	3328	247 2809	264 1536	41
20	9319	162 1779	3746	5166	5988	6159	5627	4342	40
21	145 2197	4650	6607	8018	8829	8989	8445	7147	39
22	5075	7520	9469	197 0870	214 1671	231 1819	248 1263	9952	38
23	7953	163 0390	180 2330	3722	4512	4649	4081	265 2757	37
24	146 0830	3260	5191	6573	7353	7479	6899	5561	36
25	3708	6129	8052	9425	215 0194	232 0309	9716	8366	35
26	6585	8999	181 0913	198 2276	3035	3138	249 2533	266 1170	34
27	9463	164 1868	3774	5127	5876	5967	5350	3973	33
28	147 2340	4738	6635	7978	8716	8796	8167	6777	32
29	5217	7607	9495	199 0829	216 1556	233 1625	250 0984	9581	31
30	8094	165 0476	182 2355	3679	4396	4454	3800	267 2384	30
31	148 0971	3345	5215	6530	7236	7282	6616	5187	29
32	3848	6214	8075	9380	217 0076	234 0110	9432	7989	28
33	6724	9082	183 0935	200 2230	2915	2938	251 2248	268 0792	27
34	9601	166 1951	3795	5080	5754	5766	5063	3594	26
35	149 2477	4819	6654	7930	8593	8594	7879	6396	25
36	5353	7687	9514	201 0779	218 1432	235 1421	252 0694	9198	24
37	8230	167 0556	184 2373	3629	4271	4248	3508	269 2000	23
38	150 1106	3423	5232	6478	7110	7075	6323	4801	22
39	3981	6291	8091	9327	9948	9902	9137	7602	21
40	6857	9159	185 0949	202 2176	219 2786	236 2729	253 1952	270 0403	20
41	9733	168 2026	3808	5024	5624	5555	4766	3204	19
42	151 2608	4894	6666	7873	8462	8381	7579	6004	18
43	5484	7761	9524	203 0721	220 1300	237 1207	254 0393	8805	17
44	8359	169 0628	186 2382	3569	4137	4033	3206	271 1605	16
45	152 1234	3495	5240	6418	6974	6859	6019	4404	15
46	4109	6362	8098	9265	9811	9684	8832	7204	14
47	6984	9228	187 0956	204 2113	221 2648	238 2510	255 1645	272 0003	13
48	9858	170 2095	3813	4961	5485	5335	4458	2802	12
49	153 2733	4961	6670	7808	8321	8159	7270	5601	11
50	5607	7828	9528	205 0655	222 1158	239 0984	256 0082	8400	10
51	8482	171 0694	188 2385	3502	3994	3808	2894	273 1198	9
52	154 1356	3560	5241	6349	6830	6633	5705	3997	8
53	4230	6425	8098	9195	9666	9457	8517	6794	7
54	7104	9291	189 0954	206 2042	223 2501	240 2280	257 1328	9592	6
55	9978	172 2156	3811	4888	5337	5104	4139	274 2390	5
56	155 2851	5022	6667	7734	8172	7927	6950	5187	4
57	5725	7887	9523	207 0580	224 1007	241 0751	9760	7984	3
58	8598	173 0752	190 2379	3426	3842	3574	258 2570	275 0781	2
59	156 1472	3617	5234	6272	6676	6396	5381	3577	1
60	4345	6482	8090	9117	9511	9219	8190	6374	0
′	81°	80°	79°	78°	77°	76°	75°	74°	′

NAT. COSINE.

′	8°	9°	10°	11°	12°	13°	14°	15°	′
0	140 5408	158 3844	176 3270	194 3803	212 5566	230 8682	249 3280	267 9492	60
1	8375	6826	6269	6822	8606	231 1746	6370	268 2610	59
2	141 1342	9809	9269	9841	213 1647	4811	9460	5728	58
3	4308	159 2791	177 2269	195 2861	4688	7876	250 2551	8847	57
4	7276	5774	5270	5881	7730	232 0941	5642	269 1967	56
5	142 0243	8757	8270	8901	214 0772	4007	8734	5087	55
6	3211	160 1740	178 1271	196 1922	3814	7073	251 1826	8207	54
7	6179	4724	4273	4943	6857	233 0140	4919	270 1328	53
8	9147	7708	7274	7964	9900	3207	8012	4449	52
9	143 2115	161 0692	179 0276	197 0986	215 2944	6274	252 1106	7571	51
10	5084	3677	3279	4008	5988	9342	4200	271 0694	50
11	8053	6662	6281	7031	9032	234 2410	7294	3817	49
12	144 1022	9647	9284	198 0053	216 2077	5479	253 0389	6940	48
13	3991	162 2632	180 2287	3076	5122	8548	3484	272 0064	47
14	6961	5618	5291	6100	8167	235 1617	6580	3188	46
15	9931	8603	8295	9124	217 1213	4687	9676	6313	45
16	145 2901	163 1590	181 1299	199 2148	4259	7758	254 2773	9438	44
17	5872	4576	4303	5172	7306	236 0829	5870	273 2564	43
18	8842	7563	7308	8197	218 0353	3900	8968	5690	42
19	146 1813	164 0550	182 0313	200 1222	3400	6971	255 2066	8817	41
20	4784	3537	3319	4248	6448	237 0044	5165	274 1945	40
21	7756	6525	6324	7274	9496	3116	8264	5072	39
22	147 0727	9513	9330	201 0300	219 2544	6189	256 1363	8201	38
23	3699	165 2501	183 2337	3327	5593	9262	4463	275 1330	37
24	6672	5489	5343	6354	8643	238 2336	7564	4459	36
25	9644	8478	8350	9381	220 1692	5410	257 0664	7589	35
26	148 2617	166 1467	184 1358	202 2409	4742	8485	3766	276 0719	34
27	5590	4456	4365	5437	7793	239 1560	6868	3850	33
28	8563	7446	7373	8465	221 0844	4635	9970	6981	32
29	149 1536	167 0436	185 0382	203 1494	3895	7711	258 3073	277 0113	31
30	4510	3426	3390	4523	6947	240 0788	6176	3245	30
31	7484	6417	6399	7552	9999	3864	9280	6378	29
32	150 0458	9407	9409	204 0582	222 3051	6942	259 2384	9512	28
33	3433	168 2398	186 2418	3612	6104	241 0019	5488	278 2646	27
34	6408	5390	5428	6643	9157	3097	8593	5780	26
35	9383	8381	8439	9674	223 2211	6176	260 1699	8915	25
36	151 2358	169 1373	187 1449	205 2705	5265	9255	4805	279 2050	24
37	5333	4366	4460	5737	8319	242 2334	7911	5186	23
38	8309	7358	7471	8769	224 1374	5414	261 1018	8322	22
39	152 1285	170 0351	188 0483	206 1801	4429	8494	4126	280 1459	21
40	4262	3344	3495	4834	7485	243 1575	7234	4597	20
41	7238	6338	6507	7867	225 0541	4656	262 0342	7735	19
42	153 0215	9331	9520	207 0900	3597	7737	3451	281 0873	18
43	3192	171 2325	189 2533	3934	6654	244 0819	6560	4012	17
44	6170	5320	5546	6968	9711	3902	9670	7152	16
45	9147	8314	8559	208 0003	226 2769	6984	263 2780	282 0292	15
46	154 2125	172 1309	190 1573	3038	5827	245 0068	5891	3432	14
47	5103	4304	4587	6073	8885	3151	9002	6573	13
48	8082	7300	7602	9109	227 1944	6236	264 2114	9715	12
49	155 1061	173 0296	191 0617	209 2145	5003	9320	5226	283 2857	11
50	4040	3292	3632	5181	8063	246 2405	8339	5999	10
51	7019	6288	6648	8218	228 1123	5491	265 1452	9143	9
52	9998	9285	9664	210 1255	4184	8577	4566	284 2286	8
53	156 2978	174 2282	192 2680	4293	7244	247 1663	7680	5430	7
54	5958	5279	5696	7331	229 0306	4750	266 0794	8575	6
55	8939	8277	8713	211 0369	3367	7837	3909	285 1720	5
56	157 1919	175 1275	193 1731	3407	6429	248 0925	7025	4866	4
57	4900	4273	4748	6446	9492	4013	267 0141	8012	3
58	7881	7272	7766	9486	230 2555	7102	3257	286 1159	2
59	158 0863	176 0271	194 0784	212 2525	5618	249 0191	6374	4306	1
60	3844	3270	3803	5566	8682	3280	9492	7454	0
′	81°	80°	79°	78°	77°	76°	75°	74°	′

NAT. COTAN.

NAT. SINE.

′	16°	17°	18°	19°	20°	21°	22°	23°	′
0	275 6374	292 3717	309 0170	325 5682	342 0201	358 3679	374 6066	390 7311	60
1	9170	6499	2936	8432	2935	6395	8763	9989	59
2	276 1965	9280	5702	326 1182	5668	9110	375 1459	391 2666	58
3	4761	293 2061	8468	3932	8400	359 1825	4156	5343	57
4	7556	4842	310 1234	6681	343 1133	4540	6852	8019	56
5	277 0352	7623	3999	9430	3865	7254	9547	392 0695	55
6	3147	294 0403	6764	327 2179	6597	9968	376 2243	3371	54
7	5941	3183	9529	4928	9329	360 2682	4938	6047	53
8	8736	5963	311 2294	7676	344 2060	5395	7632	9722	52
9	278 1530	8743	5058	328 0424	4791	8108	377 0327	393 1397	51
10	4324	295 1522	7822	3172	7521	361 0821	3021	4071	50
11	7118	4302	312 0586	5919	345 0252	3534	5714	6745	49
12	9911	7081	3349	8666	2982	6246	8408	9419	48
13	279 2704	9859	6112	329 1413	5712	8958	378 1101	394 2093	47
14	5497	296 2638	8875	4160	8441	362 1669	3794	4766	46
15	8290	5416	313 1638	6906	346 1171	4380	6486	7439	45
16	280 1083	8194	4400	9653	3900	7091	9178	395 0111	44
17	3875	297 0971	7163	320 2398	6628	9802	379 1870	2783	43
18	6667	3749	9925	5144	9357	363 2512	4562	5455	42
19	9459	6526	314 2686	7889	347 2085	5222	7253	8127	41
20	281 2251	9303	5448	331 0634	4812	7932	9944	396 0798	40
21	5042	298 2079	8209	3379	7540	364 0641	380 2634	3468	39
22	7833	4856	315 0969	6123	348 0267	3351	5324	6139	38
23	282 0624	7632	3730	8867	2994	6059	8014	8809	37
24	3415	299 0408	6490	332 1611	5720	8768	381 0704	397 1479	36
25	6205	3184	9250	4355	8447	365 1476	3393	4148	35
26	8995	5959	316 2010	7098	349 1173	4184	6082	6818	34
27	283 1785	8734	4770	9841	3898	6891	8770	9486	33
28	4575	300 1509	7529	333 2584	6624	9599	382 1459	398 2155	32
29	7364	4284	317 0288	5326	9349	366 2306	4147	4823	31
30	284 0153	7058	3047	8069	350 2074	5012	6834	7491	30
31	2942	9832	5805	334 0810	4798	7719	9522	399 0158	29
32	5731	301 2606	8563	3552	7523	367 0425	383 2209	2825	28
33	8520	5380	318 1321	6293	351 0246	3130	4895	5492	27
34	285 1308	8153	4079	9034	2970	5836	7582	8158	26
35	4096	302 0926	6836	335 1775	5963	8541	384 0268	400 0825	25
36	6884	3699	9593	4516	8416	368 1246	2953	3490	24
37	9671	6471	319 2350	7256	352 1139	3950	5639	6156	23
38	286 2458	9244	5106	9996	3862	6654	8324	8821	22
39	5246	303 2016	7863	336 2735	6584	9358	385 1008	401 1486	21
40	8032	4788	320 0619	5475	9306	369 2061	3693	4150	20
41	287 0819	7559	3374	8214	353 2027	4765	6377	6814	19
42	3605	304 0331	6130	337 0953	4748	7468	9060	9478	18
43	6391	3102	8885	3691	7469	370 0170	386 1744	402 2141	17
44	9177	5872	321 1640	6429	354 0190	2872	4427	4804	16
45	288 1963	8643	4395	9167	2910	5574	7110	7467	15
46	4748	305 1413	7149	338 1905	5630	8276	9792	403 0129	14
47	7533	4183	9903	4642	8350	371 0977	387 2474	2791	13
48	289 0318	6953	322 2657	7379	355 1070	3678	5156	5453	12
49	3103	9723	5411	339 0116	3789	6379	7837	8114	11
50	5887	306 2492	8164	2852	6508	9079	388 0518	404 0775	10
51	8671	5261	323 0917	5589	9226	372 1780	3199	3436	9
52	390 1455	8030	3670	8325	356 1944	4479	5880	6096	8
53	4239	307 0798	6422	340 1060	4662	7179	8560	8756	7
54	7022	3566	9174	3796	7360	9878	389 1240	405 1416	6
55	9805	6334	324 1926	6531	357 0097	373 2577	3919	4075	5
56	291 2588	9102	4678	9265	2814	5275	6598	6734	4
57	5371	308 1869	7429	341 2000	5531	7973	9277	9393	3
58	8153	4636	325 0180	4734	8248	374 0671	390 1955	406 2051	2
59	292 0935	7403	2931	7468	358 0964	3369	4633	4709	1
60	3717	309 0170	5682	342 0201	3679	6066	7311	7366	0
′	73°	72°	71°	70°	69°	68°	67°	66°	′

NAT. COSINE.

′	16°	17°	18°	19°	20°	21°	22°	23°	′
0	286 7454	305 7307	324 9197	344 3276	363 9702	383 8640	404 0262	424 4748	60
1	287 0602	306 0488	325 2413	6530	364 2997	384 1978	3646	8182	59
2	3751	3670	5630	9785	6292	5317	7031	425 1616	58
3	6900	6852	8848	345 3040	9588	8656	405 0417	5051	57
4	288 0050	307 0034	326 2066	6296	365 2885	385 1996	3804	8487	56
5	3201	3218	5284	9553	6182	5337	7191	426 1924	55
6	6352	6402	8504	346 2810	9480	8679	406 0579	5361	54
7	9503	9586	327 1724	6068	366 2779	386 2021	3968	8800	53
8	289 2655	308 2771	4944	9327	6079	5364	7358	427 2239	52
9	5808	5957	8165	347 2586	9379	8708	407 0748	5680	51
10	8961	9143	328 1387	5846	367 2680	387 2053	4139	9121	50
11	290 2114	309 2330	4610	9107	5981	5398	7531	428 2563	49
12	5269	5517	7833	348 2368	9284	8744	408 0924	6005	48
13	8423	8705	329 1056	5630	368 2587	388 2091	4318	9449	47
14	291 1578	310 1893	4281	8893	5890	5439	7713	429 2894	46
15	4734	5093	7505	349 2156	9195	8787	409 1108	6339	45
16	7890	8272	330 0731	5420	369 2500	389 2136	4504	9785	44
17	292 1047	311 1462	3957	8685	5806	5486	7901	430 3232	43
18	4205	4653	7184	350 1950	9112	8837	410 1299	6680	42
19	7363	7845	331 0411	5216	370 2420	390 2189	4697	431 0129	41
20	293 0521	312 1036	3639	8483	5728	5541	8097	3579	40
21	3680	4229	6868	351 1750	9036	8894	411 1497	7030	39
22	6839	7422	332 0097	5018	371 2346	391 2247	4898	432 0481	38
23	9999	313 0616	3327	8287	5656	5602	8300	3933	37
24	294 3160	3810	6557	352 1556	8967	8957	412 1703	7386	36
25	6321	7005	9788	4826	372 2278	392 2313	5106	433 0840	35
26	9483	314 0200	333 3020	8096	5590	5670	8510	4295	34
27	295 2645	3396	6252	353 1368	8903	9027	413 1915	7751	33
28	5808	6593	9485	4640	373 2217	393 2386	5321	434 1208	32
29	8971	9790	334 2719	7912	5532	5745	8728	4665	31
30	296 2135	315 2988	5953	354 1186	8847	9105	414 2136	8124	30
31	5299	6186	9188	4460	374 2163	394 2465	5544	435 1583	29
32	8464	9385	335 2424	7734	5479	5827	8953	5043	28
33	297 1630	316 2585	5660	355 1010	8797	9189	415 2363	8504	27
34	4796	5785	8896	4286	375 2115	395 2552	5774	436 1966	26
35	7962	8986	336 2134	7562	5433	5916	9186	5429	25
36	298 1129	317 2187	5372	356 0840	8753	9280	416 2598	8893	24
37	4297	5389	8610	4118	376 2073	396 2645	6012	437 2357	23
38	7465	8591	337 1850	7397	5394	6011	9426	5823	22
39	299 0634	318 1794	5090	357 0676	8716	9378	417 2841	9289	21
40	3803	4998	8330	3956	377 2038	397 2746	6257	438 2756	20
41	6973	8202	338 1571	7237	5361	6114	9673	6224	19
42	300 0144	319 1407	4813	358 0518	8685	9483	418 3091	9693	18
43	3315	4613	8056	3801	378 2010	398 2853	6509	439 3163	17
44	6486	7819	339 1299	7083	5335	6224	9928	6634	16
45	9658	320 1025	4543	359 0367	8661	9595	419 3348	440 0105	15
46	301 2831	4232	7787	3651	379 1988	399 2968	6769	3578	14
47	6004	7440	340 1032	6936	5315	6341	420 0190	7051	13
48	9178	321 0649	4278	360 0222	8644	9715	3613	441 0526	12
49	302 2352	3858	7524	3508	380 1973	400 3089	7036	4001	11
50	5527	7067	341 0771	6795	5302	6465	421 0460	7477	10
51	8703	322 0278	4019	361 0082	8633	9841	3885	442 0954	9
52	303 1879	3489	7267	3371	381 1964	401 3218	7311	4432	8
53	5055	6700	342 0516	6660	5296	6596	422 0738	7910	7
54	8232	9912	3765	9949	8629	9974	4165	443 1390	6
55	304 1410	323 3125	7015	362 3240	382 1962	402 3354	7594	4871	5
56	4588	6338	343 0266	6531	5296	6734	423 1023	8352	4
57	7767	9552	3518	9823	8631	403 0115	4453	444 1834	3
58	305 0946	324 2766	6770	363 3115	383 1967	3496	7884	5318	2
59	4126	5981	344 0023	6408	5303	6879	424 1316	8802	1
60	7307	9197	3276	9702	8640	404 0262	4748	445 2287	0
′	73°	72°	71°	70°	69°	68°	67°	66°	′

NAT. COTAN.

′	24°	25°	26°	27°	28°	29°	30°	31°	′
0	406 7366	422 6183	438 3711	453 9905	469 4716	484 8096	500 0000	515 0381	60
1	407 0024	8819	6326	454 2497	7284	485 0640	2519	2874	59
2	2681	423 1455	8940	5088	9852	3184	5037	5367	58
3	5337	4090	439 1553	7679	470 2419	5727	7556	7859	57
4	7993	6725	4166	455 0269	4986	8270	501 0073	516 0351	56
5	408 0649	9360	6779	2859	7553	486 0812	2591	2842	55
6	3305	424 1994	9392	5449	471 0119	3354	5107	5333	54
7	5960	4628	440 2004	8038	2685	5895	7624	7824	53
8	8615	7262	4615	456 0627	5250	8436	502 0140	517 0314	52
9	409 1269	9895	7227	3216	7815	487 0977	2655	2804	51
10	3923	425 2528	9838	5804	472 0380	3517	5170	5293	50
11	6577	5161	441 2448	8392	2944	6057	7685	7782	49
12	9230	7793	5059	457 0979	5508	8597	503 0199	518 0270	48
13	410 1883	426 0425	7668	3566	8071	488 1136	2713	2758	47
14	4536	3056	442 0278	6153	473 0634	3674	5227	5246	46
15	7189	5687	2887	8739	3197	6212	7740	7733	45
16	9841	8318	5496	458 1325	5759	8750	504 0252	519 0219	44
17	411 2492	427 0949	8104	3910	8321	489 1288	2765	2705	43
18	5144	3579	443 0712	6496	474 0882	3925	5276	5191	42
19	7795	6208	3319	9080	3443	6361	7788	7676	41
20	412 0445	8838	5927	459 1665	6004	8897	505 0298	520 0161	40
21	3096	428 1467	8534	4248	8564	490 1433	2809	2646	39
22	5745	4095	444 1140	6832	475 1124	3968	5319	5130	38
23	8395	6723	3746	9415	3683	6503	7828	7613	37
24	413 1044	9351	6352	460 1998	6242	9038	506 0338	521 0096	36
25	3693	429 1979	8957	4580	8801	491 1572	2846	2579	35
26	6342	4606	445 1562	7162	476 1359	4105	5355	5061	34
27	8990	7233	4167	9744	3917	6638	7863	7543	33
28	414 1638	9859	6771	461 2325	6474	9171	507 0370	522 0024	32
29	4285	430 2485	9375	4906	9031	492 1704	2877	2505	31
30	6932	5111	446 1978	7486	477 1588	4236	5384	4986	30
31	9579	7736	4581	462 0066	4144	6767	7890	7466	29
32	415 2226	431 0361	7184	2646	6700	9298	508 0396	9945	28
33	4872	2986	9786	5225	9255	493 1829	2901	523 2424	27
34	7517	5610	447 2388	7804	478 1810	4359	5406	4903	26
35	416 0163	8234	4990	463 0382	4364	6889	7910	7381	25
36	2808	432 0857	7591	2960	6919	9419	509 0414	9859	24
37	5453	3481	448 0192	5538	9472	494 1948	2918	524 2336	23
38	8097	6103	2792	8115	479 2026	4476	5421	4813	22
39	417 0741	8726	5392	464 0692	4579	7005	7924	7290	21
40	3385	433 1348	7992	3269	7131	9532	510 0426	9766	20
41	6028	3970	449 0591	5845	9683	495 2060	2928	525 2241	19
42	8671	6591	3190	8420	480 2235	4587	5429	4717	18
43	418 1313	9212	5789	465 0996	4786	7113	7930	7191	17
44	3956	434 1832	8387	3571	7337	9639	511 0431	9665	16
45	6597	4453	450 0984	6145	9888	496 2165	2931	526 2139	15
46	9239	7072	3582	8719	481 2438	4690	5431	4613	14
47	419 1890	9692	6179	466 1293	4987	7215	7930	7085	13
48	4521	435 2311	8775	3866	7537	9740	512 0429	9558	12
49	7161	4930	451 1372	6439	482 0086	497 2264	2927	527 2030	11
50	9801	7548	3967	9012	2634	4787	5425	4502	10
51	420 2441	436 0166	6563	467 1584	5182	7310	7923	6973	9
52	5080	2784	9158	4156	7730	9833	513 0420	9443	8
53	7719	5401	452 1753	6727	483 0277	498 2355	2916	528 1914	7
54	421 0358	8018	4347	9298	2824	4877	5413	4383	6
55	2996	437 0634	6941	468 1869	5370	7399	7908	6853	5
56	5634	3251	9535	4439	7916	9920	514 0404	9322	4
57	8272	5866	453 2128	7009	484 0462	499 2441	2899	529 1790	3
58	422 0909	8482	4721	9578	3007	4961	5393	4258	2
59	3546	438 1097	7313	469 2147	5552	7481	7887	6726	1
60	6183	3711	9905	4716	8096	500 0000	515 0381	9193	0
′	65°	64°	63°	62°	61°	60°	59°	58°	′

NAT. COSINE.

′	24°	25°	26°	27°	28°	29°	30°	31°	′
0	445 2287	466 3077	487 7326	509 5254	531 7094	554 3091	577 3503	600 8606	60
1	5773	6618	488 0927	8919	532 0826	6894	7382	601 2566	59
2	9260	467 0161	4530	510 2585	4559	555 0698	578 1262	6527	58
3	446 2747	3705	8133	6252	8293	4504	5144	602 0490	57
4	6236	7250	489 1737	9919	533 2029	8311	9027	4454	56
5	9726	468 0796	5343	511 3588	5765	556 2119	579 2912	8419	55
6	447 3216	4342	8949	7259	9503	5929	6797	603 2386	54
7	6708	7890	490 2557	512 0930	534 3242	9739	580 0684	6354	53
8	448 0200	469 1439	6166	4602	6981	557 3551	4573	604 0323	52
9	3693	4988	9775	8275	535 0723	7364	8462	4294	51
10	7187	8539	491 3386	513 1950	4465	558 1179	581 2353	8266	50
11	449 0682	470 2090	6997	5625	8208	4994	6245	605 2240	49
12	4178	5643	492 0610	9302	536 1953	8811	582 0139	6215	48
13	7675	9196	4224	514 2980	5699	559 2629	4034	606 0192	47
14	450 1173	471 2751	7838	6658	9446	6449	7930	4170	46
15	4672	6306	493 1454	515 0338	537 3194	560 0269	583 1828	8149	45
16	8171	9863	5071	4019	6943	4091	5726	607 2130	44
17	451 1672	472 3420	8689	7702	538 0694	7914	9627	6112	43
18	5173	6978	494 2308	516 1385	4445	561 1738	584 3528	608 0095	42
19	8676	473 0538	5928	5069	8198	5564	7431	4080	41
20	452 2179	4098	9549	8755	539 1952	9391	585 1335	8067	40
21	5683	7659	495 3171	517 2441	5707	562 3219	5241	609 2054	39
22	9188	474 1222	6794	6129	9464	7048	9148	6043	38
23	453 2694	4785	496 0418	9818	540 3221	563 0879	586 3056	610 0034	37
24	6201	8349	4043	518 3508	6980	4710	6965	4026	36
25	9709	475 1914	7669	7199	541 0740	8543	587 0876	8019	35
26	454 3218	5481	497 1297	519 0891	4501	564 2376	4788	611 2014	34
27	6728	9048	4925	4584	8263	6213	8702	6011	33
28	455 0238	476 2616	8554	8278	542 2027	565 0050	588 2616	612 0008	32
29	3750	6185	498 2185	520 1974	5791	3888	6533	4007	31
30	7263	9755	5816	5671	9557	7728	589 0450	8006	30
31	456 0776	477 3326	9449	9368	543 3324	566 1568	4369	613 2010	29
32	4290	6899	499 3082	521 3067	7092	5410	8289	6013	28
33	7806	478 0472	6717	6767	544 0862	9254	590 2211	614 0018	27
34	457 1322	4046	500 0352	522 0468	4632	567 3098	6134	4024	26
35	4839	7621	3989	4170	8404	6944	591 0058	8032	25
36	8357	479 1197	7627	7874	545 2177	568 0791	3984	615 2041	24
37	458 1877	4774	501 1266	523 1578	5951	4639	7910	6052	23
38	5397	8352	4906	5284	9727	8488	592 1839	616 0064	22
39	8918	480 1932	8547	8990	546 3503	569 2339	5768	4077	21
40	459 2439	5512	502 2189	524 2698	7281	6191	9699	8092	20
41	5962	9093	5832	6407	547 1060	570 0045	593 3632	617 2108	19
42	9486	481 2675	9476	525 0117	4840	3899	7565	6126	18
43	460 3011	6258	503 3121	3829	8621	7755	594 1501	618 0145	17
44	6537	9842	6768	7541	548 2404	571 1612	5437	4166	16
45	461 0063	482 3427	504 0415	526 1255	6188	5471	9375	8188	15
46	3591	7014	4063	4969	9973	9331	595 3314	619 2211	14
47	7119	483 0601	7713	8685	549 3759	572 3192	7255	6236	13
48	462 0649	4189	505 1363	527 2402	7547	7054	596 1196	620 0263	12
49	4179	7778	5015	6120	550 1335	573 0918	5140	4291	11
50	7710	484 1368	8668	9839	5125	4783	9084	8320	10
51	463 1243	4959	506 2322	528 3560	8916	8649	597 3030	621 2351	9
52	4776	8552	5977	7281	551 2708	574 2516	6978	6383	8
53	8310	485 2145	9633	529 1004	6502	6385	598 0926	622 0417	7
54	464 1845	5739	507 3290	4727	552 0297	575 0255	4877	4452	6
55	5382	9334	6948	8452	4093	4126	8828	8488	5
56	8919	486 2931	508 0607	530 2178	7890	7999	599 2781	623 2527	4
57	465 2457	6528	4267	5906	553 1688	576 1873	6735	6566	3
58	5996	487 0126	7929	9634	5488	5748	600 0691	624 0607	2
59	9536	3726	509 1591	531 3364	9288	9625	4648	4650	1
60	466 3077	7326	5254	7094	554 3091	577 3503	8606	8694	0
′	65°	64°	63°	62°	61°	60°	59°	58°	′

NAT. COTAN.

′	32°	33°	34°	35°	36°	37°	38°	39°	′
0	529 9193	544 6390	559 1929	573 5764	587 7853	601 8150	615 6615	629 3204	60
1	530 1659	8830	4340	8147	588 0206	602 0473	8907	5464	59
2	4125	545 1269	6751	574 0529	2558	2795	616 1198	7724	58
3	6591	3707	9162	2911	4910	5117	3489	9983	57
4	9057	6145	560 1572	5292	7262	7439	5780	630 2242	56
5	531 1521	8583	3981	7672	9613	9760	8069	4500	55
6	3986	546 1020	6390	575 0053	589 1964	603 2080	617 0359	6758	54
7	6450	3456	8798	2432	4314	4400	2648	9015	53
8	8913	5892	561 1206	4811	6663	6719	4936	631 1272	52
9	532 1376	8328	3614	7190	9012	9038	7224	3528	51
10	3839	547 0763	6021	9568	590 1361	604 1356	9511	5784	50
11	6301	3198	8428	576 1946	3709	3674	618 1798	8039	49
12	8763	5632	562 0834	4323	6057	5991	4084	632 0293	48
13	533 1224	8060	3239	6700	8404	8308	6370	2547	47
14	3685	548 0499	5645	9076	591 0750	605 0624	8655	4800	46
15	6145	2932	8049	577 1452	3096	2940	619 0939	7053	45
16	8605	5365	563 0453	3827	5442	5255	3224	9306	44
17	534 1065	7797	2857	6202	7787	7570	5507	633 1557	43
18	3523	549 0228	5260	8576	592 0132	9684	7790	3809	42
19	5982	2659	7663	578 0950	2476	606 2198	620 0073	6059	41
20	8440	5090	564 0066	3323	4819	4511	2355	8310	40
21	535 0898	7520	2467	5696	7163	6824	4636	634 0559	39
22	3355	9950	4869	8069	9505	9136	6917	2808	38
23	5812	550 2379	7270	579 0440	593 1847	607 1447	9198	5057	37
24	8268	4807	9670	2812	4189	3758	621 1478	7305	36
25	536 0724	7236	565 2070	5183	6530	6069	3757	9553	35
26	3179	9663	4469	7553	8871	8379	6036	635 1900	34
27	5634	551 2091	6868	9923	594 1211	608 0689	8314	4046	33
28	9089	4518	9267	580 2292	3550	2998	622 0592	6292	32
29	537 0543	6944	566 1665	4661	5889	5306	2870	8537	31
30	2996	9370	4062	7030	8228	7614	5146	636 0782	30
31	5449	552 1795	6459	9397	595 0566	9922	7423	3026	29
32	7902	4220	8856	581 1765	2904	609 2229	9698	5270	28
33	538 0354	6645	567 1252	4132	5241	4535	623 1974	7513	27
34	2806	9069	3648	6498	7577	6841	4248	9756	26
35	5257	553 1492	6043	8864	9913	9147	6522	637 1998	25
36	7708	3915	8437	582 1230	596 2249	610 1452	8796	4240	24
37	539 0158	6338	568 0832	3595	4584	3756	624 1069	6481	23
38	2608	8760	3225	5959	6918	6060	3342	8721	22
39	5058	554 1182	5619	8323	9252	8363	5614	638 0961	21
40	7507	3603	8011	583 0687	597 1586	611 0666	7885	3201	20
41	9955	6024	569 0403	3050	3919	2969	625 0156	5440	19
42	540 2403	8444	2795	5412	6251	5270	2427	7678	18
43	4851	555 0864	5187	7774	8583	7572	4696	9916	17
44	7298	3283	7577	584 0136	598 0915	9873	6966	639 2153	16
45	9745	5702	9968	2497	3246	612 2173	9235	4390	15
46	541 2191	8121	570 2357	4857	5577	4473	626 1503	6626	14
47	4637	556 0539	4747	7217	7906	6772	3771	8862	13
48	7082	2956	7136	9577	599 0236	9071	6038	640 1097	12
49	9527	5373	9524	585 1936	2565	613 1369	8305	3332	11
50	542 1971	7790	571 1912	4294	4893	3666	627 0571	5566	10
51	4415	557 0206	4299	6652	7221	5964	2837	7799	9
52	6859	2621	6686	9010	9549	8260	5102	641 0032	8
53	9302	5036	9073	586 1367	600 1876	614 0556	7366	2264	7
54	543 1744	7451	572 1459	3724	4202	2852	9631	4496	6
55	4187	9865	3844	6080	6528	5147	628 1894	6728	5
56	6628	558 2279	6229	8435	8854	7442	4157	8958	4
57	9069	4692	8614	587 0790	601 1179	9736	6420	642 1189	3
58	544 1510	7105	573 0998	3145	3503	615 2029	8682	3418	2
59	3951	9517	3381	5499	5827	4322	629 0943	5647	1
60	6390	559 1929	5764	7853	8150	6615	3204	7876	0
′	57°	56°	55°	54°	53°	52°	51°	50°	′

NAT. COSINE.

′	32°	33°	34°	35°	36°	37°	38°	39°	′
0	624 8694	649 4076	674 5085	700 2075	726 5425	753 5541	781 2856	809 7840	60
1	625 2739	8212	9318	6411	9871	754 0102	7542	810 2658	59
2	6786	650 2350	675 3553	701 0749	727 4318	4666	782 2229	7478	58
3	626 0834	6490	7790	5089	8767	9232	6919	811 2300	57
4	4884	651 0631	676 2029	9430	728 3218	755 3799	783 1611	7124	56
5	8935	4774	6268	702 3773	7671	8369	6305	812 1951	55
6	627 2988	8918	677 0509	8118	729 2125	756 2941	784 1002	6780	54
7	7042	652 3064	4752	703 2464	6582	7514	5700	813 1611	53
8	628 1098	7211	8997	6813	730 1041	757 2090	785 0400	6444	52
9	5155	653 1360	678 3243	704 1163	5501	6668	5103	814 1280	51
10	9214	5511	7492	5515	9963	758 1248	9808	6118	50
11	629 3274	9663	679 1741	9869	731 4428	5829	786 4515	815 0958	49
12	7336	654 3817	5993	705 4224	8894	759 0413	9224	5801	48
13	630 1399	7972	680 0246	8581	732 3362	4999	787 3935	816 0646	47
14	5464	655 2129	4501	706 2940	7832	9587	8649	5493	46
15	9530	6287	8758	7301	733 2303	760 4177	788 3364	817 0343	45
16	631 3598	656 0447	681 3016	707 1664	6777	8769	8082	5195	44
17	7667	4609	7276	6028	734 1253	761 3363	789 2802	818 0049	43
18	632 1738	8772	682 1537	708 0395	5730	7959	7524	4905	42
19	5810	657 2937	5801	4763	735 0210	762 2557	790 2248	9764	41
20	9883	7103	683 0066	9133	4691	7157	6975	819 4625	40
21	633 3959	658 1271	4333	709 3504	9174	763 1759	791 1703	9486	39
22	8035	5441	8601	7878	736 3660	6363	6434	820 4354	38
23	634 2113	9612	684 2871	710 2253	8147	764 0969	792 1167	9222	37
24	6193	659 3785	7143	6630	737 2636	5577	5902	821 4093	36
25	635 0274	7960	685 1416	711 1009	7127	765 0188	793 0640	8965	35
26	4357	660 2136	5692	5390	738 1620	4800	5379	822 3640	34
27	8441	6313	9969	9772	6115	9414	794 0121	8718	33
28	636 2527	661 0492	686 4247	712 4157	739 0611	766 4031	4865	823 3597	32
29	6614	4673	8528	8543	5110	8649	9611	8479	31
30	637 0703	8856	687 2810	713 2931	9611	767 3270	795 4359	824 3364	30
31	4793	662 3040	7093	7320	740 4113	7893	9110	8251	29
32	8885	7225	688 1379	714 1712	8618	768 2517	796 3862	825 3140	28
33	638 2978	663 1413	5666	6106	741 3124	7144	8617	8031	27
34	7073	5601	9955	715 0501	7633	769 1773	797 3374	826 2925	26
35	639 1169	9792	689 4246	4898	742 2143	6404	8134	7821	25
36	5267	664 3984	8538	9297	6655	770 1037	798 2895	827 2719	24
37	9366	8178	690 2832	716 3698	743 1170	5672	7659	7620	23
38	640 3467	665 2373	7128	8100	5686	771 0309	799 2425	828 2523	22
39	7569	6570	691 1425	717 2505	744 0204	4948	7193	7429	21
40	641 1673	666 0769	5725	6911	4724	9589	800 1963	829 2337	20
41	5779	4969	692 0026	718 1319	9246	772 4233	6736	7247	19
42	9886	9171	4328	5729	745 3770	8878	801 1511	830 2160	18
43	642 3994	667 3374	8633	719 0141	8296	773 3526	6288	7075	17
44	8105	7580	693 2939	4554	746 2824	8176	802 1067	831 1992	16
45	643 2216	668 1786	7247	8970	7354	774 2827	5849	6912	15
46	6329	5995	694 1557	720 3387	747 1886	7481	803 0632	832 1834	14
47	644 0444	669 0205	5868	7806	6420	775 2137	5418	6759	13
48	4560	4417	695 0181	721 2227	748 0956	6795	804 0206	833 1686	12
49	8678	8630	4496	6650	5494	776 1455	4997	6615	11
50	645 2797	670 2845	8813	722 1075	749 0033	6118	9790	834 1547	10
51	6918	7061	696 3131	5502	4575	777 0782	805 4584	6481	9
52	646 1041	671 1290	7451	9930	9119	5448	9382	835 1418	8
53	5165	5500	697 1773	723 4361	750 3665	778 0117	806 4181	6357	7
54	9290	9721	6097	8793	8212	4788	8983	836 1298	6
55	647 3417	672 3944	698 0422	724 3227	751 2762	9460	807 3787	6242	5
56	7546	8169	4749	7663	7314	779 4135	8593	837 1188	4
57	648 1676	673 2396	9078	725 2101	752 1867	8812	808 3401	6136	3
58	5808	6624	699 3409	6540	6423	780 3492	8212	838 1087	2
59	9941	674 0854	7741	726 0982	753 0981	8173	809 3025	6041	1
60	649 4076	5085	700 2075	5425	5541	781 2856	7840	839 0996	0
′	57°	56°	55°	54°	53°	52°	51°	50°	′

NAT. COTAN.

′	40°	41°	42°	43°	44°	45°	46°	47°	′
0	642 7876	656 0590	669 1306	681 9984	694 6584	707 1068	719 3398	731 3537	60
1	643 0104	2785	3468	682 2111	8676	3124	5418	5521	59
2	2332	4980	5628	4237	695 0767	5180	7438	7503	58
3	4559	7174	7789	6363	2858	7236	9457	9486	57
4	6785	9367	9948	8489	4949	9291	720 1476	732 1467	56
5	9011	657 1560	670 2108	683 0613	7039	708 1345	3494	3449	55
6	644 1236	3752	4266	2738	9128	3398	5511	5429	54
7	3461	5944	6424	4861	696 1217	5451	7528	7409	53
8	5685	8135	8582	6984	3305	7504	9544	9388	52
9	7909	658 0326	671 0739	9107	5392	9556	721 1559	733 1367	51
10	645 0132	2516	2895	684 1229	7479	709 1607	3574	3345	50
11	2355	4706	5051	3350	9565	3657	5589	5322	49
12	4577	6895	7206	5471	697 1651	5707	7602	7299	48
13	6798	9083	9361	7591	3736	7757	9615	9275	47
14	9019	659 1271	672 1515	9711	5821	9806	722 1628	734 1250	46
15	646 1240	3458	3668	685 1830	7905	710 1854	3640	3225	45
16	3460	5645	5821	3948	9988	3901	5651	5199	44
17	5679	7831	7973	6066	698 2071	5948	7661	7173	43
18	7898	660 0017	673 0125	8184	4153	7995	9671	9146	42
19	647 0116	2202	2276	686 0300	6234	711 0041	723 1681	735 1118	41
20	2334	4386	4427	2416	8315	2086	3690	3090	40
21	4551	6570	6577	4532	699 0396	4130	5698	5061	39
22	6767	8754	8727	6647	2476	6174	7705	7032	38
23	8984	661 0936	674 0876	8761	4555	8218	9712	9002	37
24	648 1199	3119	3024	687 0875	6633	712 0260	724 1719	736 0971	36
25	3414	5300	5172	2988	8711	2303	3724	2940	35
26	5628	7482	7319	5101	700 0789	4344	5729	4908	34
27	7842	9662	9466	7213	2866	6385	7734	6875	33
28	649 0056	662 1842	675 1612	9325	4942	8426	9738	8842	32
29	2268	4022	3757	688 1435	7018	713 0465	725 1741	737 0808	31
30	4480	6200	5902	3546	9093	2504	3744	2773	30
31	6692	8379	8046	5655	701 1167	4543	5746	4738	29
32	8903	663 0557	676 0190	7765	3241	6581	7747	6703	28
33	650 1114	2734	2333	9873	5314	8618	9748	8666	27
34	3324	4910	4476	689 1981	7387	714 0655	726 1748	738 0629	26
35	5533	7087	6618	4089	9459	2691	3748	2592	25
36	7742	9262	8760	6195	702 1531	4727	5747	4553	24
37	9951	664 1437	677 0901	8302	3601	6762	7745	6515	23
38	651 2158	3612	3041	690 0407	5672	8796	9743	6475	22
39	4366	5785	5181	2512	7741	715 0830	727 1740	739 0435	21
40	6572	7959	7320	4617	9811	2863	3736	2394	20
41	8778	665 0131	9459	6721	703 1879	4895	5732	4353	19
42	652 0984	2304	678 1597	8824	3947	6927	7728	6311	18
43	3189	4475	3734	691 0927	6014	8959	9722	8268	17
44	5394	6646	5871	3029	8081	716 0989	728 1716	740 0225	16
45	7598	8817	8007	5131	704 0147	3019	3710	2181	15
46	9801	666 0987	679 0143	7232	2213	5049	5703	4137	14
47	653 2004	3156	2278	9332	4278	7078	7695	6092	13
48	4206	5325	4413	692 1432	6342	9106	9686	8046	12
49	6408	7493	6547	3531	8406	717 1134	729 1677	741 0000	11
50	8609	9661	8681	5630	705 0469	3161	3668	1953	10
51	654 0810	667 1828	680 0813	7728	2532	5187	5657	3905	9
52	3010	3994	2946	9825	4594	7213	7646	5857	8
53	5209	6160	5078	693 1922	6655	9238	9635	7808	7
54	7408	8326	7209	4018	8716	718 1263	730 1623	9758	6
55	9607	668 0490	9339	6114	706 0776	3287	3610	742 1708	5
56	655 1804	2655	681 1469	8209	2835	5310	5597	3658	4
57	4002	4818	3599	694 0304	4894	7333	7583	5606	3
58	6198	6981	5728	2398	6953	9355	9568	7554	2
59	8395	9144	7856	4491	9011	719 1377	731 1553	9502	1
60	656 0590	669 1306	9984	6584	707 1068	3398	3537	743 1448	0
′	49°	48°	47°	46°	45°	44°	43°	42°	′

NAT. COSINE.

′	40°	41°	42°	43°	44°	45°	46°	47°	′
0	8390996	8692867	9004040	9325151	9656888	1·0000000	1·0355303	1·0723687	60
1	5955	7976	9309	9330591	9662511	05819	61333	29943	59
2	8400915	8703087	9014580	6034	8137	11642	67367	36203	58
3	5878	8200	9854	9341479	9673767	17469	73404	42467	57
4	8410844	8713316	9025131	6928	9399	23298	79445	48734	56
5	5812	8435	9030411	9352380	9685035	29131	85489	55006	55
6	8420782	8723556	5693	7834	9690674	34968	91538	61282	54
7	5755	8680	9040979	9363292	6316	40807	97589	67561	53
8	8430730	8733806	6267	8753	9701962	46651	1·0403645	73845	52
9	5708	8935	9051557	9374216	7610	52497	09704	80132	51
10	8440688	8744067	6851	9683	9713262	58348	15767	86423	50
11	5670	9201	9062147	9385153	8917	64201	21833	92718	49
12	8450655	8754338	7446	9390625	9724575	70058	27904	99018	48
13	5643	9478	9072748	6101	9730236	75918	33977	1·0805321	47
14	8460633	8764620	8053	9401579	5901	81782	40055	11628	46
15	5625	9765	9083360	7061	9741569	87649	46136	17939	45
16	8470620	8774912	8671	9412545	7240	93520	52221	24254	44
17	5617	8780062	9093984	8033	9752914	99394	58310	30573	43
18	8480617	5215	9300	9423523	8591	1·0105272	64402	36896	42
19	5619	8790370	9104919	9017	9764272	11153	70498	43223	41
20	8490624	5528	9940	9434513	9956	17036	76598	49554	40
21	5631	8800688	9115265	9440013	9775643	22925	82702	55889	39
22	8500640	5852	9120592	5516	9781333	28817	88809	62228	38
23	5653	8811017	5922	9451021	7027	34712	94920	68571	37
24	8510667	6186	9131255	6530	9792724	40610	1·0501034	74918	36
25	5684	8821357	6591	9462042	8424	46512	07153	81269	35
26	8520704	6531	9141929	7556	9804127	52418	13275	87624	34
27	5726	8831707	7270	9473074	9833	58326	19401	93984	33
28	8530750	6886	9152615	8595	9815543	64239	25531	1·0900347	32
29	5777	8842068	7962	9484119	9821256	70155	31664	06714	31
30	8540807	7253	9163312	9646	6973	76074	37801	13085	30
31	5839	8852440	8665	9495176	9832692	81997	43942	19460	29
32	8550873	7630	9174020	9500709	8415	87923	50087	25840	28
33	5910	8862822	9379	6245	9844141	93853	56235	32223	27
34	8560950	8017	9184740	9511784	9871	99786	62388	38610	26
35	5992	8873215	9190104	7326	9855603	1·0205723	68544	45002	25
36	8571037	8415	5471	9522871	9861339	11664	74704	51397	24
37	6084	8883619	9200841	8420	7079	17608	80867	57797	23
38	8581133	8825	6214	9533971	9872821	23555	87035	64201	22
39	6185	8894033	9211590	9526	8567	29506	93206	70609	21
40	8591240	9244	6969	9545083	9884316	35461	99381	77020	20
41	6297	8904458	9222350	9550644	9890069	41419	1·0605560	83436	19
42	8601357	9675	7734	6208	5825	47381	11742	89857	18
43	6419	8914894	9233122	9561774	9901584	53346	17929	96281	17
44	8611484	8920116	8512	7344	7346	59315	24119	1·1002709	16
45	6551	5341	9243905	9572917	9913112	65287	30313	09141	15
46	8621621	8930569	9301	8494	8881	71263	36511	15578	14
47	6694	5799	9254700	9584073	9924654	77243	42713	22019	13
48	8631768	8941032	9260102	9655	9930429	83226	48918	28463	12
49	6846	6268	5506	9595241	6208	89212	55128	34912	11
50	8641926	8951506	9270914	9600829	9941991	95203	61341	41365	10
51	7009	6747	6324	6421	7777	1·0301196	67558	47823	9
52	8652094	8961991	9281738	9612016	9953566	07194	73779	54284	8
53	7181	7238	7154	7614	9358	13195	80004	60750	7
54	8662272	8972487	9292573	9623215	9965154	19199	86233	67219	6
55	7365	7739	7996	8819	9970953	25208	92466	73693	5
56	8672460	8982994	9303421	9634427	6756	31220	98702	80171	4
57	7558	8251	8849	9640037	9982562	37235	1·0704943	86653	3
58	8682659	8993512	9314280	5651	8371	43254	11187	93140	2
59	7762	8775	9714	9651268	9994184	49277	17435	99630	1
60	8692867	9004040	9325151	6888	1·0000000	55303	23687	1·1106125	0
′	49°	48°	47°	46°	45°	44°	43°	42°	′

NAT. COTAN.

′	48°	49°	50°	51°	52°	53°	54°	55°	′
0	743 1448	754 7096	766 0444	777 1460	788 0108	798 6355	809 0170	819 1520	60
1	3394	9004	2314	3290	1898	8105	1879	3189	59
2	5340	755 0911	4183	5120	3688	9855	3588	4856	58
3	7285	2818	6051	6949	5477	799 1604	5296	6523	57
4	9229	4724	7918	8777	7266	3352	7004	8189	56
5	744 1173	6630	9785	778 0604	9054	5100	8710	9854	55
6	3115	8535	767 1652	2431	789 0841	6847	810 0416	820 1519	54
7	5058	756 0439	3517	4258	2627	8593	2122	3183	53
8	6999	2342	5382	6084	4413	800 0338	3826	4846	52
9	8941	4246	7246	7909	6198	2083	5530	6509	51
10	745 0881	6148	9110	9733	7983	3827	7234	8170	50
11	2821	8050	768 0973	779 1557	9767	5571	8936	9832	49
12	4760	9951	2835	3380	790 1550	7314	811 0638	821 1492	48
13	6699	757 1851	4697	5202	3333	9056	2339	3152	47
14	8636	6751	6558	7024	5115	801 0797	4040	4811	46
15	746 0574	5650	8418	8845	6896	2538	5740	6469	45
16	2510	7548	769 0278	780 0665	8676	4278	7439	8127	44
17	4446	9446	2137	2485	791 0456	6018	9137	9784	43
18	6382	758 1343	3996	4304	2235	7756	812 0835	822 1440	42
19	8317	3240	5853	6123	4014	9495	2532	3096	41
20	747 0251	5136	7710	7940	5792	802 1232	4229	4751	40
21	2184	7031	9567	9757	7569	2969	5925	6405	39
22	4117	8926	770 1423	781 1574	9345	4705	7620	8059	38
23	6049	759 0820	3278	3390	792 1121	6440	9314	9712	37
24	7981	2713	5132	5205	2896	8175	813 1008	823 1364	36
25	9912	4606	6986	7019	4671	9909	2701	3015	35
26	748 1842	6498	8840	8833	6445	803 1642	4393	4666	34
27	3772	8389	771 0692	782 0646	8218	3375	6084	6316	33
28	5701	760 0280	2544	2459	9990	5107	7775	7965	32
29	7629	2170	4395	4270	793 1762	6838	9466	9614	31
30	9557	4060	6246	6082	3533	8569	814 1155	824 1262	30
31	749 1484	5949	8096	7892	5304	804 0299	2844	2909	29
32	3411	7837	9945	9702	7074	2028	4532	4556	28
33	5337	9724	772 1794	783 1511	8843	3756	6220	6202	27
34	7262	761 1611	3642	3320	794 0611	5484	7906	7847	26
35	9187	3497	5489	5127	2379	7211	9593	9491	25
36	750 1111	5383	7336	6935	4146	8938	815 1278	825 1135	24
37	3034	7268	9182	8741	5913	805 0664	2963	2778	23
38	4957	9152	773 1027	784 0547	7678	2389	4647	4420	22
39	6879	762 1036	2872	2352	9444	4113	6330	6062	21
40	8800	2919	4716	4157	795 1208	5837	8013	7703	20
41	751 0721	4802	6559	5961	2972	7560	9695	9343	19
42	2641	6683	8402	7764	4735	9283	816 1376	826 0983	18
43	4561	8564	774 0244	9566	6497	806 1005	3056	2622	17
44	6480	763 0445	2086	785 1368	8259	2726	4736	4260	16
45	8398	2325	3926	3169	796 0020	4446	6416	5897	15
46	752 0316	4204	5767	4970	1780	6166	8094	7534	14
47	2233	6082	7606	6770	3540	7885	9772	9170	13
48	4149	7960	9445	8569	5299	9603	817 1449	[illegible]27 0806	12
49	6065	9838	775 1283	786 0367	7058	807 1321	3125	2440	11
50	7980	764 1714	3121	2165	8815	3038	4801	4074	10
51	9894	3590	4957	3963	797 0572	4754	6476	5708	9
52	753 1808	5465	6794	5759	2329	6470	8151	7340	8
53	3721	7340	8629	7555	4084	8185	9824	8972	7
54	5634	9214	776 0464	9350	5839	9899	818 1497	828 0603	6
55	7546	765 1087	2298	787 1145	7594	808 1612	3169	2234	5
56	9457	2960	4132	2939	9347	3325	4841	3864	4
57	754 1368	4832	5965	4732	798 1100	5037	6512	5493	3
58	3278	6704	7797	6524	2853	6749	8182	7121	2
59	5187	8574	9629	8316	4604	8460	9852	8749	1
60	7096	766 0444	777 1460	788 0108	6355	809 0170	819 1520	829 0376	0
′	41°	40°	39°	38°	37°	36°	35°	34°	′

NAT. COSINE.

′	48°	49°	50°	51°	52°	53°	54°	55°	′
0	1·1106125	1·1503684	1·1917536	1·2 48972	1·2799416	1·3270448	1·3763819	1·4281480	60
1	12624	10445	24579	56319	1·2807094	78483	72242	90326	59
2	19127	17210	31626	63672	14776	86524	80672	99178	58
3	25635	23979	38679	71030	22465	94571	89108	1·4308039	57
4	32146	30754	45736	78393	30160	1·3302624	97551	16906	56
5	38662	37532	52799	85762	37860	10684	1·3806001	25781	55
6	45182	44316	59866	93136	45566	18750	14458	34664	54
7	51706	51104	66938	1·2400515	53277	26822	22922	43554	53
8	58235	57896	74015	07900	60995	34900	31392	52451	52
9	64768	64693	81097	15290	68718	42984	39869	61356	51
10	71305	71495	88184	22685	76447	51075	48353	70268	50
11	77846	78301	95276	30086	84182	59172	56844	79187	49
12	84391	85112	1·2002373	37492	91922	67276	65342	88114	48
13	90941	91927	09475	44903	99669	75386	73847	97049	47
14	97495	98747	16581	52320	1·2907421	83502	82358	1·4405991	46
15	1·1204053	1·1605571	23693	59742	15179	91624	90876	14940	45
16	10616	12400	30810	67169	22943	99753	99401	23897	44
17	17183	19234	37932	74602	30713	1·3407888	1·3907934	32862	43
18	23754	26073	45058	82040	38488	16029	16473	41834	42
19	30329	32916	52190	89484	46270	24177	25019	50814	41
20	36909	39763	59327	96933	54057	32331	33571	59801	40
21	43493	46615	66468	1·2504388	61850	40492	42131	68796	39
22	50081	53472	73615	11848	69649	48658	50698	77798	38
23	56674	60334	80767	19313	77454	56832	59272	86808	37
24	63271	67200	87924	26784	85265	65011	67852	95825	36
25	69872	74071	95065	34260	93081	73198	76440	1·4504850	35
26	76478	80947	1·2102252	41742	1·3000904	81390	85034	13883	34
27	83088	87827	09424	49229	08733	89589	93636	22923	33
28	89702	94712	16601	56721	16567	97794	1·4002245	31971	32
29	96321	1·1701601	23783	64219	24407	1·3506006	10860	41027	31
30	1·1302944	08496	30970	71723	32254	14224	19483	50090	30
31	09571	15395	38162	79232	40106	22449	28113	59161	29
32	16203	22298	45359	86747	47964	30680	36749	68240	28
33	22839	29207	52562	94267	55828	38918	45393	77326	27
34	29479	36120	59769	1·2601792	63699	47162	54044	86420	26
35	36124	43038	66982	09323	71575	55413	62702	95522	25
36	42773	49960	74199	16860	79457	63670	71367	1·4604632	24
37	49427	56888	81422	24402	87345	71934	80039	13749	23
38	56085	63820	88650	31950	95239	80204	88718	22874	22
39	62747	70756	95883	39503	1·3103140	88481	97405	32007	21
40	69414	77698	1·2203121	47062	11046	96764	1·4106098	41147	20
41	76086	84644	10364	54626	18958	1·3605054	14799	50296	19
42	82761	91595	17613	62196	26876	13350	23506	59452	18
43	89441	98551	24866	69772	34801	21653	32221	68616	17
44	96126	1·1805512	32125	77353	42731	29963	40943	77788	16
45	1·1402815	12477	39389	84940	50668	38279	49673	86967	15
46	09508	19447	46658	92532	58610	46602	58409	96155	14
47	16206	26422	53932	1·2700130	66559	54931	67153	1·4705350	13
48	22908	33402	61211	07733	74513	63267	75904	14553	12
49	29615	40387	68496	15342	82474	71610	84662	23764	11
50	36326	47376	75786	22957	90441	79959	93427	32983	10
51	43041	54370	83081	30578	98414	88315	1·4202200	42210	9
52	49762	61369	90381	38204	1·3206393	96678	10979	51445	8
53	56486	68373	97687	45835	14379	1·3705047	19766	60688	7
54	63215	75382	1·2304997	53473	22370	13423	28561	69938	6
55	69949	82395	12313	61116	30368	21806	37362	79197	5
56	76687	89414	19634	68765	38371	30195	46171	88463	4
57	83429	96437	26961	76419	46381	38591	54988	97738	3
58	90176	1·1903465	34292	84079	54397	46994	63811	1·4907021	2
59	96928	10498	41629	91745	62420	55403	72642	16311	1
60	1·1503684	17536	48972	99416	70448	63819	81480	25610	0
′	41°	40°	39°	38°	37°	36°	35°	34°	′

NAT. COTAN.

′	56°	57°	58°	59°	60°	61°	62°	63°	′
0	829 0376	838 6706	848 0481	857 1673	866 0254	874 6197	882 9476	891 0065	60
1	2002	8290	2022	3171	1708	7607	883 0841	1385	59
2	3628	9873	3562	4668	3161	9016	2206	2705	58
3	5252	839 1455	5102	6164	4614	875 0425	3569	4024	57
4	6877	3037	6641	7660	6066	1832	4933	5342	56
5	8500	4618	8179	9155	7517	3239	6295	6659	55
6	830 0123	6199	9717	858 0649	8967	4645	7656	7975	54
7	1745	7778	849 1254	2143	867 0417	6051	9017	9291	53
8	3366	9357	2790	3635	1866	7455	884 0377	892 0606	52
9	4987	840 0936	4325	5127	3314	8859	1736	1920	51
10	6607	2513	5860	6619	4762	876 0263	3095	3234	50
11	8226	4090	7394	8109	6209	1665	4453	4546	49
12	9845	5666	8927	9599	7655	3067	5810	5858	48
13	831 1463	7241	850 0459	859 1088	9100	4468	7166	7169	47
14	3080	8816	1991	2576	868 0544	5868	8522	8480	46
15	4696	841 0390	3522	4064	1988	7268	9876	9789	45
16	6312	1963	5053	5551	3431	8666	885 1230	893 1098	44
17	7927	3536	6582	7037	4874	877 0064	2584	2406	43
18	9541	5108	8111	8523	6315	1462	3936	3714	42
19	832 1155	6679	9639	860 0007	7756	2858	5288	5021	41
20	2768	8249	851 1167	1491	9196	4254	6639	6326	40
21	4380	9819	2693	2975	869 0636	5649	7989	7632	39
22	5991	842 1388	4219	4457	2074	7043	9339	8936	38
23	7602	2956	5745	5939	3512	8437	886 0688	894 0240	37
24	9212	4524	7269	7420	4949	9830	2036	1542	36
25	833 0822	6091	8793	8901	6386	878 1222	3383	2844	35
26	2430	7657	852 0316	861 0380	7821	2613	4730	4146	34
27	4038	9222	1839	1859	9256	4004	6075	5446	33
28	5646	843 0787	3360	3337	870 0691	5394	7420	6746	32
29	7252	2351	4881	4815	2124	6783	8765	8045	31
30	8858	3914	6402	6292	3557	8171	887 0108	9344	30
31	834 0463	5477	7921	7768	4989	9559	1451	895 0641	29
32	2068	7039	9440	9243	6420	879 0946	2793	1938	28
33	3672	8600	853 0958	862 0717	7851	2332	4134	3234	27
34	5275	844 0161	2475	2191	9281	3717	5475	4529	26
35	6877	1720	3992	5664	871 0710	5102	6815	5824	25
36	8479	3279	5508	5137	2138	6486	8154	7118	24
37	835 0080	4838	7023	6608	3566	7869	9492	8411	23
38	1680	6395	8538	8079	4993	9251	888 0830	9703	22
39	3279	7952	854 0051	9549	6419	880 0633	2166	896 0994	21
40	4878	9508	1564	863 1019	7844	2014	3503	2285	20
41	6476	845 1064	3077	2488	9269	3394	4838	3575	19
42	8074	2618	4588	3956	872 0693	4774	6172	4864	18
43	9670	4172	6099	5423	2116	6152	7506	6153	17
44	836 1266	5726	7609	6889	3538	7530	8839	7440	16
45	2862	7278	9119	8355	4960	8907	889 0171	8727	15
46	4456	8830	855 0627	9820	6381	881 0284	1503	897 0014	14
47	6050	846 0381	2135	864 1284	7801	1660	2834	1299	13
48	7643	1932	3643	2748	9221	3035	4164	2584	12
49	9236	3481	5149	4211	873 0640	4409	5493	3868	11
50	837 0827	5030	6655	5673	2058	5782	6822	5151	10
51	2418	6579	8160	7134	3475	7155	8149	6433	9
52	4009	8126	9664	8595	4891	8527	9476	7715	8
53	5598	9673	856 1168	865 0055	6307	9898	890 0803	8996	7
54	7187	847 1219	2671	1514	7722	882 1269	2128	898 0276	6
55	8775	2765	4173	2973	9137	2638	3453	1555	5
56	838 0363	4309	5674	4430	874 0550	4007	4777	2834	4
57	1950	5853	7175	5887	1963	5376	6100	4112	3
58	3536	7397	8675	7344	3375	6743	7423	5389	2
59	5121	8939	857 0174	8799	4786	8110	8744	6665	1
60	6706	848 0481	1673	866 0254	6197	9476	891 0065	7940	0
′	33°	32°	31°	30°	29°	28°	27°	26°	′

NAT. COSINE.

′	56°	57°	58°	59°	60°	61°	62°	63°	′
0	1·4825610	1·5398650	1·6003345	1·6642795	1·7320508	1·8040478	1·8807265	1·9626105	60
1	34916	1·5408460	13709	53766	32149	52860	20470	40227	59
2	44231	18280	24082	64748	43803	65256	33690	54364	58
3	53554	28108	34465	75741	55468	77664	46924	68518	57
4	62884	37946	44858	86744	67144	90086	60172	82688	56
5	72223	47792	55260	97758	78833	1·8102521	73436	96874	55
6	81570	57647	65672	1·6708782	90533	14969	86713	1·9711077	54
7	90925	67510	76094	19818	1·7402245	27430	1·8900006	25296	53
8	1·4900288	77383	86525	30864	13969	39904	13313	39531	52
9	09659	87264	96966	41921	25705	52391	26635	53782	51
10	19039	97155	1·6107417	52988	37453	64892	39971	68050	50
11	28426	1·5507054	17878	64067	49213	77405	53322	82334	49
12	37822	16963	28349	75156	60984	89932	66688	96635	48
13	47225	26880	38829	86256	72768	1·8202473	80068	1·9810952	47
14	56637	36806	49320	97367	84564	15026	93464	25286	46
15	66058	46741	59820	1·6808489	96371	27593	1·9006874	39636	45
16	75486	56685	70330	19621	1·7508191	40173	20299	54003	44
17	84923	66639	80850	30765	20023	52767	33738	68387	43
18	94367	76601	91380	41919	31866	65374	47193	82787	42
19	1·5003821	86572	1·6201920	53085	43722	77994	60663	97204	41
20	13282	96552	12469	64261	55590	90628	74147	1·9911637	40
21	22751	1·5606542	23029	75449	67470	1·8303275	87647	26087	39
22	32229	16540	33599	86647	79362	15936	1·9101162	40554	38
23	41716	26548	44178	97856	91267	28610	14691	55038	37
24	51210	36564	54768	1·6909077	1·7603183	41297	28236	69539	36
25	60713	46590	65368	20308	15112	53999	41795	84056	35
26	70224	56625	75977	31550	27053	66713	55370	98590	34
27	79743	66669	86597	42804	39007	79442	68960	2·0013142	33
28	89271	76722	97227	54069	50972	92184	82565	27710	32
29	98807	86784	1·6307867	65344	62950	1·8404940	96186	42295	31
30	1·5108352	96856	18517	76631	74940	17709	1·9209821	56897	30
31	17905	1·5706936	29177	87929	86943	30492	23472	71516	29
32	27466	17020	39847	99238	98958	43289	37138	86153	28
33	37036	27126	50528	1·7010559	1·7710985	56099	50819	2·0100806	27
34	46614	37234	61218	21890	23024	68923	64516	15477	26
35	56201	47352	71919	33233	35076	81761	78228	30164	25
36	65796	57479	82630	44587	47141	94613	91956	44869	24
37	75400	67615	93351	55953	59218	1·8507479	1·9305699	59592	23
38	85012	77760	1·6404082	67329	71307	20358	19457	74331	22
39	94632	87915	14824	78717	83409	33252	33231	89088	21
40	1·5204261	98079	25576	90116	95524	46159	47020	2·0203862	20
41	13899	1·5808253	36338	1·7101527	1·7807651	59080	60825	18654	19
42	23545	18436	47111	12949	19790	72015	74645	33462	18
43	33200	28628	57893	24382	31943	84965	88481	48289	17
44	42863	38830	68687	35827	44107	97928	1·9402333	63133	16
45	52535	49041	79490	47283	56285	1·8610905	16200	77994	15
46	62215	59261	90304	58751	68475	23896	30083	92873	14
47	71904	69491	1·6501128	70230	80678	36902	43981	2·0107769	13
48	81602	79731	11963	81720	92893	49921	57896	22683	12
49	91308	89979	22808	93222	1·7905121	62955	71826	37615	11
50	1·5301023	1·5900238	33663	1·7204736	17362	76003	85772	52565	10
51	10746	10505	44529	16261	29616	89065	99733	67532	9
52	20479	20783	55405	27797	41883	1·8702141	1·9513711	82517	8
53	30219	31070	66292	39346	54162	15231	27704	97519	7
54	39969	41366	77189	50905	66454	28336	41713	2·0412540	6
55	49727	51672	88097	62477	78759	41455	55739	27578	5
56	59494	61987	99016	74060	91077	54588	69780	42634	4
57	69270	72312	1·6609945	85654	1·8003408	67736	83837	57708	3
58	79054	82647	20884	97260	15751	80898	97910	72800	2
59	88848	92991	31834	1·7308878	28108	94074	1·9612000	87910	1
60	98650	1·6003345	42795	20508	40478	1·8807265	26105	2·0503038	0
′	33°	32°	31°	30°	29°	28°	27°	26°	′

NAT. COTAN.

′	64°	65°	66°	67°	68°	69°	70°	71°	′
0	898 7940	906 3078	913 5455	920 5049	927 1839	933 5804	939 6926	945 5186	60
1	9215	4307	6637	6185	2928	6846	7921	6132	59
2	899 0489	5535	7819	7320	4016	7888	8914	7078	58
3	1763	6762	9001	8455	5104	8928	9907	8023	57
4	3035	7989	914 0181	9589	6191	9968	940 0899	8968	56
5	4307	9215	1361	921 0722	7277	934 1007	1891	9911	55
6	5578	907 0440	2540	1854	8363	2045	2881	946 0854	54
7	6848	1665	3718	2986	9447	3082	3871	1795	53
8	8117	2888	4895	4116	928 0531	4119	4860	2736	52
9	9386	4111	6072	5246	1614	5154	5848	3677	51
10	900 0654	5333	7247	6375	2696	6189	6835	4616	50
11	1921	6554	8422	7504	3778	7223	7822	5555	49
12	3188	7775	9597	8632	4858	8257	8808	6493	48
13	4453	8995	915 0770	9758	5938	9289	9793	7430	47
14	5718	908 0214	1943	922 0884	7017	935 0321	941 0777	8366	46
15	6982	1432	3115	2010	8096	1352	1760	9301	45
16	8246	2649	4286	3134	9173	2382	2743	947 0236	44
17	9508	3866	5456	4258	929 0250	3412	3724	1170	43
18	901 0770	5082	6626	5381	1326	4440	4705	2103	42
19	2031	6297	7795	6503	2401	5468	5686	3035	41
20	3292	7511	8963	7624	3475	6495	6665	3966	40
21	4551	8725	916 0130	8745	4549	7521	7644	4897	39
22	5810	9938	1297	9865	5622	8547	8621	5827	38
23	7068	909 1150	2462	923 0984	6694	9571	9598	6756	37
24	8325	2361	3627	2102	7765	936 0595	942 0575	7684	36
25	9582	3572	4791	3220	8835	1618	1550	8612	35
26	902 0838	4781	5955	4336	9905	2641	2525	9538	34
27	2092	5990	7118	5452	930 0974	3662	3498	948 0464	33
28	3347	7199	8279	6567	2042	4683	4471	1389	32
29	4600	8406	9440	7682	3109	5703	5444	2313	31
30	5853	9613	917 0601	8795	4176	6722	6415	3237	30
31	7105	910 0819	1760	9908	5241	7740	7386	4159	29
32	8356	2024	2919	924 1020	6306	8758	8355	5081	28
33	9606	3228	4077	2131	7370	9774	9324	6002	27
34	903 0856	4432	5234	3242	8434	937 0790	943 0293	6922	26
35	2105	5635	6391	4351	9496	1806	1260	7842	25
36	3353	6837	7546	5460	931 0558	2820	2227	8760	24
37	4600	8038	8701	6568	1619	3833	3192	9678	23
38	5847	9238	9855	7676	2679	4846	4157	949 0595	22
39	7093	911 0438	918 1009	8782	3739	5858	5122	1511	21
40	8338	1637	2161	9888	4797	6869	6085	2426	20
41	9582	2835	3313	925 0993	5855	7880	7048	3341	19
42	904 0825	4033	4464	2097	6912	8889	8010	4255	18
43	2068	5229	5614	3201	7969	9898	8971	5168	17
44	3310	6425	6763	4303	9024	938 0906	9931	6080	16
45	4551	7620	7912	5405	932 0079	1913	944 0890	6991	15
46	5792	8815	9060	6506	1133	2920	1849	7902	14
47	7032	912 0008	919 0207	7606	2186	3925	2807	8812	13
48	8271	1201	1353	8706	3238	4930	3764	9721	12
49	9509	2393	2499	9805	4290	5934	4720	950 0629	11
50	905 0746	3584	3644	926 0902	5340	6938	5675	1536	10
51	1983	4775	4788	2000	6390	7940	6630	2443	9
52	3219	5965	5931	3096	7439	8942	7584	3348	8
53	4454	7154	7073	4192	8488	9943	8537	4253	7
54	5688	8342	8215	5286	9535	939 0943	9489	5157	6
55	6922	9529	9356	6380	933 0582	1942	945 0441	6061	5
56	8154	913 0716	920 0496	7474	1628	2940	1391	6963	4
57	9386	1902	1635	8566	2673	3938	2341	7865	3
58	906 0618	3087	2774	9658	3718	4935	3290	8766	2
59	1848	4271	3912	927 0748	4761	5931	4238	9666	1
60	3078	5455	5049	1839	5804	6926	5186	951 0565	0
′	25°	24°	23°	22°	21°	20°	19°	18°	′

NAT. COSINE.

′	64°	65°	66°	67°	68°	69°	70°	71°	′
0	2·0503038	2·1445069	2·2460368	2·3558524	2·4750869	2·6050891	2·7474774	2·9042109	60
1	18185	61366	77962	77590	71612	73558	99661	69576	59
2	33349	77683	95580	96683	92386	96259	2·7524588	97089	58
3	48531	94021	2·2513221	2·3615801	2·4813190	2·6118995	49554	2·9124649	57
4	63732	2·1510378	30885	34946	34023	41766	74561	52256	56
5	78950	26757	48572	54118	54887	64571	99608	79909	55
6	94187	43156	66283	73316	75781	87411	2·7624695	2·9207610	54
7	2·0609442	59575	84016	92540	96706	2·6210286	49822	35358	53
8	24716	76015	2·2601773	2·3711791	2·4917660	33196	74990	63152	52
9	40008	92476	19554	31068	38645	56141	2·7700199	90995	51
10	55318	2·1608958	37357	50372	59661	79121	25448	2·9318885	50
11	70646	25460	55184	69703	80707	2·6302136	50738	46822	49
12	85994	41983	73035	89060	2·5001784	25186	76069	74807	48
13	2·0701359	58527	90909	2·3808444	22891	48271	2·7801440	2·9402840	47
14	16743	75091	2·2708807	27855	44029	71392	26853	30921	46
15	32146	91677	26729	47293	65198	94549	52307	59050	45
16	47567	2·1708283	44674	66758	86398	2·6417741	77802	87227	44
17	63007	24911	62643	86250	2·5107629	40969	2·7903339	2·9515453	43
18	78465	41559	80636	2·3905769	28890	64232	28917	43727	42
19	93942	58229	98653	25316	50183	87531	54537	72050	41
20	2·0809438	74920	2·2816693	44889	71507	2·6510867	80198	2·9600422	40
21	24953	91631	34758	64490	92863	34238	2·8005901	28842	39
22	40487	2·1808364	52846	84118	2·5214249	57645	31646	57312	38
23	56039	25119	70959	2·4003774	35667	81089	57433	85831	37
24	71610	41894	89096	23457	57117	2·6604569	83263	2·9714399	36
25	87200	58691	2·2907257	43168	78598	28085	2·8109164	43016	35
26	2·0902809	75510	25442	62906	2·5300111	51638	35048	71083	34
27	18437	92349	43651	82672	21655	75227	61004	2·9800400	33
28	34085	2·1909210	61885	2·4102465	43231	98853	87003	29167	32
29	49751	26093	80143	22296	64839	2·6722516	2·8213045	57983	31
30	65436	42997	98425	42136	86479	46215	39129	86850	30
31	81140	59923	2·3016732	62013	2·5408151	69951	65256	2·9915766	29
32	96864	76871	35064	81918	29855	93725	91426	44734	28
33	2·1012607	93840	53420	2·4201851	51591	2·6817535	2·8317639	73751	27
34	28369	2.2010831	71801	21812	73359	41383	43896	3·0002820	26
35	44150	27843	90206	41801	95160	65267	70196	31939	25
36	59951	44878	2·3108637	61819	2·5516992	89190	96539	61109	24
37	75771	61934	27092	81864	38858	2·6913149	2·8422926	90330	23
38	91611	79012	45571	2·4301938	60756	37147	49356	3·0119603	22
39	2·1107470	96112	64076	22041	82686	61181	75831	48926	21
40	23348	2·2113234	82606	42172	2·5604649	85254	2·8502349	78301	20
41	39246	30379	2·3201160	62331	26645	2·7009364	28911	3·0207728	19
42	55164	47545	19740	82519	48674	33513	55517	37207	18
43	71101	64733	38345	2·4402736	70735	57699	82168	66737	17
44	87057	81944	56975	22982	92830	81923	2·8608863	96320	16
45	2·1203034	99177	75630	43256	2·5714957	2·7106186	35602	3·0325954	15
46	19030	2·2216432	94311	63559	37118	30487	62386	55641	14
47	35046	33709	2·3313017	83891	59312	54826	89215	85381	13
48	51082	51009	31748	2·4504252	81539	79204	2·8716088	3·0415173	12
49	67137	68331	50505	24642	2·5803800	2·7203620	43007	45018	11
50	83213	85676	69287	45061	26094	28076	69970	74915	10
51	99308	2·2303043	88095	65510	48421	52569	96979	3·0504866	9
52	2·1315423	20433	2·3406928	85987	70782	77102	2·8824033	34870	8
53	31559	37845	25787	2·4606494	93177	2·7301674	51132	64928	7
54	47714	55280	44672	27030	2·5915606	26284	78277	95038	6
55	63890	72738	63582	47596	38068	50934	2·8905467	3·0625203	5
56	80085	90218	82519	68191	60564	75623	32704	55421	4
57	96301	2·2407721	2·3501481	88816	83095	2·7400352	59986	85694	3
58	2·1412537	25247	20469	2·4709470	2·6005659	25120	87314	3·0716020	2
59	28793	42796	39483	30155	28258	49927	2·9014688	46400	1
60	45069	60368	58524	50869	50891	74774	42109	76835	0
′	25°	24°	23°	22°	21°	20°	19°	18°	′

NAT. COTAN.

′	72°	73°	74°	75°	76°	77°	78°	79°	′
0	951 0565	956 3048	961 2617	965 9258	970 2957	974 3701	978 1476	981 6272	60
1	1464	3898	3418	966 0011	3661	4355	2080	6826	59
2	2361	4747	4219	0762	4363	5008	2684	7380	58
3	3258	5595	5019	1513	5065	5660	3287	7933	57
4	4154	6443	5818	2263	5766	6311	3889	8485	56
5	5050	7290	6616	3012	6466	6962	4490	9037	55
6	5944	8136	7413	3761	7165	7612	5090	9587	54
7	6838	8981	8210	4508	7863	8261	5689	982 0137	53
8	7731	9825	9005	5255	8561	8909	6288	0686	52
9	8623	957 0669	9800	6001	9258	9556	6886	1234	51
10	9514	1512	962 0594	6746	9953	975 0203	7483	1781	50
11	952 0404	2354	1387	7490	971 0649	0849	8079	2327	49
12	1294	3195	2180	8234	1343	1494	8674	2873	48
13	2183	4035	2972	8977	2036	2138	9268	3417	47
14	3071	4875	3762	9718	2729	2781	9862	3961	46
15	3958	5714	4552	967 0459	3421	3423	979 0455	4504	45
16	4844	6552	5342	1200	4112	4065	1047	5046	44
17	5730	7389	6130	1939	4802	4706	1638	5587	43
18	6615	8225	6917	2678	5491	5345	2228	6128	42
19	7499	9060	7704	3415	6180	5985	2818	6668	41
20	8382	9895	8490	4152	6867	6623	3406	7206	40
21	9264	958 0729	9275	4888	7554	7260	3994	7744	39
22	953 0146	1562	963 0060	5624	8240	7897	4581	8282	38
23	1027	2394	0843	6358	8926	8533	5167	8818	37
24	1907	3226	1626	7092	9610	9168	5752	9353	36
25	2786	4056	2408	7825	972 0294	9802	6337	9888	35
26	3664	4886	3189	8557	0976	976 0435	6921	983 0422	34
27	4542	5715	3969	9288	1658	1068	7504	0955	33
28	5418	6543	4748	968 0018	2339	1699	8086	1487	32
29	6294	7371	5527	0748	3020	2330	8668	2019	31
30	7170	8197	6305	1476	3699	2960	9247	2549	30
31	8044	9023	7081	2204	4378	3589	9827	3079	29
32	8917	9848	7858	2931	5056	4218	980 0405	3608	28
33	9790	959 0672	8633	3658	5733	4845	0983	4136	27
34	954 0662	1496	9407	4383	6409	5472	1560	4663	26
35	1533	2318	964 0181	5108	7084	6098	2136	5189	25
36	2403	3140	0954	5832	7759	6723	2712	5715	24
37	3273	3961	1726	6555	8432	7347	3286	6239	23
38	4141	4781	2497	7277	9105	7970	3860	6763	22
39	5009	5600	3268	7998	9777	8593	4433	7286	21
40	5876	6418	4037	8719	973 0449	9215	5005	7808	20
41	6743	7236	4806	9438	1119	9836	5576	8330	19
42	7608	8053	5574	969 0157	1789	977 0456	6147	8850	18
43	8473	8869	6341	0875	2458	1075	6716	9370	17
44	9336	9684	7108	1593	3125	1693	7285	9889	16
45	955 0199	960 0499	7873	2309	3793	2311	7853	984 0407	15
46	1062	1312	8638	3025	4459	2928	8420	0924	14
47	1923	2125	9402	3740	5124	3544	8986	1441	13
48	2784	2937	965 0165	4453	5789	4159	9552	1956	12
49	3643	3748	0927	5167	6453	4773	981 0116	2471	11
50	4502	4558	1689	5879	7116	5387	0680	2985	10
51	5361	5368	2449	6591	7778	5999	1243	3498	9
52	6218	6177	3209	7301	8439	6611	1805	4010	8
53	7074	6984	3968	8011	9100	7222	2366	4521	7
54	7930	7792	4726	8720	9760	7832	2927	5032	6
55	8785	8598	5484	9428	974 0419	8442	3486	5542	5
56	9639	9403	6240	970 0136	1077	9050	4045	6050	4
57	956 0492	961 0208	6996	0842	1734	9658	4603	6558	3
58	1345	1012	7751	1548	2390	978 0265	5160	7066	2
59	2197	1815	8505	2253	3046	0871	5716	7572	1
60	3048	2617	9258	2957	3701	1476	6272	8078	0
′	17°	16°	15°	14°	13°	12°	11°	10°	′

NAT. COSINE.

′	72°	73°	74°	75°	76°	77°	78°	′
0	3·0776835	3·2708526	3·4874144	3·7320508	4·0107809	4·3314759	4·7046301	60
1	3·0807325	42588	3·4912470	63980	57570	72316	4·7113686	59
2	37869	76715	50874	3·7407546	4·0207446	4·3430018	81256	58
3	68468	3·2810907	89356	51207	57440	87866	4·7249012	57
4	99122	45164	3·5027916	94963	4·0307550	4·3545861	4·7316954	56
5	3·0929831	79487	66555	3·7538815	57779	4·3604003	85083	55
6	60596	3·2913876	3·5105273	82763	4·0408125	62293	4·7453401	54
7	91416	48330	44070	3·7626807	58590	4·3720731	4·7521907	53
8	3·1022291	82851	82946	70947	4·0509174	79317	90603	52
9	53223	3·3017438	3·5221902	3·7715185	59877	4·3838054	4·7659410	51
10	84210	52091	60938	59519	4·0610700	96940	4·7728568	50
11	3·1115254	86811	3·5300054	3·7803951	61643	4·3955 77	97837	49
12	46353	3·3121598	39251	48481	4·0712707	4·4015164	4·7867300	48
13	77509	56452	78528	93109	63892	74504	4·7936957	47
14	3·1208722	91373	3·5417886	3·7937835	4·0815199	4·4133996	4·8006808	46
15	39991	3·3226362	57325	82661	66627	93641	76854	45
16	71317	61419	96846	3·8027585	4·0918178	4·4253439	4·8147096	44
17	3·1302701	96543	3·5526449	72609	69852	4·4313392	4·8217536	43
18	34141	3·3331736	76133	3·8117733	4·1021649	73500	88174	42
19	65639	66997	3·5615900	62957	73569	4·4433762	4·8359010	41
20	97194	3·3402326	55749	3·8208281	4·1125614	94181	4·8430045	40
21	3·1428807	37724	95681	53707	77784	4·4554756	4·8501282	39
22	60478	73191	3·5735696	99233	4·1230079	4·4615489	72719	38
23	92207	3·3508728	75794	3·8344861	82499	76379	4·8644359	37
24	3·1523994	44333	3·5815975	90591	4·1335046	4·4737428	4·8716201	36
25	55840	80008	56241	3·8436424	87719	98636	88248	35
26	87744	3·3615753	96590	82358	4·1440519	4·4860004	4·8860499	34
27	3·1619706	51568	3·5937024	3·8528396	93446	4·4921532	4·8932956	33
28	51728	87453	77543	74537	4·1546501	83221	4·9005620	32
29	83808	3·3723408	3·6018146	3·8620782	99665	4·5045072	78491	31
30	3·1715948	59434	58835	67131	4·1652998	4·5107085	4·9151570	30
31	48147	95531	99609	3·8713584	4·1706440	69261	4·9224859	29
32	80406	3·3831699	3·6140469	60142	60011	4·5231601	98358	28
33	3·1812724	67938	81415	3·8806805	4·1813713	94105	4·9372068	27
34	45102	3·3904249	3·6222447	53574	67546	4·5356773	4·9445990	26
35	77540	40631	63566	3·8900448	4·1921510	4·5419608	4·9520125	25
36	3·1910039	77085	3·6304771	47429	75606	82608	94474	24
37	42598	3·4013612	46064	94516	4·2029835	4·5545776	4·9669037	23
38	75217	50210	87444	3·9041710	84196	4·5609111	4·9743817	22
39	3·2007897	86882	3·6428911	89011	4·2138690	72615	4·9818813	21
40	40638	3·4123626	70467	3·9136420	93318	4·5736287	94027	20
41	73440	60443	3·6512111	83937	4·2248080	4·5800129	4·9969459	19
42	3·2106304	97333	53844	3·9231563	4·2302977	64141	5·0045111	18
43	39228	3·4234297	95665	79297	58009	4·5928325	5·0120984	17
44	72215	71334	3·6637575	3·9327141	4·2413177	92680	97078	16
45	3·2205263	3·4308446	79575	75094	68482	4·6057207	5·0273395	15
46	38373	45631	3·6721665	3·9423157	4·2523923	4·6121908	5·0349935	14
47	71546	82891	63845	71331	79501	86783	5·0426700	13
48	3·2304780	3·4420226	3·6806115	3·9519615	4·2635218	4·6251832	5·0503690	12
49	38078	57635	48475	68011	91072	4·6317056	80907	11
50	71438	95120	90927	3·9616518	4·2747066	82457	5·0658352	10
51	3·2404860	3·4532679	3·6933469	65137	4·2803199	4·6448034	5·0736025	9
52	38346	70315	76104	3·9713868	59472	4·6513788	5·0813928	8
53	71895	3·4608026	3·7018830	62712	4·2915885	79721	92061	7
54	3·2505508	45813	61648	3·9811669	72440	4·6645832	5·0970426	6
55	39184	83676	3·7104558	60739	4·3029136	4·6712124	5·1049024	5
56	72924	3·4721616	47561	3·9909924	85974	78595	5·1127855	4
57	3·2606728	59632	90658	59223	4·3142955	4·6845248	5·1206921	3
58	40596	97726	3·7233847	4·0008636	4·3200079	4·6912083	86224	2
59	74529	3·4835896	77131	58165	57347	79100	5·1365763	1
60	3·2708526	74144	3·7320508	4·0107809	4·3314759	4·7046301	5·1445540	0
′	17°	16°	15°	14°	13°	12°	11°	′

NAT. COTAN.

′	80°	81°	82°	83°	84°	85°	86°	87°	′
0	9848 078	9876 883	9902 681	9925 462	9945 219	9961 947	9975 641	9986 295	60
1	582	9877 338	9903 085	816	523	9962 200	843	447	59
2	9849 086	792	489	9926 169	825	452	9976 045	598	58
3	589	9878 245	891	521	9946 127	704	245	748	57
4	9850 091	697	9904 293	873	428	954	445	898	56
5	593	9879 148	694	9927 224	729	9963 204	645	9987 046	55
6	9851 093	599	9905 095	573	9947 028	453	843	194	54
7	593	9880 048	494	922	327	701	9977 040	340	53
8	9852 092	497	893	9928 271	625	948	237	486	52
9	590	945	9906 290	618	921	9964 195	433	631	51
10	9853 087	9881 392	687	965	9948 217	440	627	775	50
11	583	838	9907 083	9929 310	513	685	821	919	49
12	9854 079	9882 284	478	655	807	929	9978 015	9988 061	48
13	574	728	873	999	9949 101	9965 172	207	203	47
14	9855 068	9883 172	9908 266	9930 342	393	414	399	344	46
15	561	615	659	685	685	655	589	484	45
16	9856 053	9884 057	9909 051	9931 026	976	895	779	623	44
17	544	498	442	367	9950 266	9966 135	968	761	43
18	9857 035	939	832	706	556	374	9979 156	899	42
19	524	9885 378	9910 221	9932 045	844	612	343	9989 035	41
20	9858 013	817	610	384	9951 132	849	530	171	40
21	501	9886 255	997	721	419	9967 085	716	306	39
22	988	692	9911 384	9933 057	705	321	900	440	38
23	9859 475	9887 128	770	393	990	555	9980 084	573	37
24	960	564	9912 155	728	9952 274	789	267	706	36
25	9860 445	998	540	9934 062	557	9968 022	450	837	35
26	929	9888 432	923	395	840	254	631	968	34
27	9861 412	865	9913 306	727	9953 122	485	811	9990 098	33
28	894	9889 297	688	9935 058	403	715	991	227	32
29	9862 375	728	9914 069	389	683	945	9981 170	355	31
30	856	9890 159	449	719	962	9969 173	348	482	30
31	9863 336	588	828	9936 047	9954 240	401	525	609	29
32	815	9891 017	9915 206	375	518	628	701	734	28
33	9864 293	445	584	703	795	854	877	859	27
34	770	872	961	9937 029	9955 070	9970 080	9982 052	983	26
35	9865 246	9892 298	9916 337	355	345	304	225	9991 106	25
36	722	723	712	679	620	528	398	228	24
37	9866 196	9893 148	9917 086	9938 003	893	750	570	350	23
38	670	572	459	326	9956 165	972	742	470	22
39	9867 143	994	832	648	437	9971 193	912	590	21
40	615	9894 416	9918 204	969	708	413	9983 082	709	20
41	9868 087	838	574	9939 290	978	633	250	827	19
42	557	9895 258	944	610	9957 247	851	418	944	18
43	9869 027	677	9919 314	928	515	9972 069	585	9992 060	17
44	496	9896 096	682	9940 246	783	286	751	176	16
45	964	514	9920 049	563	9958 049	502	917	290	15
46	9870 431	931	416	880	315	717	9984 081	404	14
47	897	9897 347	782	9941 195	580	931	245	517	13
48	9871 363	762	9921 147	510	844	9973 145	408	629	12
49	827	9898 177	511	823	9959 107	357	570	740	11
50	9872 291	590	874	9942 136	370	569	731	851	10
51	754	9899 003	9922 237	448	631	780	891	960	9
52	9873 216	415	599	760	892	990	9985 050	9993 069	8
53	678	826	959	9943 070	9960 152	9974 199	209	177	7
54	9874 138	9900 237	9923 319	379	411	408	367	284	6
55	598	646	679	688	669	615	524	390	5
56	9875 057	9901 055	9924 037	996	926	822	680	495	4
57	514	462	394	9944 303	9961 183	9975 028	835	600	3
58	972	869	751	609	438	233	989	704	2
59	9876 428	9902 275	9925 107	914	693	437	9986 143	806	1
60	883	681	462	9945 219	947	641	295	908	0
′	9°	8°	7°	6°	5°	4°	3°	2°	′

NAT. COSINE.

′	79°	80°	81°	82°	83°	84°	85°	′
0	5·1445540	5·6712818	6·3137515	7·1153697	8·1443464	9·5143645	11·430052	60
1	525557	809446	256601	304190	639786	410613	468474	59
2	605813	906394	376126	455308	837041	679068	507154	58
3	686311	5·7003663	496092	607056	8·2035239	949022	546093	57
4	767051	101256	616502	759437	234384	9·6220486	585294	56
5	848035	199173	737359	912456	434485	493475	624761	55
6	929264	297416	858665	7·2066116	635547	768000	664495	54
7	5·2010738	395988	980422	220422	837579	9·7044075	704500	53
8	092459	494889	6·4102633	375378	8·3040586	321713	744779	52
9	174428	594122	225301	530987	244577	600927	785333	51
10	256647	693688	348428	687255	449558	881732	826167	50
11	339116	793588	472017	844184	655536	9·8164140	867282	49
12	421836	893825	596070	7·3001780	862519	448166	908682	48
13	504809	994400	720591	160047	8·4070515	733823	950370	47
14	588035	5·8095315	845581	318989	279531	9·9021125	992349	46
15	671517	196572	971043	478610	489573	310088	12·034622	45
16	755255	298172	6·5096981	638916	700651	600724	077192	44
17	839251	400117	223396	799909	912772	893050	120062	43
18	923505	502410	350293	961595	8·5125943	10·018708	163236	42
19	5·3008018	605051	477672	7·4123978	340172	048283	206716	41
20	092793	708042	605538	297064	555468	078031	250505	40
21	177830	811386	733892	450855	771838	107954	294609	39
22	263131	915084	862739	615357	989290	138054	339028	38
23	348696	5·9019138	992080	780576	8·6207833	168332	383768	37
24	434527	123550	6·6121919	946514	427475	198789	428831	36
25	520626	228322	252258	7·5113178	648223	229428	474221	35
26	606993	333455	383100	280571	870088	260249	519942	34
27	693630	438952	514449	448699	8·7093077	291255	565997	33
28	780538	544815	646307	617567	317198	322447	612390	32
29	867718	651045	778677	787179	542461	353827	659125	31
30	955172	757644	911562	957541	768874	385397	706205	30
31	5·4042901	864614	6·7044966	7·6128657	996446	417158	753634	29
32	130906	971957	178891	300533	8·8225186	449112	801417	28
33	219188	6·0079676	313341	473174	455103	481261	849557	27
34	307750	187772	448318	646584	686206	513607	898058	26
35	396592	296247	583826	820769	918505	546151	946924	25
36	485715	405103	719867	995735	8·9152009	578895	996160	24
37	575121	514343	856446	7·7171486	386726	611841	13·045769	23
38	664812	623967	993565	348028	622668	644992	095757	22
39	754788	733979	6·8131227	525366	859843	678348	146127	21
40	845052	844381	269437	703506	9·0098261	711913	196883	20
41	935604	955174	408196	882453	337933	745687	248031	19
42	5·5026446	6·1066360	547508	7·8062212	578867	779673	299574	18
43	117579	177943	687378	242790	821074	813872	351518	17
44	209005	289923	827807	424191	9·1064564	848288	403867	16
45	300724	402303	968799	606423	309348	882921	456625	15
46	392740	515085	6·9110359	789489	555436	917775	599799	14
47	485052	628272	252489	973396	802838	952850	563391	13
48	577663	741865	395192	7·9158151	9·2051564	988150	617409	12
49	670574	855867	538473	343758	301627	11·023676	671856	11
50	763786	970279	682335	530224	553035	059431	726738	10
51	857302	6·2085106	826781	717555	805802	095416	782060	9
52	951121	200347	971806	905756	9·3059936	131635	837827	8
53	5·6045247	316007	7·0117441	8·0094835	315450	168089	894045	7
54	139680	432086	263662	284796	572355	204780	950719	6
55	234421	548588	410482	475647	830663	241712	14·007856	5
56	329474	665515	557905	667394	9·4090384	278885	065459	4
57	424838	782868	705934	860042	351531	316304	123536	3
58	520516	900651	854573	8·1053599	614116	353970	182092	2
59	616509	6·3018866	7·1003826	248071	878149	391885	241134	1
60	712818	137515	153697	443464	9·5143645	430052	300666	0
′	10°	9°	8°	7°	6°	5°	4°	′

NAT. COTAN.

′	88°	89°	′
0	9993 908	9998 477	60
1	9994 009	527	59
2	110	577	58
3	209	625	57
4	308	673	56
5	405	720	55
6	502	766	54
7	598	812	53
8	693	856	52
9	788	900	51
10	881	942	50
11	974	984	49
12	9995 066	9999 025	48
13	157	065	47
14	247	105	46
15	336	143	45
16	424	181	44
17	512	218	43
18	599	254	42
19	684	289	41
20	770	323	40
21	854	357	39
22	937	389	38
23	9996 020	421	37
24	101	452	36
25	182	482	35
26	262	511	34
27	341	539	33
28	419	567	32
29	497	593	31
30	573	619	30
31	649	644	29
32	724	668	28
33	798	602	27
34	871	714	26
35	943	736	25
36	9997 015	756	24
37	086	776	23
38	156	795	22
39	224	813	21
40	292	831	20
41	360	847	19
42	426	863	18
43	492	878	17
44	556	892	16
45	620	905	15
46	683	917	14
47	745	928	13
48	807	939	12
49	867	949	11
50	927	958	10
51	986	966	9
52	9998 044	973	8
53	101	979	7
54	157	985	6
55	213	989	5
56	267	993	4
57	321	996	3
58	374	998	2
59	426	1·0000 000	1
60	477	000	0
′	1°	0°	′

NAT. COSINE.

′	86°	87°	88°	89°	′
0	14·300666	19·081137	28·636253	57·289962	60
1	360696	187930	877089	58·261174	59
2	421230	295922	29·122005	59·265872	58
3	482273	405133	371106	60·305820	57
4	543833	515584	624499	61·382905	56
5	605916	627296	882299	62·499154	55
6	668529	740291	30·144619	63·656741	54
7	731679	854591	411580	64·858008	53
8	795372	970219	683307	66·105473	52
9	859616	20·087199	959928	67·401854	51
10	924417	205553	31·241577	68·750087	50
11	989784	325308	528392	70·153346	49
12	15·055723	446486	820516	71·615070	48
13	122242	569115	32·118099	73·138991	47
14	189349	693220	421295	74·729165	46
15	257052	818828	730264	76·390009	45
16	325358	945966	33·045173	78·126342	44
17	394276	21·074664	366194	79·943430	43
18	463814	204949	693509	81·847041	42
19	533981	336851	34·027303	83·843507	41
20	604784	470401	367771	85·939791	40
21	676233	605630	715115	88·143572	39
22	748337	742569	35·069546	90·463336	38
23	821105	881251	431282	92·908487	37
24	894545	22·021710	800553	95·489475	36
25	968667	163980	36·177596	98·217943	35
26	16·043482	308097	562659	101·10690	34
27	118998	454096	956001	104·17094	33
28	195225	602015	37·357892	107·42648	32
29	272174	751892	768613	110·89205	31
30	349855	903766	38·188459	114·58865	30
31	428279	23·057677	617738	118·54018	29
32	507456	213666	39·056771	122·77396	28
33	587396	371777	505895	127·32134	27
34	668112	532052	965460	132·21851	26
35	749614	694537	40·435837	137·50745	25
36	831915	859277	917412	143·23712	24
37	915025	24·026320	41·410588	149·46502	23
38	998957	195714	915790	156·25908	22
39	17·083724	367509	42·433464	163·70019	21
40	169337	541758	964077	171·88540	20
41	255809	718512	43·508122	180·93220	19
42	343155	897826	44·066113	190·98419	18
43	431385	25·079757	638596	202·21875	17
44	520516	264361	45·226141	214·85762	16
45	610559	451700	829351	229·18166	15
46	701529	641832	46·448862	245·55198	14
47	793442	834823	47·085343	264·44080	13
48	886310	26·030736	739501	286·47773	12
49	980150	229638	48·412084	312·52137	11
50	18·074977	431600	49·103881	343·77371	10
51	170807	636690	815726	381·97099	9
52	267654	844984	50·548506	429·71757	8
53	365537	27·056557	51·303157	491·10600	7
54	464471	271486	52·080673	572·95721	6
55	564473	489853	882109	687·54887	5
56	665562	711740	53·708587	859·43630	4
57	767754	937233	54·561300	1145·9153	3
58	871068	28·166422	55·441517	1718·8732	2
59	975523	399397	56·350590	3437·7467	1
60	19·081137	636253	57·289962	Infinite.	0
′	3°	2°	1°	0°	′

NAT. COTAN.

TRAVERSE TABLE

TO EVERY QUARTER POINT OF THE COMPASS.

Dif. of lat. and dep. for ¼ Point. | *Dif. of lat. and dep. for ½ Point.*

dist.	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*
1	01·0	00·0	61	60·9	03·0	1	01·0	00·1	61	60·7	06·0
2	02·0	00·1	62	61·9	03·0	2	02·0	00·2	62	61·7	06·1
3	03·0	00·1	63	62·9	03·1	3	03·0	00·3	63	62·7	06·2
4	04·0	00·2	64	63·9	03·1	4	04·0	00·4	64	63·7	06·3
5	05·0	00·2	65	64·9	03·2	5	05·0	00·5	65	64·7	06·4
6	06·0	00·3	66	65·9	03·2	6	06·0	00·6	66	65·7	06·5
7	07·0	00·3	67	66·9	03·3	7	07·0	00·7	67	66·7	06·6
8	08·0	00·4	68	67·9	03·3	8	08·0	00·8	68	67·7	06·7
9	09·0	00·4	69	68·9	03·4	9	09·0	00·9	69	68·7	06·8
10	10·0	00·5	70	69·9	03·4	10	10·0	01·0	70	69·7	06·9
11	11·0	00·5	71	70·9	03·5	11	10·9	01·1	71	70·7	07·0
12	12·0	00·6	72	71·9	03·5	12	11·9	01·2	72	71·7	07·1
13	13·0	00·6	73	72·9	03·6	13	12·6	01·3	73	72·7	07·2
14	14·0	00·7	74	73·9	03·6	14	13·9	01·4	74	73·6	07·3
15	15·0	00·7	75	74·9	03·7	15	14·9	01·5	75	74·6	07·4
16	16·0	00·8	76	75·9	03·7	16	15·9	01·6	76	75·6	07·4
17	17·0	00·8	77	76·9	03·8	17	16·9	01·7	77	76·6	07·5
18	18·0	00·9	78	77·9	03·8	18	17·9	01·8	78	77·6	07·6
19	19·0	00·9	79	78·9	03·9	19	18·9	01·9	79	78·6	07·7
20	20·0	01·0	80	79·9	03·9	20	19·9	02·0	80	79·6	07·8
21	21·0	01·0	81	80·9	04·0	21	20·9	02·1	81	80·6	07·9
22	22·0	01·1	82	81·9	04·0	22	21·9	02·2	82	81·6	08·0
23	23·0	01·1	83	82·9	04·1	23	22·9	02·3	83	82·6	08·1
24	24·0	01·2	84	83·9	04·1	24	23·9	02·4	84	83·6	08·2
25	25·0	01·2	85	84·9	04·2	25	24·9	02·4	85	84·6	08·3
26	26·0	01·3	86	85·9	04·2	26	25·9	02·5	86	85·6	08·4
27	27·0	01·3	87	86·9	04·3	27	26·9	02·6	87	86·6	08·5
28	28·0	01·4	88	87·9	04·3	28	27·9	02·7	88	87·6	08·6
29	29·0	01·4	89	88·9	04·4	29	28·9	02·8	89	88·6	08·7
30	30·0	01·5	90	89·9	04·4	30	29·9	02·9	90	89·6	08·8
31	31·0	01·5	91	90·9	04·5	31	30·9	03·0	91	90·6	08·9
32	32·0	01·6	92	91·9	04·5	32	31·8	03·1	92	91·6	09·0
33	33·0	01·6	93	92·9	04·6	33	32·8	03·2	93	92·6	09·1
34	34·0	01·7	94	93·9	04·6	34	33·8	03·3	94	93·6	09·2
35	35·0	01·7	95	94·9	04·7	35	34·8	03·4	95	94·5	09·3
36	36·0	01·8	96	95·9	04·7	36	35·8	03·5	96	95·5	09·4
37	37·0	01·8	97	96·9	04·8	37	36·8	03·6	97	96·5	09·5
38	38·0	01·9	98	97·9	04·8	38	37·8	03·7	98	97·5	09·6
39	39·0	01·9	99	98·9	04·9	39	38·8	03·8	99	98·5	09·7
40	40·0	02·0	100	99·9	04·9	40	39·8	03·9	100	99·5	09·8
41	41·0	02·0	101	100·9	05·0	41	40·8	04·0	101	100·5	09·9
42	41·9	02·1	102	101·9	05·0	42	41·8	04·1	102	101·5	10·0
43	42·9	02·1	103	102·9	05·1	43	42·8	04·2	103	102·5	10·1
44	43·9	02·2	104	103·9	05·1	44	43·8	04·3	104	103·5	10·2
45	44·9	02·2	105	104·9	05·2	45	44·8	04·4	105	104·5	10·3
46	45·9	02·3	106	105·9	05·2	46	45·8	04·5	106	105·5	10·4
47	46·9	02·3	107	106·9	05·3	47	46·8	04·6	107	106·5	10·5
48	47·9	02·4	108	107·9	05·3	48	47·8	04·7	108	107·5	10·6
49	48·9	02·4	109	108·9	05·4	49	48·8	04·8	109	108·5	10·7
50	49·9	02·5	110	109·9	05·4	50	49·8	04·9	110	109·5	10·8
51	50·9	02·5	111	110·9	05·5	51	50·8	05·0	111	110·5	10·9
52	51·9	02·6	112	111·9	05·5	52	51·7	05·1	112	111·5	11·0
53	52·9	02·6	113	112·9	05·5	53	52·7	05·2	113	112·5	11·1
54	53·9	02·7	114	113·9	05·6	54	53·7	05·3	114	113·5	11·2
55	54·9	02·7	115	114·9	05·6	55	54·7	05·4	115	114·5	11·3
56	55·9	02·8	116	115·9	05·7	56	55·7	05·5	116	115·4	11·4
57	56·9	02·8	117	116·9	05·7	57	56·7	05·6	117	116·4	11·5
58	57·9	02·9	118	117·9	05·8	58	57·7	05·7	118	117·4	11·6
59	58·9	02·9	119	118·9	05·8	59	58·7	05·8	119	118·4	11·7
60	59·9	02·9	120	119·9	05·9	60	59·7	05·9	120	119·4	11·8
dist.	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*

For 7¾ Points. | For 7½ Points

dist.	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*
1	01·0	00·1	61	60·3	08·9	1	01·0	00·2	61	59·8	11·9
2	02·0	00·3	62	61·3	09·1	2	02·0	00·4	62	60·8	12·1
3	03·0	00·4	63	62·3	09·2	3	02·9	00·6	63	61·8	12·3
4	04·0	00·6	64	63·3	09·4	4	03·9	00·8	64	62·8	12·5
5	04·9	00 7	65	64·3	09·5	5	04·9	01·0	65	63·7	12·7
6	05·9	00·9	66	65·3	09·7	6	05·9	01·2	66	64·7	12·9
7	06·9	01·0	67	66·3	09·8	7	06·9	01·4	67	65·7	13·1
8	07·9	01·2	68	67·3	10·0	8	07·8	01·6	68	66·7	13·3
9	08·9	01·3	69	68·2	10·1	9	08·8	01·8	69	67·7	13·5
10	09·9	01·5	70	69·2	10·3	10	09·8	02·0	70	68·7	13·7
11	10·9	01·6	71	70·2	10·4	11	10·8	02·1	71	69·6	13·9
12	11·9	01·8	72	71·2	10·6	12	11·8	02·3	72	70·6	14·0
13	12·9	01·9	73	72·2	10·7	13	12·7	02·5	73	71·6	14·2
14	13·8	02·1	74	73·2	10·9	14	13·7	02·7	74	72·6	14·4
15	14·8	02·2	75	74·2	11·0	15	14·7	02·9	75	73·6	14·6
16	15·8	02·3	76	75·2	11·1	16	15·7	03·1	76	74·5	14·8
17	16·8	02·5	77	76·2	11·3	17	16·7	03·3	77	75·5	15·0
18	17·8	02·6	78	77·2	11·4	18	17·7	03·5	78	76·5	15·2
19	18·8	02·8	79	78·1	11·6	19	18·6	03·7	79	77·5	15·4
20	19·8	02·9	80	79·1	11·7	20	19·6	03·9	80	78·5	15·6
21	20·8	03·1	81	80·1	11·9	21	20·6	04·1	81	79·4	15·8
22	21·8	03·2	82	81·1	12·0	22	21·6	04·3	82	80·4	16·0
23	22·7	03·4	83	82·1	12·2	23	22·6	04·5	83	81·4	16·2
24	23·7	03·5	84	83·1	12·3	24	23·5	04·7	84	82·4	16·4
25	24·7	03·7	85	84·1	12·5	25	24·5	04·9	85	83·4	16·6
26	25·7	03·8	86	85·1	12·6	26	25·5	05·1	86	84·3	16·8
27	26·7	04·0	87	86·1	12·8	27	26·5	05·3	87	85·3	17·0
28	27·7	04·1	88	87·0	12·9	28	27·5	05·5	88	86·3	17·2
29	28·7	04·3	89	88·0	13·1	29	28·4	05·7	89	87·3	17·4
30	29·7	04·4	90	89·0	13·2	30	29·4	05·9	90	88·3	17·6
31	30·7	04·5	91	90·0	13·3	31	30·4	06·0	91	89·2	17·8
32	31·7	04·7	92	91·0	13·5	32	31·4	06·2	92	90·2	18·0
33	32·6	04·8	93	92·0	13·6	33	32·4	06·4	93	91·2	18·1
34	33·6	05·0	94	93·0	13·8	34	33·3	06·6	94	92·2	18·3
35	34·6	05·1	95	94·0	13·9	35	34·3	06·8	95	93·2	18·5
36	35·6	05·3	96	95·0	14·1	36	35·3	07·0	96	94·2	18·7
37	36·6	05·4	97	95·9	14·2	37	36·3	07·2	97	95·1	18·9
38	37·6	05·6	98	96·9	14·4	38	37·3	07·4	98	96·1	19·1
39	38·6	05·7	99	97·9	14·5	39	38·2	07·6	90	97·1	19·3
40	39·6	05·9	100	98·9	14·7	40	39·2	07·8	100	98·1	19·5
41	40·6	06·0	101	99·9	14·8	41	40·2	08·0	101	99·1	19·7
42	41·5	06·2	102	100·9	15·0	42	41·2	08·2	102	100·0	19·9
43	42·5	06·3	103	101·9	15·1	43	42·2	08·4	103	101·0	20·1
44	43·5	06·5	104	102·9	15·3	44	43·2	08·6	104	102·0	20·3
45	44·5	06·6	105	103·9	15·4	45	44·1	08·8	105	103·0	20·5
46	45·5	06·7	106	104·8	15·5	46	45·1	09·0	106	104·0	20·7
47	46·5	06·9	107	105·8	15·7	47	46·1	09·2	107	104·9	20·9
48	47·5	07·0	108	106·8	15·8	48	47·1	09·4	108	105·9	21·1
49	48·5	07·2	109	107·8	16·0	49	48·1	09·6	109	106·9	21·3
50	49·5	07·3	110	108·8	16·1	50	49·0	09·8	110	107·9	21·5
51	50·4	07·5	111	109·8	16·3	51	50·0	10·0	111	108·9	21·7
52	51·4	07·6	112	110·8	16·4	52	51·0	10·1	112	109·8	21·9
53	52·4	07·8	113	111·8	16·6	53	52·0	10·3	113	110·8	22·0
54	53·4	07·9	114	112·8	16·7	54	53·0	10·5	114	111·8	22·2
55	54·4	08·1	115	113·7	16·9	55	53·9	10·7	115	112·8	22·4
56	55·4	08·2	116	114·7	17·0	56	54·9	10·9	116	113·8	22·6
57	56·4	08·4	117	115·7	17·2	57	55·9	11·1	117	114·7	22·8
58	57·4	08·5	118	116·7	17·3	58	56·9	11·3	118	115·7	23·0
59	58·4	08·7	119	117·4	17·5	59	57·9	11·5	119	116·7	23·2
60	59·3	08·8	120	118·7	17·6	60	58·8	11·7	120	117·7	23·4
dist.	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*

For 7¼ Points. For 7 Points.

Dif. of lat. & dep. for 1¼ Point. | *Dif. of lat. & dep. for 1½ Point.*

dist.	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*
1	01·0	00·2	61	59·2	14·8	1	01·0	00·3	61	58·4	17·7
2	01·9	00·5	62	60·1	15·1	2	01·9	00·6	62	59·3	18·0
3	02·9	00·7	63	61·1	15·3	3	02·9	00·9	63	60·3	18·3
4	03·9	01·0	64	62·1	15·6	4	03·8	01·2	64	61·2	18·6
5	04·9	01·2	65	63·1	15·8	5	04·8	01·5	65	62·2	18·9
6	05·8	01·5	66	64·0	16·0	6	05·7	01·7	66	63·2	19·2
7	06·8	01·7	67	65·0	16·3	7	06·7	02·0	67	64·1	19·4
8	07·8	01·9	68	66·0	16·5	8	07·7	02·3	68	65·1	19·7
9	08·7	02·2	69	66·9	16·8	9	08·6	02·6	69	66·0	20·0
10	09·7	02·4	70	67·9	17·0	10	09·6	02·9	70	67·0	20·3
11	10·7	02·7	71	68·9	17·3	11	10·5	03·2	71	67·9	20·6
12	11·6	02·9	72	69·8	17·5	12	11·5	03·5	72	68·9	20·9
13	12·6	03·2	73	70·8	17·7	13	12·4	03·8	73	69·9	21·2
14	13·6	03·4	74	71·8	18·0	14	13·4	04·1	74	70·8	21·5
15	14·6	03·6	75	72·8	18·2	15	14·4	04·4	75	71·8	21·8
16	15·5	03·9	76	73·7	18·5	16	15·3	04·6	76	72·7	22·1
17	16·5	04·1	77	74·7	18·7	17	16·3	04·9	77	73·7	22·3
18	17·5	04·4	78	75·7	19·0	18	17·2	05·2	78	74·6	22·6
19	18·4	04·6	79	76·6	19·2	19	18·2	05·5	79	75·6	22·9
20	19·4	04·9	80	77·6	19·4	20	19·1	05·8	80	76·6	23·2
21	20·4	05·1	81	78·6	19·7	21	20·1	06·1	81	77·5	23·5
22	21·3	05·3	82	79·5	19·9	22	21·1	06·4	82	78·5	23·8
23	22·3	05·6	83	80·5	20·2	23	22·0	06·7	83	79·4	24·1
24	23·3	05·8	84	81·5	20·4	24	23·0	07·0	84	80·4	24·4
25	24·3	06·1	85	82·5	20·7	25	23·9	07·3	85	81·3	24·7
26	25·2	06·3	86	83·4	20·9	26	24·9	07·5	86	92·3	25·0
27	26·2	06·6	87	84·4	21·1	27	25·8	07·8	87	83·3	25·2
28	27·2	06·8	88	85·4	21·4	28	26·8	08·1	88	84·2	25·5
29	28·1	07·0	89	86·3	21·6	29	27·8	08·4	89	85·2	25·8
30	29·1	07·3	90	87·3	21·9	30	28·7	08·7	90	86·1	26·1
31	30·1	07·5	91	88·3	22·1	31	29·7	09·0	91	87·1	26·4
32	31·0	07·8	92	89·2	22·4	32	30·6	09·3	92	88·0	26·7
33	32·0	08·0	93	90·2	22·6	33	31·6	09·6	93	89·0	27·0
34	33·0	08·3	94	91·2	22·8	34	32·5	09·9	94	90·0	27·3
35	34·0	08·5	95	92·2	23·1	35	33·5	10·2	95	90·9	27·6
36	34·9	08·7	96	93·1	23·3	36	34·5	10·4	96	91·9	27·9
37	35·9	09·0	97	94·1	23·6	37	35·4	10·7	97	92·8	28·2
38	36·9	09·2	98	95·1	23·8	38	36·4	11·0	98	93·8	28·4
39	37·8	09·5	99	96·0	24·1	39	37·3	11·3	99	94·7	28·7
40	38·8	09·7	100	97·0	24·3	40	38·3	11·6	100	95·7	29·0
41	39·8	10·0	101	98·0	24·5	41	39·2	11·9	101	96·7	29·3
42	40·7	10·2	102	98·9	24·8	42	40·2	12·2	102	97·6	29·6
43	41·7	10·4	103	99·9	25·0	43	41·2	12·5	103	98·6	29·9
44	42·7	10·7	104	100·9	25·3	44	42·1	12·8	104	99·5	30·2
45	43·7	10·9	105	101·9	25·5	45	43·1	13·1	105	100·5	30·5
46	44·6	11·2	106	102·8	25·8	46	44·0	13·3	106	101·4	30·8
47	45·6	11·4	107	103·8	26·0	47	45·0	13·6	107	102·4	31·1
48	46·6	11·7	108	104·8	26·2	48	45·9	13·9	108	103·4	31·4
49	47·5	11·9	109	105·7	26·5	49	46·9	14·2	109	104·3	31·6
50	48·5	12·2	110	106·7	26·7	50	47·9	14·5	110	105·3	31·9
51	49·5	12·4	111	107·7	27·0	51	48·8	14·8	111	106·2	32·2
52	50·4	12·6	112	108·6	27·2	52	49·8	15·1	112	107·2	32·5
53	51·4	12·9	113	109·6	27·5	53	50·7	15·4	113	108·1	32·8
54	52·4	13·1	114	110·6	27·7	54	51·7	15·7	114	109·1	33·1
55	53·4	13·4	115	111·6	27·9	55	52·6	16·0	115	110·1	33·4
56	54·3	13·6	116	112·5	28·2	56	53·6	16·3	116	111·0	33·7
57	55·3	13·9	117	113·5	28·4	57	54·5	16·5	117	112·0	34·0
58	56·3	14·1	118	114·5	28·7	58	55·5	16·8	118	112·9	34·2
59	57·2	14·3	119	115·4	28·9	59	56·5	17·1	119	113·9	34·5
60	58·2	14·6	120	116·4	29·2	60	57·4	17·4	120	114·8	34·8
dist.	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*

For 6¾ Points. | For 6½ Points.

T. IV.] *Dif. of lat. & dep. for 1¾ Point. Dif. of lat. & dep. for 2 Point.*

dist.	*lat.*	*dep.*	*dist.*	*lat.*	*dep*	*dist.*	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*
1	00·9	00·3	61	57·4	20·5	1	00·9	00·4	61	56·4	23·3
2	01·9	00·7	62	58·4	20·9	2	01·8	00·8	62	57·3	23·7
3	02·8	01·0	63	59·3	21·2	3	02·8	01·1	63	58·2	24·1
4	03·8	01·3	64	60·3	21·6	4	03·7	01·5	64	59·1	24·5
5	04·7	01·7	65	61·2	21·9	5	04·6	01·9	65	60·1	24·9
6	05·6	02 0	66	62·1	22·2	6	05·5	02·3	66	61·0	25·3
7	06·6	02·4	67	63·1	22·6	7	06·5	02·7	67	61·9	25·6
8	07·5	02·7	68	64·0	22·9	8	07·4	03·1	68	62·8	26·0
9	08·5	03·0	69	65·0	23·2	9	08·3	03·4	69	63·8	26·4
10	09·4	03·4	70	65·9	23·6	10	09·2	03·8	70	64·7	26·8
11	10·4	03·7	71	66·8	23·9	11	10·2	04·2	71	65·6	27·2
12	11·3	04·0	72	67·8	24·3	12	11·1	04·6	72	66·5	27·6
13	12·2	04·4	73	68·7	24·6	13	12·0	05·0	73	67·4	27·9
14	13·2	04·7	74	69·7	24·9	14	12·9	05·4	74	68·4	28·3
15	14·1	05·1	75	70·6	25·3	15	13·9	05·7	75	69·3	28·7
16	15·1	05·4	76	71·6	25·6	16	14·8	06·1	76	70·2	29·1
17	16·0	05·7	77	72·5	25·9	17	15·7	06·5	77	71·1	29·5
18	17·0	06·1	78	73·4	26·3	18	16·6	06·9	78	72·1	29·9
19	17·9	06·4	79	74·4	26·6	19	17·6	07·3	79	73·0	30·2
20	18·8	06·7	80	75·3	26·9	20	18·5	07·7	80	73·9	30·6
21	19·8	07·1	81	76·3	27·3	21	19·4	08·0	81	74·8	31·0
22	20·7	07·4	82	77·2	27·6	22	20·3	08·4	82	75·8	31·4
23	21·7	07·7	83	78·1	28·0	23	21·3	08·8	83	76·7	31·8
24	22·6	08·1	84	79·1	28·3	24	22·2	09·2	84	77·6	32·1
25	23·5	08·4	85	80·0	28·6	25	23·1	09·6	85	78·5	32·5
26	24·5	08·8	86	81·0	29·0	26	24·0	10·0	86	79·5	32·9
27	25·4	09·1	87	81·9	29·3	27	24·9	10·3	87	80·4	33·3
28	26·4	09·4	88	82·9	29·6	28	25·9	10·7	88	81·3	33·7
29	27·3	09·8	89	83·8	30·0	29	26·8	11·1	89	82·2	34·1
30	28·2	10·1	90	84·7	30·3	30	27·7	11·5	90	83·2	34·4
31	29·2	10·4	91	85·7	30·7	31	28·6	11·9	91	84·1	34·8
32	30·1	10·8	92	86·6	31·0	32	29·6	12·2	92	85·0	35·2
33	31·1	11·1	93	87·6	31·3	33	30·5	12·6	93	85·9	35·6
34	32·0	11·5	94	88·5	31·7	34	31·4	13·0	94	86·8	36·0
35	33·0	11·8	95	89·4	32·0	35	32·3	13·4	95	87·8	36·4
36	33·9	12·1	96	90·4	32·3	36	33·3	13·3	96	88·7	36·7
37	34·8	12·5	97	91·3	32·7	37	34·2	14·2	97	89·6	37·1
38	35·8	12·8	98	92·3	33·0	38	35·1	14·5	98	90·5	37·5
39	36·7	13·1	99	93·2	33·3	39	36·0	14·9	99	91·5	37·9
40	37·7	13·5	100	94·2	33·7	40	37·0	15·3	100	92·4	38·3
41	38·6	13·8	101	95·1	34·0	41	37·9	15·7	101	93·3	38·7
42	39·5	14·1	102	96·0	34·4	42	38·8	16·1	102	94·2	39·0
43	40·5	14·5	103	97·0	34·7	43	39·7	16·5	103	95·2	39·4
44	41·4	14·8	104	97·9	35·0	44	40·6	16·8	104	96·1	39·8
45	42·4	15·2	105	98·9	35·4	45	41·6	17·2	105	97·0	40·2
46	43·3	15·5	106	99·8	35·7	46	42·5	17·6	106	97·9	40·6
47	44·3	15·8	107	100·7	36·0	47	43·4	18·0	107	98·9	41·0
48	45·2	16·2	108	101·7	36·4	48	44·4	18·4	108	99·8	41·3
49	46·1	16·5	109	102·6	36·7	49	45·3	18·8	109	100·7	41·7
50	47·1	16·8	110	103·6	37·1	50	46·2	19·1	110	101·6	42·1
51	48·0	17·2	111	104·5	37·4	51	47·1	19·5	111	102·6	42·5
52	49·0	17·5	112	105·4	37·7	52	48·0	19·9	112	103·5	42·9
53	49·9	17·9	113	106·4	38·1	53	49·0	20·3	113	104·4	43·2
54	50·8	18·2	114	107·3	38·4	54	49·9	20·7	114	105·3	43·6
55	51·8	18 5	115	108·3	38·7	55	50·8	21·0	115	106·3	44·0
56	52·7	18·9	116	109·2	39·1	56	51·7	21·4	116	107·2	44·4
57	53·7	19·2	117	110·2	39·4	57	52·7	21·8	117	108·1	44·8
58	54·6	19·5	118	111·1	39·7	58	53·6	22·2	118	109·0	45·2
59	55·5	19·9	119	112·0	40·1	59	54·5	22·6	119	109·9	45·5
60	56·5	20·2	120	113·0	40·4	60	55·4	23·0	120	110·9	45·9
dist.	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*

For 6¼ Points. For 6 Points.

Dif. of lat. & dep. for 2¼ Point. | *Dif. of lat. & dep. for 2½ Point.*

dist.	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*
1	00·9	00·4	61	55·1	26·1	1	00·9	00·5	61	53·8	28·8
2	01·8	00·9	62	56·0	26·5	2	01·8	00·9	62	54·7	29·2
3	02·7	01·3	63	57·0	26·9	3	02·6	01·0	63	55·6	29·7
4	03·6	01·7	64	57·9	27·4	4	03·5	01·9	64	56·4	30·2
5	04·5	02·1	65	58·8	27·8	5	04·4	02·4	65	57·3	30·6
6	05·4	02·6	66	59·7	28·2	6	05·3	02·8	66	58·2	31·1
7	06·3	03·0	67	60·6	28·6	7	06·2	03·3	67	59·1	31·6
8	07·2	03·4	68	61·5	29·1	8	07·1	03·8	68	60·0	32·1
9	08·1	03·8	69	62·4	29·5	9	07·9	04·2	69	60·9	32·5
10	09·0	04·3	70	63·3	29·9	10	08·8	04·7	70	61·7	33·0
11	09·9	04·7	71	64·2	30·4	11	09·7	05·2	71	62·6	33·5
12	10·8	05·1	72	65·1	30·8	12	10·6	05·7	72	63·5	33·9
13	11·8	05·6	73	66·0	31·2	13	11·5	06·1	73	64·4	34·4
14	12·7	06·0	74	66·9	31·6	14	12·3	06·6	74	65·3	34·9
15	13·6	06·4	75	67·8	32·1	15	13·2	07·1	75	66·1	35·4
16	14·5	06·8	76	68·7	32·5	16	14·1	07·5	76	67·0	35·8
17	15·4	07·3	77	69·6	32·9	17	15·0	08·0	77	67·9	36·3
18	16·3	07·7	78	70·5	33·4	18	15·9	08·5	78	68·8	36·8
19	17·2	08·1	79	71·4	33·8	19	16·8	09·0	79	69·7	37·2
20	18·1	08·6	80	72·3	34·2	20	17·6	09·4	80	70·6	37·7
21	19·0	09·0	81	73·2	34·6	21	18·5	09·9	81	71·4	38·2
22	19·9	09·4	82	74·1	35·1	22	19·4	10·4	82	72·3	38·6
23	20·8	09·8	83	75·0	35·5	23	20·3	10·8	83	73·2	39·1
24	21·7	10·3	84	75·9	35·9	24	21·2	11·3	84	74·1	39·6
25	22·6	10·7	85	76·8	36·3	25	22·1	11·8	85	75·0	40·1
26	23·5	11·1	86	77·7	36·8	26	22·9	12·3	86	75·9	40·5
27	24·4	11·5	87	78·6	37·2	27	23·8	12·7	87	76·7	41·0
28	25·3	12·0	88	79·6	37·6	28	24·7	13·2	88	77·6	41·5
29	26·2	12·4	89	80·5	38·1	29	25·6	13·7	89	78·5	41·9
30	27·1	12·8	90	81·4	38·5	30	26·5	14·1	90	79·4	42·4
31	28·0	13·3	91	82·3	38·9	31	27·3	14·6	91	80·3	42·9
32	28·9	13·7	92	83·2	39·3	32	28·2	15·1	92	81·1	43·4
33	29·8	14·1	93	84·1	39·8	33	29·1	15·6	93	82·0	43·8
34	30·7	14·5	94	85·0	40·2	34	30·0	16·0	94	82·9	44·3
35	31·6	15·0	95	85·9	40·6	35	30·9	16·5	95	83·8	44·8
36	32·5	15·4	96	86·8	41·1	36	31·8	17·0	96	84·7	45·2
37	33·4	15·8	97	87·7	41·5	37	32·6	17·4	97	85·6	45·7
38	34·4	16·2	98	88·6	41·9	38	33·5	17·9	98	86·4	46·2
39	35·3	16·7	99	89·5	42·3	39	34·4	18·4	99	87·3	46·7
40	36·2	17·1	100	90·4	42·8	40	35·3	18·9	100	88·2	47·1
41	37·1	17·5	101	91·3	43·2	41	36·2	19·3	101	89·1	47·6
42	38·0	18·0	102	92·2	43·6	42	37·0	19·8	102	90·0	48·1
43	38·9	18·4	103	93·1	44·0	43	37·9	20·3	103	90·8	48·5
44	39·8	18·8	104	94·0	44·5	44	38·8	20·7	104	91·7	49·0
45	40·7	19·2	105	94·9	44·9	45	39·7	21·2	105	92·6	49·5
46	41·6	19·7	106	95·8	45·3	46	40·6	21·7	106	93·5	50·0
47	42·5	20·1	107	96·7	45·8	47	41·5	22·2	107	94·4	50·4
48	43·4	20·5	108	97·6	46·2	48	42·3	22·6	108	95·3	50·9
49	44·3	21·0	109	98·5	46·6	49	43·2	23·1	109	96·1	51·4
50	45·2	21·4	110	99·4	47·0	50	44·1	23·6	110	97·0	51·8
51	46·1	21·8	111	100·3	47·5	51	45·0	24·0	111	97·9	52·3
52	47·0	22·2	112	101·2	47·9	52	45·9	24·5	112	98·8	52·8
53	47·9	22·7	113	102·1	48·3	53	46·7	25·0	113	99·7	53·3
54	48·8	23·1	114	103·1	48·7	54	47·6	25·5	114	100·5	53·7
55	49·7	23·5	115	104·0	49·2	55	48·5	25·9	115	101·4	54·2
56	50·6	23·9	116	104·9	49·6	56	49·4	26·4	116	102·3	54·7
57	51·5	24·4	117	105·8	50·0	57	50·3	26·9	117	103·2	55·1
58	52·4	24·8	118	106·7	50·5	58	51·2	27·3	118	104·1	55·6
59	53·3	25·2	119	107·6	50·9	59	52·0	27·8	119	105·0	56·1
60	54·2	25·7	120	108·5	51·3	60	52·9	28·3	120	105·8	56·6
dist.	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*

For 5¾ Points. | For 5½ Points.

Dif. of lat. & dep. for 2¾ Points. *Dif. of lat. & dep. for 3 Points.*

dist.	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*
1	00·0	00·5	61	52·3	31·4	1	00·8	00·6	61	50·7	33·9
2	01·7	01·0	62	53·2	31·9	2	01·7	00·1	62	51·5	34·4
3	02·6	01·5	63	54·0	32·4	3	02·5	01·7	63	52·4	35·0
4	03·4	02·1	64	54·9	32·9	4	03·3	02·2	64	53·2	35·6
5	04·3	02·6	65	55·8	33·4	5	04·2	02·8	65	54·0	36·1
6	05·1	03·1	66	56·6	33·9	6	05·0	03·3	66	54·9	36·7
7	06·0	03·6	67	57·5	34·4	7	05·8	03·9	67	55·7	37·2
8	06·9	04·1	68	58·3	35·0	8	06·7	04·4	68	56·5	37·8
9	07·7	04·6	69	59·2	35·5	9	07·5	05·0	69	57·4	38·3
10	08·6	05·1	70	60·0	36·0	10	08·3	05·6	70	58·2	38·9
11	09·4	05·7	71	60·9	36·5	11	09·1	06·1	71	59·0	39·4
12	10·3	06·2	72	61·8	37·0	12	10·0	06·7	72	59·9	40·0
13	11·2	06·7	73	62·6	37·5	13	10·8	07·2	73	60·7	40·6
14	12·0	07·2	74	63·5	38·0	14	11·6	07·8	74	61·5	41·1
15	12·9	07·7	75	64·3	38·6	15	12·5	08·3	75	62·4	41·7
16	13·7	08·2	76	65·2	39·1	16	13·3	08·9	76	63·2	42·2
17	14·6	08·7	77	66·0	39·6	17	14·1	09·4	77	64·0	42·8
18	15·4	09·3	78	66·9	40·1	18	15·0	10·0	78	64·8	43·3
19	16·3	09·8	79	67·8	40·6	19	15·8	10·6	79	65·7	43·9
20	17·2	10·3	80	68·6	41·1	20	16·6	11·1	80	66·5	44·4
21	18·0	10·8	81	69·5	41·6	21	17·5	11·7	81	67·3	45·0
22	18·9	11·3	82	70·3	42·2	22	18·3	12·2	82	68·2	45·6
23	19·7	11·8	83	71·2	42·7	23	19·1	12·8	83	69·0	46·1
24	20·6	12·3	84	72·0	43·2	24	20·0	13·3	84	69·8	46·7
25	21·4	12·9	85	72·9	43·7	25	20·8	13·9	85	70·7	47·2
26	22·3	13·4	86	73·8	44·2	26	21·6	14·4	86	71·5	47·8
27	23·2	13·9	87	74·6	44·7	27	22·4	15·0	87	72·3	48·3
28	24·0	14·4	88	75·5	45·2	28	23·3	15·6	88	73·2	48·9
29	24·9	14·9	89	76·3	45·7	29	24·1	16·1	89	74·0	49·4
30	25·7	15·4	90	77·2	46·3	30	24·9	16·7	90	74·8	50·0
31	26·6	15·9	91	78·1	46·8	31	25·8	17·2	91	75·7	50·6
32	27·4	16·4	92	78·9	47·3	32	26·6	17·8	92	76·5	51·1
33	28·3	17·0	93	79·8	47·8	33	27·4	18·3	93	77·3	51·7
34	29·2	17·5	94	80·6	48·3	34	28·3	18·9	94	78·2	52·2
35	30·0	18·0	95	81·5	48·8	35	29·1	19·4	95	79·0	52·8
36	30·9	18·5	96	82·3	49·3	36	29·9	20·0	96	79·8	53·3
37	31·7	19·0	97	83·2	49·9	37	30·8	20·6	97	80·6	53·9
38	32·6	19·5	98	84·1	50·4	38	31·6	21·1	98	81·5	54·4
39	33·5	20·0	99	84·9	50·9	39	32·4	21·7	99	82·3	55·0
40	34·3	20·6	100	85·8	51·5	40	33·3	22·2	100	83·1	55·6
41	35·2	21·1	101	86·6	51·9	41	34·1	22·8	101	84·0	56·1
42	36·0	21·6	102	87·5	52·4	42	34·9	23·3	102	84·8	56·7
43	36·9	22·1	103	88·3	52·9	43	35·8	23·9	103	85·6	57·2
44	37·7	22·6	104	89·2	53·5	44	36·6	24·4	104	86·5	57·8
45	38·6	23·1	105	90·1	54·0	45	37·4	25·0	105	87·3	58·3
46	39·5	23·6	106	90·9	54·5	46	38·2	25·6	106	88·1	58·9
47	40·3	24·2	107	91·8	55·0	47	39·1	26·1	107	89·0	59·4
48	41·2	24·7	108	92·6	55·5	48	39·9	26·7	108	89·8	60·0
49	42·0	25·2	109	93·5	56·0	49	40·7	27·2	109	90·6	60·6
50	42·9	25·7	110	94·3	56·5	50	41·6	27·8	110	91·5	61·1
51	43·7	26·2	111	95·2	57·1	51	42·4	28·3	111	92·3	61·7
52	44·6	26·7	112	96·1	57·6	52	43·2	28·9	112	93·1	62·2
53	45·5	27·2	113	96·9	58·1	53	44·1	29·4	113	94·0	62·8
54	46·3	27·8	114	97·8	58·6	54	44·9	30·0	114	94·8	63·3
55	47·2	28·3	115	98·6	59·1	55	45·7	30·6	115	95·6	63·9
56	48·0	28·8	116	99·5	59·6	56	46·6	31·1	116	96·4	64·4
57	48·9	29·3	117	100·4	60·1	57	47·4	31·7	117	97·3	65·0
58	49·7	29·8	118	101·2	60·7	58	48·2	32·2	118	98·1	65·6
59	50·6	30·3	119	102·1	61·2	59	49·1	32·8	119	98·9	66·1
60	51·5	30·8	120	102·9	61·7	60	49·9	33·3	120	99·8	66·7
dist.	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*

For 5¼ Points.

For 5 Points.

Dif. of lat. & dep. for 3¼ Points. | *Dif. of lat. & dep. for 3½ Points.*

dist.	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*
1	00·8	00·6	61	49·0	36·3	1	00·8	00·6	61	47·1	38·7
2	01·6	01·2	62	49·8	36·9	2	01·5	01·3	62	47·9	39·3
3	02·4	01·8	63	50·6	37·5	3	02·3	01·9	63	48·7	40·0
4	03·2	02·4	64	51·4	38·1	4	03·1	02·5	64	49·5	40·6
5	04·0	03·0	65	52·2	38·7	5	03·9	03·2	65	50·2	41·2
6	04·8	03·6	66	53·0	39·3	6	64·6	03·8	66	51·0	41·9
7	05·6	04·2	67	53·8	39·9	7	05·4	04·4	67	51·8	42·5
8	06·4	04·8	68	54·6	40·5	8	06·2	05·1	68	52·6	43·1
9	07·2	05·4	69	55·4	41·1	9	07·0	05·7	69	53·3	43·8
10	08·0	06·0	70	56·2	41·7	10	07·7	06·3	70	54·1	44·4
11	08·8	06·6	71	57·0	42·3	11	08·5	07·0	71	54·9	45·0
12	09·6	07·1	72	57·8	42·9	12	09·3	07·6	72	55·7	45·7
13	10·4	07·7	73	58·6	43·5	13	10·1	08·2	73	56·4	46·3
14	11·2	08·3	74	59·4	44·1	14	10·8	08·9	74	57·2	46·9
15	12·0	08·9	75	60·2	44·7	15	11·6	09·5	75	58·0	47·6
16	12·8	09·5	76	61·0	45·3	16	12·4	10·1	76	58·7	48·2
17	13·7	10·1	77	61·8	45·9	17	13·1	10·8	77	59·5	48·8
18	14·5	10·7	78	62·6	46·5	18	13·9	11·4	78	60·3	49·5
19	15·3	11·3	79	63·4	47·1	19	14·7	12·0	79	61·1	50·1
20	16·1	11·9	80	64·3	47·7	20	15·5	12·7	80	61·8	50·7
21	16·9	12·5	81	65·1	48·3	21	16·2	13·3	81	62·6	51·4
22	17·7	13·1	82	65·9	48·9	22	17·0	14·0	82	63·4	52·0
23	18·5	13·7	83	66·7	49·4	23	17·8	14·6	83	64·2	52·7
24	19·3	14·3	84	67·5	50·0	24	18·5	15·2	84	64·9	53·3
25	20·1	14·9	85	68·3	50·6	25	19·3	15·9	85	65·7	53·9
26	20·9	15·5	86	69·1	51·2	26	20·1	16·5	86	66·5	54·6
27	21·7	16·1	87	69·9	51·8	27	20·9	17·1	87	67·2	55·2
28	22·5	16·7	88	70·7	52·4	28	21·6	17·8	88	68·0	55·8
29	23·3	17·3	89	71·5	53·0	29	22·4	18·4	89	68·8	56·5
30	24·1	17·9	90	72·3	53·6	30	23·2	19·0	90	69·6	57·1
31	24·9	18·5	91	73·1	54·2	31	24·0	19·7	91	70·3	57·7
32	25·7	19·1	92	73·9	54·8	32	24·7	20·3	92	71·1	58·4
33	26·5	19·7	93	74·7	55·4	33	25·5	20·9	93	71·9	59·0
34	27·3	20·3	94	75·5	56·0	34	26·3	21·6	94	72·7	59·6
35	28·1	20·9	95	76·3	56·6	35	27·1	22·2	95	73·4	60·3
36	28·9	21·4	96	77·1	57·2	36	27·8	24·8	96	74·2	60·9
37	29·7	22·0	97	77·9	57·8	37	28·6	23·5	97	75·0	61·5
38	30·5	22·6	98	78·7	58·4	38	29·4	24·1	98	75·7	62·2
39	31·3	23·2	99	79·5	59·0	39	30·1	24·7	99	76·5	62·8
40	32·1	23·8	100	80·3	59·6	40	30·9	25·4	100	77·3	63·4
41	32·9	24·4	101	81·1	60·2	41	31·7	26·0	101	78·1	64·1
42	33·7	25·0	102	81·9	60·8	42	32·5	26·6	102	78·8	64·7
43	34·5	25·6	103	82·7	61·4	43	33·2	27·3	103	79·6	65·3
44	35·3	26·2	104	83·5	62·0	44	34·0	27·9	104	80·4	66·0
45	36·1	26·8	105	84·3	62·6	45	34·8	28·5	105	81·2	66·6
46	36·9	27·4	106	85·1	63·1	46	35·6	29·2	106	81·9	67·2
47	37·7	28·0	107	85·9	63·7	47	36·3	29·8	107	82·7	67·9
48	38·6	28·6	108	86·7	64·3	48	37·1	30·4	108	83·5	68·5
49	39·4	29·2	109	87·5	64·9	49	37·9	31·1	109	84·3	69·1
50	40·2	29·8	110	88·4	65·5	50	38·6	31·7	110	85·0	69·8
51	41·0	30·4	111	89·2	66·1	51	39·4	32·3	111	85·8	70·4
52	41·8	31·0	112	90·0	66·7	52	40·2	33·0	112	86·6	71·0
53	42·6	31·6	113	90·8	67·3	53	41·0	33·6	113	87·3	71·7
54	43·4	32·2	114	91·6	67·9	54	41·7	34·3	114	88·1	72·3
55	44·2	32·8	115	92·4	68·5	55	42·5	34·9	115	88·9	73·0
56	45·0	33·4	116	93·2	69·1	56	43·3	35·5	116	89·7	73·6
57	45·8	34·0	117	94·0	69·7	57	44·1	36·2	117	90·4	74·2
58	46·6	34·6	118	94·8	70·3	58	44·8	36·8	118	91·2	74·9
59	47·4	35·1	119	95·6	70·9	59	45·6	37·4	119	92·0	75·5
60	48·2	35·7	120	96·4	71·5	60	46·4	38·1	120	92·8	76·1
dist.	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*

For 4¾ Points. | For 4½ Points.

dist.	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*	*dist.*	*lat.*	*dep.*
1	00·7	00·7	61	45·2	41·0	1	00·7	00·7	61	43·1	43·1
2	01·5	01·3	62	45·9	41·6	2	01·4	01·4	62	43·8	43·8
3	02·2	02·0	63	46·7	42·3	3	02·1	02·1	63	44·5	44·5
4	03·0	02·7	64	47·4	43·0	4	02·8	02·8	64	45·3	45·3
5	03·7	03·4	65	48·2	43·6	5	03·5	03·5	65	46·0	46·0
6	04·4	04·0	66	48·9	44·3	6	04·2	04·2	66	46·7	46·7
7	05·2	04·7	67	49·6	45·0	7	04·9	04·9	67	47·4	47·4
8	05·9	05·4	68	50·4	45·7	8	05·7	05·7	68	48·1	48·1
9	06·7	06·0	69	51·1	46·3	9	06·4	06·4	69	48·8	48·8
10	07·4	06·7	70	51·9	47·0	10	07·1	07·1	70	49·5	49·5
11	08·2	07·4	71	52·6	47·7	11	07·8	07·8	71	50·2	50·2
12	08·9	08·1	72	53·3	48·3	12	08·5	08·5	72	50·9	50·9
13	09·6	08·7	73	54·1	49·0	13	09·2	09·2	73	51·6	51·6
14	10·4	09·4	74	54·8	49·7	14	09·9	09·9	74	52·3	52·3
15	11·1	10·1	75	55·6	50·4	15	10·6	10·6	75	53·0	53·0
16	11·9	10·7	76	56·3	51·0	16	11·3	11·3	76	53·7	53·7
17	12·6	11·4	77	57·0	51·7	17	12·0	12·0	77	54·4	54·4
18	13·3	12·1	78	57·8	52·4	18	12·7	12·7	78	55·2	55·2
19	14·1	12·8	79	58·5	53·0	19	13·4	13·4	79	55·9	55·9
20	14·8	13·4	80	59·3	53·7	20	14·1	14·1	80	56·6	56·6
21	15·6	14·1	81	60·0	54·4	21	14·8	14·8	81	57·3	57·3
22	16·3	14·8	82	60·8	55·1	22	15·6	15·6	82	58·0	58·0
23	17·0	15·4	83	61·5	55·7	23	16·3	16·3	83	58·7	58·7
24	17·8	16·1	84	62·2	56·4	24	17·0	17·0	84	59·4	59·4
25	18·5	16·8	85	63·0	57·1	25	17·7	17·7	85	60·1	60·1
26	19·3	17·5	86	63·7	57·7	26	18·4	18·4	86	60·8	60·8
27	20·0	18·1	87	64·5	58·4	27	19·1	19·1	87	61·5	61·5
28	20·7	18·8	88	65·2	59·1	28	19·8	19·8	88	62·2	62·2
29	21·5	19·5	89	65·9	59·8	29	20·5	20·5	89	62·9	62·9
30	22·2	20·1	90	66·7	60·4	30	21·2	21·2	90	63·6	63·6
31	23·0	20·8	91	67·4	61·1	31	21·9	21·9	91	64·3	64·3
32	23·7	21·5	92	68·2	61·8	32	22·6	22·6	92	65·1	65·1
33	24·4	22·2	93	68·9	62·4	33	23·3	23·3	93	65·8	65·8
34	25·2	22·8	94	69·6	63·1	34	24·0	24·0	94	66·5	66·5
35	25·9	23·5	95	70·4	63·8	35	24·7	24·7	95	67·2	67·2
36	26·7	24·2	96	71·1	64·5	36	25·5	25·5	96	67·9	67·9
37	27·4	24·8	97	71·9	65·1	37	26·2	26·2	97	68·6	68·6
38	28·2	25·5	98	72·6	65·8	38	26·9	26·9	98	69·3	69·3
39	28·9	26·2	99	73·3	66·5	39	27·6	27·6	99	70·0	70·0
40	29·6	26·9	100	74·1	67·2	40	28·3	28·3	100	70·7	70·7
41	30·4	27·5	101	74·8	67·8	41	29·0	29·0	101	71·4	71·4
42	31·1	28·2	102	75·6	68·5	42	29·7	29·7	102	72·1	72·1
43	31·9	28·9	103	76·3	69·2	43	30·4	30·4	103	72·8	72·8
44	32·6	29·5	104	77·1	69·8	44	31·1	31·1	104	73·5	73·5
45	33·3	30·2	105	77·8	70·5	45	31·8	31·8	105	74·2	74·2
46	34·1	30·9	106	78·5	71·2	46	32·5	32·5	106	75·0	75·0
47	34·8	31·6	107	79·3	71·8	47	33·2	33·2	107	75·7	75·7
48	35·6	32·2	108	80·0	72·5	48	33·9	33·9	108	76·4	76·4
49	36·3	32·9	109	80·8	73·2	49	34·6	34·6	109	77·1	77·1
50	37·0	33·6	110	81·5	73·9	50	35·4	35·4	110	77·8	77·8
51	37·8	34·2	111	82·2	74·5	51	36·1	36·1	111	78·5	78·5
52	38·5	34·9	112	83·0	75·2	52	36·8	36·8	112	79·2	79·2
53	39·3	35·6	113	83·7	75·9	53	37·5	37·5	113	79·9	79·9
54	40·0	36·3	114	84·5	76·5	54	38·2	38·2	114	80·6	80·6
55	40·7	36·9	115	85·2	77·2	55	38·9	38·9	115	81·3	81·3
56	41·5	37·6	116	85·9	77·9	56	39·6	39·6	116	82·0	82·0
57	42·2	38·3	117	86·7	78·6	57	40·3	40·3	117	82·7	82·7
58	43·0	38·9	118	87·4	79·2	58	41·0	41·0	118	83·4	83·4
59	43·7	39·6	119	88·2	79·9	59	41·7	41·7	119	84·1	84·1
60	44·5	40·3	120	88·9	80·6	60	42·4	42·4	120	84·8	84·8
dist.	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*	*dist.*	*dep.*	*lat.*

For 4¼ Points. For 4 Points.

TABLE V.

A TABLE OF RUMBS,

SHOWING

THE DEGREES, MINUTES, AND SECONDS, THAT EVERY POINT AND QUARTER-POINT OF THE COMPASS MAKES WITH THE MERIDIAN.

NORTH.		Pt.	qr.	°	′	″	Pt.	qr.	SOUTH.	
		0	1	2	48	45	0	1		
		0	2	5	37	30	0	2		
		0	3	8	26	15	0	3		
N. by E.	N. by W.	1	0	11	15	0	1	0	S. by E.	S. by W.
		1	1	14	3	45	1	1		
		1	2	16	52	30	1	2		
		1	3	19	41	15	1	3		
N.N.E.	N.N.W.	2	0	22	30	0	2	0	S.S.E.	S.S.W.
		2	1	25	18	45	2	1		
		2	2	28	7	30	2	2		
		2	3	30	56	15	2	3		
N.E. by N.	N.W.by N.	3	0	33	45	0	3	0	S. E. by S.	S. W. by S.
		3	1	36	33	45	3	1		
		3	2	39	22	30	3	2		
		3	3	42	11	15	3	3		
N.E.	N.W.	4	0	45	0	0	4	0	S.E.	S.W.
		4	1	47	48	45	4	1		
		4	2	50	37	30	4	2		
		4	3	53	26	15	4	3		
N.E. by E.	N.W.byW.	5	0	56	15	0	5	0	S.E. by E.	S.W.by W.
		5	1	59	3	45	5	1		
		5	2	61	52	30	5	2		
		5	3	64	41	15	5	3		
E.N.E.	W.N.W.	6	0	67	30	0	6	0	E.S.E.	W.S.W.
		6	1	70	18	45	6	1		
		6	2	73	7	30	6	2		
		6	3	75	56	15	6	3		
E by N.	W. by N.	7	0	78	45	0	7	0	E. by S	W. by S.
		7	1	81	33	45	7	1		
		7	2	84	22	30	7	2		
		7	3	87	11	15	7	3		
East.	West.	8	0	90	0	0	8	0	East	West.

WORKMAN'S TABLE,

FOR CORRECTING THE MIDDLE LATITUDE.

Mid. Lat.	3°	4°	5°	6°	7°	8°	9°	10°	11°
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
15	0 02	0 03	0 04	0 06	0 09	0 12	0 15	0 19	0 23
16	0 02	0 03	0 04	0 06	0 09	0 12	0 15	0 18	0 22
17	0 02	0 03	0 04	0 06	0 08	0 11	0 14	0 17	0 21
18	0 02	0 03	0 04	0 06	0 08	0 11	0 14	0 17	0 20
19	0 02	0 03	0 04	0 06	0 07	0 10	0 13	0 16	0 19
20	0 02	0 03	0 04	0 06	0 07	0 09	0 12	0 15	0 18
21	0 02	0 03	0 04	0 06	0 07	0 09	0 12	0 15	0 18
22	0 02	0 03	0 04	0 06	0 07	0 09	0 12	0 15	0 17
23	0 02	0 03	0 04	0 06	0 07	0 09	0 12	0 15	0 17
24	0 02	0 03	0 04	0 06	0 07	0 09	0 11	0 14	0 16
25	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 16
26	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 16
27	0 02	0 03	0 04	0 05	0 07	0 08	0 11	0 14	0 16
28	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15
29	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15
30	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15
31	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15
32	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15
33	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15
34	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15
35	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15
36	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15
37	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15
38	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15
39	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15
40	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15
41	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15
42	0 02	0 03	0 04	0 05	0 06	0 08	0 10	0 13	0 15
43	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 16
44	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 16
45	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 16
46	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 16
47	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 16
48	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 16
49	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 17
50	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 17
51	0 02	0 03	0 04	0 05	0 07	0 09	0 11	0 14	0 17
52	0 02	0 03	0 04	0 05	0 07	0 09	0 12	0 15	0 18
53	0 02	0 03	0 04	0 06	0 07	0 09	0 12	0 15	0 18
54	0 02	0 03	0 04	0 06	0 08	0 10	0 13	0 16	0 19
55	0 02	0 03	0 04	0 06	0 08	0 10	0 13	0 16	0 19
56	0 02	0 03	0 04	0 06	0 08	0 10	0 13	0 16	0 20
57	0 02	0 03	0 04	0 06	0 08	0 11	0 14	0 17	0 20
58	0 02	0 03	0 04	0 06	0 09	0 11	0 14	0 17	0 21
59	0 02	0 03	0 04	0 06	0 09	0 12	0 15	0 18	0 22
60	0 02	0 03	0 04	0 06	0 09	0 12	0 15	0 19	0 23
61	0 02	0 03	0 05	0 07	0 09	0 12	0 15	0 19	0 23
62	0 02	0 03	0 05	0 07	0 09	0 12	0 16	0 20	0 24
63	0 02	0 04	0 05	0 07	0 09	0 13	0 16	0 20	0 24
64	0 02	0 04	0 06	0 08	0 09	0 13	0 17	0 21	0 25
65	0 02	0 04	0 06	0 08	0 10	0 13	0 17	0 21	0 25
66	0 02	0 04	0 06	0 08	0 10	0 14	0 18	0 22	0 26
67	0 02	0 04	0 06	0 08	0 11	0 15	0 18	0 23	0 27
68	0 02	0 04	0 06	0 08	0 11	0 15	0 19	0 24	0 28
69	0 02	0 05	0 06	0 09	0 12	0 16	0 20	0 25	0 30
70	0 03	0 05	0 06	0 09	0 13	0 17	0 21	0 26	0 31
71	0 04	0 06	0 07	0 09	0 13	0 18	0 22	0 27	0 33
72	0 04	0 06	0 08	0 10	0 14	0 19	0 23	0 29	0 35

Mid. Lat.	12°	13°	14°	15°	16°	17°	18°	19°	20°
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
15	0 27	0 31	0 35	0 40	0 45	0 51	0 58	1 06	1 14
16	0 26	0 30	0 34	0 38	0 43	0 49	0 56	1 03	1 11
17	0 25	0 28	0 32	0 37	0 42	0 48	0 54	1 01	1 08
18	0 24	0 27	0 31	0 36	0 41	0 46	0 52	0 58	1 06
19	0 23	0 26	0 30	0 34	0 40	0 45	0 50	0 56	1 03
20	0 22	0 25	0 29	0 33	0 38	0 43	0 48	0 54	1 00
21	0 21	0 25	0 29	0 33	0 37	0 42	0 47	0 53	0 58
22	0 20	0 24	0 28	0 32	0 36	0 41	0 46	0 51	0 56
23	0 20	0 24	0 28	0 32	0 36	0 40	0 45	0 50	0 55
24	0 19	0 23	0 27	0 31	0 35	0 39	0 44	0 48	0 53
25	0 19	0 23	0 27	0 31	0 35	0 39	0 43	0 47	0 52
26	0 19	0 22	0 26	0 30	0 34	0 38	0 42	0 47	0 52
27	0 19	0 22	0 26	0 30	0 33	0 38	0 42	0 46	0 51
28	0 18	0 21	0 25	0 29	0 33	0 37	0 41	0 46	0 51
29	0 18	0 21	0 25	0 29	0 32	0 36	0 41	0 45	0 50
30	0 18	0 21	0 25	0 28	0 32	0 36	0 41	0 45	0 50
31	0 18	0 21	0 25	0 28	0 32	0 36	0 41	0 45	0 50
32	0 18	0 21	0 25	0 28	0 31	0 36	0 41	0 45	0 50
33	0 18	0 21	0 24	0 27	0 31	0 35	0 40	0 44	0 49
34	0 18	0 21	0 24	0 27	0 31	0 35	0 40	0 44	0 49
35	0 18	0 21	0 24	0 27	0 31	0 35	0 40	0 44	0 49
36	0 18	0 21	0 24	0 27	0 31	0 35	0 40	0 44	0 49
37	0 18	0 21	0 24	0 27	0 31	0 35	0 40	0 44	0 49
38	0 18	0 21	0 24	0 27	0 31	0 36	0 40	0 45	0 50
39	0 18	0 21	0 25	0 28	0 32	0 36	0 41	0 45	0 50
40	0 18	0 22	0 25	0 28	0 32	0 36	0 41	0 45	0 50
41	0 18	0 22	0 25	0 28	0 32	0 37	0 41	0 45	0 50
42	0 18	0 22	0 26	0 29	0 33	0 37	0 42	0 46	0 51
43	0 19	0 23	0 26	0 30	0 34	0 38	0 42	0 46	0 51
44	0 19	0 23	0 27	0 30	0 34	0 38	0 43	0 47	0 52
45	0 19	0 23	0 27	0 31	0 35	0 39	0 43	0 47	0 52
46	0 19	0 23	0 27	0 31	0 35	0 39	0 44	0 48	0 53
47	0 20	0 23	0 27	0 31	0 35	0 40	0 44	0 49	0 54
48	0 20	0 23	0 27	0 31	0 35	0 40	0 45	0 50	0 55
49	0 21	0 24	0 28	0 32	0 36	0 41	0 45	0 51	0 57
50	0 21	0 24	0 28	0 32	0 36	0 41	0 46	0 52	0 58
51	0 21	0 24	0 28	0 32	0 37	0 42	0 47	0 53	0 59
52	0 22	0 25	0 29	0 33	0 37	0 42	0 48	0 54	1 00
53	0 22	0 25	0 29	0 33	0 38	0 43	0 49	0 55	1 01
54	0 23	0 26	0 30	0 34	0 39	0 44	0 50	0 56	1 02
55	0 23	0 26	0 30	0 35	0 40	0 45	0 51	0 57	1 03
56	0 24	0 27	0 31	0 36	0 41	0 46	0 52	0 58	1 04
57	0 24	0 28	0 32	0 37	0 42	0 48	0 54	1 00	1 06
58	0 25	0 29	0 33	0 38	0 44	0 50	0 55	1 02	1 08
59	0 26	0 30	0 34	0 39	0 45	0 51	0 57	1 04	1 10
60	0 27	0 31	0 35	0 40	0 46	0 52	0 59	1 06	1 13
61	0 27	0 31	0 36	0 41	0 47	0 54	1 01	1 08	1 15
62	0 28	0 32	0 37	0 42	0 49	0 56	1 03	1 10	1 18
63	0 29	0 33	0 39	0 44	0 51	0 58	1 05	1 12	1 21
64	0 29	0 34	0 40	0 46	0 53	1 00	1 07	1 14	1 24
65	0 30	0 35	0 41	0 48	0 55	1 02	1 09	1 17	1 27
66	0 31	0 37	0 43	0 50	0 58	1 05	1 12	1 21	1 31
67	0 33	0 38	0 45	0 53	1 00	1 07	1 16	1 25	1 35
68	0 34	0 40	0 48	0 55	1 02	1 10	1 19	1 30	1 39
69	0 36	0 44	0 50	0 58	1 05	1 13	1 23	1 34	1 44
70	0 38	0 44	0 52	1 00	1 08	1 17	1 28	1 39	1 50
71	0 40	0 46	0 55	1 03	1 12	1 22	1 32	1 44	1 56
72	0 42	0 49	0 58	1 06	1 16	1 27	1 38	1 50	2 04

A TABLE
OF
ATMOSPHERICAL REFRACTIONS,

WITH CORRECTIONS FOR THE HEIGHT OF THE BAROMETER, AND THERMOMETER, TAKEN FROM THE NAUTICAL ALMANACK.

TABLE VII.

App. Altitude.	*Refr. Br.* 30. *Th.* 50°.	*Diff. for* 1′ *Alt.*	*Diff. for* +1′ *B.*	*Diff. for* -1° *Fah.*	*App. Altitude.*	*Refr. Br.* 30 *Th.* 50°.	*Diff. for* 1′ *Alt.*	*Diff. for* +1 *B.*	*Diff. for* -1° *Fah.*
° ′	′ ″	″	″	″	° ′	′ ″	″	′ ″	″
4 0	11 52	2·2	24·1	1·70	9 0	5 54	·6	11·9	·76
10	11 30	2·1	23·4	1·64	10	5 47	·6	11·7	·74
20	11 10	2·0	22·7	1·58	20	5 41	·6	11·5	·73
30	10 50	1·9	22·0	1·53	30	5 36	·6	11·3	·71
40	10 32	1·8	21·3	1·48	40	5 30	·5	11·1	·71
50	10 15	1·7	20·7	1·43	50	5 25	·5	11·0	·70
5 0	9 58	1·6	20·6	1·38	10 0	5 20	·5	10·8	·69
10	9 42	1·5	19·1	1·34	10	5 15	·5	10·6	.67
20	9 27	1·5	19·1	1·30	20	5 10	·5	10·4	·65
30	9 11	1·4	18·6	1·26	30	5 5	·5	10·2	·64
40	8 58	1·3	18·1	1·22	40	5 0	·5	10·1	·63
50	8 45	1·3	17.6	1·19	50	4 56	·4	9·9	·62
6 0	8 32	1·2	17·2	1·15	11 0	4 51	·4	9·8	·60
10	8 20	1·2	16·8	1·11	10	4 47	·4	9·6	·59
20	8 9	1·1	16·4	1·09	20	4 43	·4	9·5	·58
30	7 58	1·1	16·0	1·06	30	4 39	·4	9·4	·57
40	7 47	1·0	15·7	1·03	40	4 35	·4	9·2	·56
50	7 37	1·0	15·3	1·00	50	4 31	·4	9·1	·55
7 0	7 27	1·0	15·0	·98	12 0	4 28·1	·38	9·00	·556
10	7 17	·9	14·6	·95	10	4 24·4	·37	8·86	·548
20	7 8	·9	14·3	·93	20	4 20·8	·36	8·74	·541
30	6 59	·8	14·1	·91	30	4 17·3	·35	8:63	·533
40	6 51	·8	13·8	·89	40	4 13·9	33	8·51	·524
50	6 43	·8	13·5	·87	50	4 10·7	·32	8·41	·517
8 0	6 35	·7	13·3	·85	13 0	4 7·5	·31	8·30	·509
10	6 28	·7	13·1	·83	10	4 4·4	·31	8·20	·503
20	6 21	·7	12·8	·82	20	4 1.4	·30	8·10	·496
30	6 14	·7	12·6	·80	30	3 58·4	·30	8·00	·490
40	6 7	·7	12·3	·79	40	3 55·5	·29	7·89	·482
50	6 0	·6	12·1	·77	50	3 52·6	·29	7·79	·476

TABLE VII.

TABLE OF REFRACTIONS.

App. Altitude.	Refr. Br. 30. Th. 50°.	Diff. for 1′ Alt.	Diff. for +1′ B.	Diff. for -1° F.	App. Altitude.	Refr. Br. 30 Th. 50°.	Diff. for 1′ Alt.	Diff. for +1 B.	Diff. for -1° F.
° ′	′ ″	″	″	″	° ′	″	″	″	″
14 0	3 49·9	·28	7·76	·469	48 0	52·3	·031	1·75	·104
10	3 47·1	·28	7·61	·464	49 0	50·5	·030	1·69	·101
20	3 44·4	·27	7·52	·458	50 0	48·8	·029	1·63	·097
30	3 41·8	·26	7·43	·453	51 0	47·1	·028	1·58	·094
40	3 39·2	·26	7·34	·448	52 0	45·4	·027	1·52	·090
50	3 36·7	·25	7·26	·444	53 0	43·8	·026	1·47	·088
15 0	3 34·3	·24	7·18	·439	54 0	42·2	·026	1·41	·085
30	3 27·3	·22	6·95	·424	55 0	40·8	·025	1·36	·082
16 0	3 20·6	·21	6·73	·411	56 0	39·3	·025	1·31	·079
30	3 14·4	·20	6·51	·399	57 0	37·8	·025	1·26	·076
17 0	3 8·5	·19	6·31	·386	58 0	36·4	·024	1·22	·073
30	3 2·9	·18	6·12	·374	59 0	35·0	·024	1·17	·070
18 0	2 57·6	·17	5·98	·362	60 0	33·6	·023	1·12	·067
19 0	2 47·7	·16	5·61	·340	61 0	32·3	·022	1·08	·065
20 0	2 38·7	·15	5·31	·322	62 0	31·0	·022	1·04	·062
21 0	2 30·5	·13	5·04	·305	63 0	29·7	·021	·99	·050
22 0	2 23·2	·12	4·79	·290	64 0	28·4	·021	·95	·057
23 0	2 16·5	·11	4·57	·276	65 0	27·2	·020	·91	·055
24 0	2 10·1	·10	4·35	·264	66 0	25·9	·020	·87	·052
25 0	2 4·2	·09	4·16	·252	67 0	24·7	·020	·83	·060
26 0	1 58·8	·09	3·97	·241	68 0	23·5	·020	·79	·047
27 0	1 53·8	·08	3·81	·230	69 0	22·4	·020	·75	·045
28 0	1 49·1	·08	3·65	·219	70 0	21·2	·020	·71	·043
29 0	1 44·7	·07	3·50	·209	71 0	19·9	·020	·67	·040
30 0	1 40·5	·07	3·36	·201	72 0	18·8	·019	·63	·038
31 0	1 36·6	·06	3·23	·193	73 0	17·7	·018	·59	·036
32 0	1 33·0	·06	3·11	·186	74 0	16·6	·018	·56	·033
33 0	1 29·5	·06	2·99	·179	75 0	15·5	·018	·52	·031
34 0	1 26·1	·05	2·88	·173	76 0	14·4	·018	·48	·029
35 0	1 23·0	·05	2·78	·167	77 0	13·4	·017	·45	·027
36 0	1 20·0	·05	2·68	·161	78 0	12·3	·017	·41	·025
37 0	1 17·1	·05	2·58	·155	79 0	11·2	·017	·38	·023
38 0	1 14·4	·05	2·49	·149	80 0	10·2	·017	·34	·021
39 0	1 11·8	·04	2·40	·144	81 0	9·2	·017	·31	·018
40 0	1 9·3	·04	2·32	·139	82 0	8·2	·017	·27	·016
41 0	1 6·9	·04	2·24	·134	83 0	7·1	·017	·24	·014
42 0	1 4·6	·038	2·16	·130	84 0	6·1	·017	·20	·012
43 0	1 2·4	·036	2·09	·125	85 0	5·1	·017	·17	·010
44 0	1 0·3	·034	2·02	·120	86 0	4·1	·017	·14	·008
45 0	58·1	·034	1·94	·117	87 0	3·1	·017	·10	·006
46 0	56·1	·033	1·88	·112	88 0	2·0	·017	·07	·004
47 0	54·2	·032	1·81	·108	89 0	1·0	·017	·03	·002

TABLE VIII.

Dip. of the Horizon.		
Height.	Dip.	
Feet.		
1	0′	58″
2	1	21
3	1	40
4	1	56
5	2	9
6	2	21
7	2	33
8	2	44
9	2	53
10	3	2
11	3	10
12	3	19
13	3	27
14	3	36
15	3	42
16	3	50
17	3	57
18	4	4
19	4	11
20	4	17
21	4	23
22	4	30
23	4	36
24	4	42
26	4	52
28	5	5
30	5	15
35	5	39
40	6	4
45	6	27
50	6	46
60	7	25
70	8	1
80	8	34
90	9	6
100	9	35

TABLE IX.

Sun's par. in Alt.	
Alt.	Par.
0°	9″
10	9
20	8
30	8
40	7
50	6
55	5
60	4
65	4
70	3
75	2
80	2
85	1
90	0

TABLE X.

Diminution of the vertical semidiam. of ⊙ or ☽, on account of Refraction.	
Alt.	Dim. of semidi.
5°	25″
6	19
7	14
8	11
9	9
10	8
11	7
12	6
13	5
14	4
15	4
16	3
18	3
20	2
30	1
45	1

TABLE XI.

Augmentation of the ☽'s semidiam.	
Alt.	Aug.
0°	0″
5	1
10	3
15	4
20	6
25	7
30	8
35	9
40	10
45	11
50	12
55	13
60	14
70	15
80	15
90	16

TABLE XII.

Reduction of the ☽'s Equatorial parallax for the spheroidal figure of the Earth.

Lat.	Horizontal Parallax.				
	54′	56′	58′	60′	62′
0°	0·0″	0·0″	0·0″	0·0″	0·0″
8	0·2	0·2	0·2	0·2	0·2
16	0·8	0·8	0·9	0·9	0·9
20	1·3	1·3	1·4	1·4	1·5
24	1·8	1·9	1·9	2·0	2·0
28	2·4	2·5	2·6	2·6	2·7
33	3·0	3·1	3·3	3·4	3·5
36	3·7	3·9	4·0	4·1	4·3
40	4·5	4·6	4·8	5·0	5·1
44	5·2	5·4	5·6	5·8	6·0
48	6·0	6·2	6·3	6·6	6·8
52	6·7	7·0	7·2	7·4	7·6
56	7·4	7·7	8·0	8·2	8·5
60	8·1	8·4	8·7	9·0	9·3
64	8·7	9·1	9·4	9·7	10·0
68	9·3	9·6	10·0	10·3	10·6
72	9·8	10·1	10·4	10·8	11·2
76	10·2	10·6	10·9	11·3	11·7
84	10·7	11·1	11·5	11·9	12·0
90	10·8	11·2	11·6	12·0	12·4

$y \quad \frac{1}{2}x) =$ [illegible]

$x = 39°18'$ — $\log\tan 84°18' = 11.00$

$1 = 2.3 \times 1 \times 57.3° = 131.79°$

[illegible]

$= 79157\{$ [illegible] $- \log\tan$ [illegible]

$=$ [illegible] $\}$ [illegible]

$= \{10 - 9\} = 79157 \times 1 = 79157$

[illegible]